FUNDAMENTALS OF GEOPHYSICAL FLUID DYNAMICS

Earth's atmosphere and oceans exhibit complex patterns of fluid motion over a vast range of space and time scales. On the planetary scale they combine to establish the climate in response to solar radiation that is inhomogeneously absorbed by the materials comprising air, water, and land. Spontaneous, energetic variability arises from instabilities in the planetary-scale circulations, appearing in many different forms such as waves, jets, vortices, boundary layers, and turbulence. Geophysical fluid dynamics (GFD) is the science of all these types of fluid motion.

This textbook is a concise and accessible introduction to GFD for intermediate to advanced students of the physics, chemistry, and/or biology of Earth's fluid environment. The book was developed from the author's many years of teaching a first-year graduate course at the Department of Atmospheric and Oceanic Sciences, University of California, Los Angeles. Readers are expected to be familiar with physics and mathematics at the level of general dynamics (mechanics) and partial differential equations.

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Preface

Earth's atmosphere and oceans exhibit complex patterns of fluid motion over a vast range of space and time scales. On the planetary scale they combine to establish the climate in response to solar radiation that is inhomogeneously absorbed by the materials comprising air, water, and land. Spontaneous, energetic variability arises from instabilities in the planetary-scale circulations, appearing in many different forms such as waves, jets, vortices, boundary layers, and turbulence. Geophysical fluid dynamics (GFD) is the science of all these types of fluid motion. It seeks to identify and analyze the essential dynamical processes that lie behind observed phenomena. As with any other theoretical science of complex nonlinear dynamics, mathematical analysis and computational modeling are essential research methodologies, and there is a continuing search for more powerful, accurate, and efficient techniques.

This book is an introduction to GFD for readers interested in doing research in the physics, chemistry, and/or biology of Earth's fluid environment. It is a product of teaching a first-year graduate course at the Department of Atmospheric and Oceanic Sciences, University of California, Los Angeles (UCLA) for many years. It is only an introduction to the subject; additional, more specialized GFD courses are required to fully prepare for practicing research in the subject. Nevertheless, to stimulate students' enthusiasm, the contents are a mixture of rudimentary mathematical analyses and somewhat complex dynamical outcomes. Students in this course are expected to be familiar with physics and mathematics at the level of general dynamics (mechanics) and partial differential equations. In the present graduate curriculum at UCLA, students are first exposed to one course on basic fluid dynamics and thermodynamics and another course on the principal phenomena of winds and currents and their underlying conceptual models. This background comprises the starting point for the book.

GFD is a mature subject, having had its adolescence in the middle of the last century. Consequently many meritorious books already exist. Most of them are specialized in their material, but several of the more general ones are usefully complementary to this book, e.g., Cushman-Roisin (1994), Gill (1982), Holton (2004), Pedlosky (1987), Salmon (1998), and Stern (1975).

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Symbols

Symbols	Name	First usage
a	Earth's radius	Section 2.4
—*	boundary location	Eq. (4.61)
$\mathbf{a}$	initial position of a parcel	Eq. (2.1)
A	absolute momentum	Before Eq. (4.54)
—	wind gyre forcing amplitude	Eq. (6.62)
$\mathcal{A}$	horizontal area within $\mathcal{C}$	Eq. (3.17)
APE	available potential energy	Eq. (4.20)
b	pycnocline depth	Eq. (4.9)
—	buoyancy, $-g\rho/\rho_0$	Eq. (5.9)
B	topographic elevation	Eq. (4.1)
$\mathcal{B}$	Burger number	Eq. (4.105)
c, C	phase speed	Eqs. (3.94) and (5.57)
$\mathbf{c}_{\mathrm{g}}$	wave group velocity	Eq. (4.34)
c_{p}	heat capacity (constant pressure)	Eq. (2.38)
$\mathbf{c}_{\mathrm{p}}$	wave phase velocity	Eq. (4.33)
c_{v}	heat capacity (constant volume)	After Eq. (2.12)
C	circulation	Eq. (2.27)
C_{s}	speed of sound	Eq. (2.41)
$\mathcal{C}$	closed line	Eq. (2.27)
D	western boundary layer width	Eq. (6.62)
$\mathcal{D}$	isopycnal form stress	Eq. (5.87)
$\mathcal{D}_{\mathrm{bot}}$	topographic form stress	Eq. (5.86)
D/Dt or D_t	substantial derivative	Eq. (2.3)
e	internal energy	Eq. (2.9)
$\hat{\mathbf{e}}$	unit vector	Eq. (2.57)

* the dash symbol denotes the same symbol with a different meaning.

Symbols	Name	First usage
E	volume-integrated total energy	Eq. (2.23)
—	Ekman number	Eq. (6.44)
$\mathbf{E}$	Eliassen–Palm flux	Eq. (5.103)
$\mathcal{E}$	local total energy density	Eq. (2.22)
Ens	enstrophy	Eq. (3.111)
f	Coriolis frequency	Eq. (2.89)
f_{h}	horizontal Coriolis frequency	Eq. (2.114)
$F(p)$	pressure coordinate	Eqs. (2.74) and (2.75)
$\mathbf{F}$	non-conservative force	Eq. (2.2)
F	boundary function	Eq. (2.13)
$\mathcal{F}$	$\hat{\mathbf{z}} \cdot \boldsymbol{\nabla} \times \mathbf{F}$	Eq. (3.24)
Fr	Froude number	Eq. (4.42)
g	gravitational acceleration	After Eq. (2.2)
g'	reduced gravity	Eq. (4.12)
g'_{I}	two-layer reduced gravity	Eq. (5.2)
$g'_{n+0.5}$	N-layer reduced gravity	Eq. (5.20)
G	pressure function	Eq. (2.85)
$G_m(n)$, $G_m(z)$	modal transformation function	Eq. (5.29)
h	free-surface height	Eq. (2.17)
—	layer thickness	Eq. (4.1)
—	boundary-layer thickness	Section 6.1
h_{ek}	Ekman layer depth	After Eq. (6.44)
$h_{\mathrm{pycnocline}}$	depth of oceanic pycnocline	After Eq. (6.78)
h_*	sea-level with a rigid-lid approximation	Eq. (2.44)
—	turbulent Ekman layer thickness	Eq. (6.45)
H	oceanic depth	Section 2.2.3
—	atmospheric height	Eqs. (2.64)–(2.66)
—	vertical scale	Section 2.3.4
—	Hamiltonian function	Eq. (3.69)
H_{I}	oceanic interior thickness	Eq. (6.54)
i	$\sqrt{-1}$	After Eq. (2.70)
I	identity matrix	Eq. (5.42)
$\mathbf{I}$	identity vector	Eq. (5.41)
$\mathcal{I}$	vorticity angular momentum	Eq. (3.71)
J	Jacobian operator	Eq. (3.26)
k	x wavenumber	Eq. (3.32)
—	wavenumber vector magnitude, $\lvert\mathbf{k}\rvert$	After Eq. (3.113)

Symbols	Name	First usage
k_E	energy centroid wavenumber	Eq. (3.116)
$\mathbf{k}$	wavenumber vector	Eq. (3.112)
$\mathbf{k}_*$	dominant wavenumber component	Eq. (4.34)
K	wavenumber magnitude	Eq. (4.37)
—	von Karmen's constant	Eq. (6.49)
KE	kinetic energy	Eq. (3.2)
l, ℓ	y wavenumber	Eq. (3.32)
L	(horizontal) length scale	Before Eq. (2.5)
L_β	Rhines scale	Eq. (4.127)
—	inertial western boundary current width	Eq. (6.78)
L_x	zonal domain width	After Eq. (6.64)
L_y	meridional domain width	Section 5.3.1
L_τ	horizontal scale of wind stress	Section 5.3.1
m	azimuthal wavenumber	Eq. (3.76)
—	vertical mode number	Eq. (5.29)
M	Mach number	Eq. (2.41)
—	mass	Eq. (4.14)
n	vertical layer number	Eq. (5.18)
$\hat{\mathbf{n}}$	unit vector in normal direction	After Eq. (2.15)
$N(z)$	buoyancy frequency	Eq. (2.69)
N	number of vertical layers	Before Eq. (5.18)
$\mathcal{N}(z)$	buoyancy frequency	After Eq. (5.28)
$\mathbf{r}$	trajectory	Near Eq. (2.1)
p	pressure	Eq. (2.2)
P	oscillation period	After Eq. (2.70)
—	centrifugal pressure	Eq. (2.97)
—	potential vorticity matrix operator	Eq. (5.42)
PE	potential energy	Eq. (4.19)
$\mathcal{P}$	discriminant for baroclinic instabilty	Eq. (5.64)
q	specific humidity	After Eq. (2.12)
—	potential vorticity	Eqs. (3.28) and (4.24)
q_{QG}	quasigeostrophic potential vorticity	Eq. (4.113)
q_E	Ertel potential vorticity	Eq. (5.25)
q_{IPE}	isentropic potential vorticity for primitive equations	Eq. (5.24)
Q	potential vorticity	Eq. (4.56)
$\mathcal{Q}$	heating rate	Eq. (2.9)
$\tilde{\mathcal{Q}}$	potential heating rate	Eq. (2.52)

Symbols	Name	First usage
r	radial coordinate	Eq. (3.44)
—	damping rate	Eq. (5.104)
R	gas constant	Eq. (2.47)
—	deformation radius	Eq. (4.43)
R_e	external deformation radius	After Eq. (2.111)
R_m	deformation radius for mode m	Eq. (5.39)
Re	Reynolds number	Eq. (2.5)
Re_e	eddy Reynolds number	After Eq. (6.24)
Re_g	grid Reynolds number	Section 6.1.7
Ro	Rossby number	Eq. (2.102)
$\mathcal{R}$	horizontal Reynolds stress	After Eq. (3.98)
—	dispersion-to-advection ratio for Rossby waves	Eq. (4.124)
s	streamline coordinate	After Eq. (2.1)
—	instability growth rate	Eq. (3.88)
S	salinity	After Eq. (2.12)
—	strain rate	Fig. (2.3) and Eq. (3.51)
—	spectrum	Eq. (3.113)
—	stretching vorticity matrix operator	Eq. (5.46)
$\mathcal{S}$	non-conservative material source	Eq. (2.7)
—	material surface	Eq. (2.25)
$\mathcal{S}_f$	sign of f	Eq. (6.27)
t	time coordinate	Before Eq. (2.1)
t_d	spin-down time	Eq. (6.43)
T	time scale	After Eq. (2.5)
—	temperature	Eq. (2.11)
$\mathbf{T}$	depth-integrated horizontal column transport	Eq. (6.21)
$\mathbf{T}_{ek}$	Ekman layer horizontal column transport	Eq. (6.52)
$T_\perp$	horizontal volume transport	Eq. (6.74)
u	eastward velocity component	Before Eq. (2.2)
u_*	friction velocity	Eq. (6.45)
$\mathbf{u}$	vector velocity	Before Eq. (2.1)
$\mathbf{u}_g$	geostrophic horizontal velocity	Eq. (2.103)
$\mathbf{u}_a$	ageostrophic horizontal velocity	Before Eq. (4.112)
$\mathbf{u}^{st}$	Stokes drift	Eq. (4.95)
U	radial velocity	Eq. (3.45)

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Symbols	Name	First usage
U	rotating-frame velocity	Eq. (2.93)
—	mean zonal velocity	Eq. (3.96)
—	depth-averaged zonal velocity	Eq. (6.59)
$\mathbf{U}^*$	eddy-induced velocity	After (5.98)
v	northward velocity component	Before Eq. (2.2)
V	(horizontal) velocity scale	Before Eq. (2.5)
—	rotating-frame velocity	Eq. (2.93)
—	azimuthal velocity	Eq. (3.45)
—	depth-averaged meridional velocity	Eq. (6.59)
V^*	northward eddy-induced velocity	Eq. (5.97)
$\mathcal{V}$	material volume	Eq. (2.25)
w	upward (vertical) velocity component	Before Eq. (2.2)
w_*	upward surface velocity with a rigid-lid approximation	Eq. (2.44)
w_{ek}	Ekman pumping velocity	Eq. (6.22)
w_{QG}	quasigeostrophic vertical velocity	Eq. (5.49)
W	vertical velocity scale	Section 2.3.4
W^*	upward eddy-induced velocity	Eq. (5.98)
x	eastward coordinate	Before Eq. (2.2)
$\mathbf{x}$	spatial position vector	Before Eq. (2.1)
$\hat{\mathbf{x}}$	unit eastward vector	Section 2.1.2
X	divergent velocity potential	Eq. (2.29)
$\mathbf{X}$	streamline	After Eq. (2.1)
$\mathbf{X} = (X, Y)$	rotating coordinate vector	Eq. (2.91)
—	streamfunction horizontal-centroid	Eq. (4.126)
$\mathcal{X}$	vorticity x-centroid	Eq. (3.71)
y	northward coordinate	Before Eq. (2.2)
$\hat{\mathbf{y}}$	unit northward vector	Section 2.1.2
$\mathcal{Y}$	vorticity y-centroid	Eq. (3.71)
z	upward coordinate	Before Eq. (2.2)
z_o	roughness length	Eq. (6.49)
$\hat{\mathbf{z}}$	unit upward vector	Section 2.1.2
Z	geopotential height	After Eq. (2.38)
—	isentropic height	Eq. (5.24)
α	thermal expansion coefficient	Eq. (2.34)
—	point vortex index	Eq. (3.60)
β	haline contraction coefficient	Eq. (2.35)

Symbols	Name	First usage
β	Coriolis frequency gradient	Eq. (2.89)
—	point vortex index	Eq. (3.63)
γ	pressure expansion coefficient	Eq. (2.36)
—	gas constant ratio	After Eq. (2.51)
—	Reimann invariant	Eq. (4.85)
Γ	solution of characteristic equation	After Eq. (4.85)
δ	divergence	Eq. (2.24)
$\delta,\ \Delta$	incremental change	After Eq. (2.28), Fig. 2.3
$\delta_{p,q}$	discrete delta function	Eq. (3.108)
ϵ	wave steepness	Section 4.4
—	small expansion parameter	Eq. (4.106)
ϵ_{bot}	bottom damping coefficient	Before Eq. (5.80)
$\zeta,\ \zeta^z$	vertical vorticity	Eq. (3.5)
$\boldsymbol{\zeta}$	vector vorticity	Eq. (2.26)
η	entropy	Eq. (2.11)
—	interface height	Eq. (4.1)
θ	potential temperature	Eq. (2.51)
—	latitude	Eq. (2.87)
—	azimuthal coordinate	Eq. (3.44)
—	complex phase angle	Eq. (5.72)
Θ	wave phase function	After Eq. (4.92)
κ	diffusivity	After Eq. (2.8)
—	gas constant ratio	After Eq. (2.51)
λ	wavelength	After Eq. (4.33)
—	inverse Ekman layer depth	Eq. (6.29)
λ_0	phase constant	Eq. (2.120)
μ	chemical potential	Eq. (2.11)
—	$(KR)^{-2}$	Eq. (5.71)
ν	viscosity	After Eq. (2.2)
ν_{e}	eddy viscosity	Eqs. (3.102) and (6.23)
$\nu_{\mathrm{h}}, \nu_{\mathrm{v}}$	horizontal, vertical eddy viscosity	Eq. (5.80)
ξ	Lagrangian parcel displacement	Eq. (4.58)
—	characteristic coordinate	Eq. (4.86)
—	western boundary layer coordinate	Eq. (6.68)
ρ	density	Eq. (2.2)
ρ_{pot}	potential density	Eq. (2.51)
σ	instability growth rate	Eq. (3.79)
τ	material concentration	Eq. (2.7)

Symbols	Name	First usage
$\boldsymbol{\tau}_s$	surface stress	Before Eq. (5.80)
ϕ, Φ	geopotential function	Eqs. (2.38) and (2.80)
Φ	force potential	Eq. (2.2)
χ	divergent velocity potential	Eq. (2.29)
ψ	streamfunction	Eq. (2.29)
Ψ	transport streamfunction	Eq. (6.59)
ω	cross-isobaric velocity	Eq. (2.79)
—	oscillation frequency	Eq. (3.32)
Ω, $\boldsymbol{\Omega}$	rotation rate, vector	Eq. (2.87)
$\boldsymbol{\Omega}_e$	Earth's rotation vector	Eq. (2.87)
$\boldsymbol{\nabla}$	gradient operator	After Eq. (2.2)
$\boldsymbol{\nabla}_h$	horizontal gradient operator	Eq. (2.31)
$\frac{\partial}{\partial z}$ or ∂_z	partial derivative with respect to, e.g., z	After Eq. (2.30)
$\overline{\cdot}$	averaging operator	Before Eq. (2.67)
$\langle \cdot \rangle$	zonal averaging operator	Eq. (3.97)
$\cdot^*$	complex conjugate	Eq. (3.66)
$\cdot'$	fluctuation operator	Eq. (3.72)
$\tilde{\cdot}$	modal coefficient	Eq. (5.29)