1

Purposes and value of geophysical fluid dynamics

In this book we will address a variety of topics that, taken together, comprise an introduction to *geophysical fluid dynamics* (GFD). The discussion is intended to be more about the concepts and methods of the subject rather than the specific formulae or observed phenomena. I hope they will be of both present interest and future utility to those who intend to work in Earth Sciences but do not expect to become specialists in the theory of dynamics, as well as to those who do have that expectation and for whom this is only a beginning.

Before starting I would like to make some preliminary remarks about the scope, purposes, and value of GFD.

The subject matter of GFD is motion in the fluid media on Earth and the distributions of material properties, such as mass, temperature, ozone, and plankton. (By common custom, planetary and astrophysical fluids are also included in GFD, since many of the scientific issues are similar, but it is awkward to use a more accurate title that explicitly includes all of these media. This book will not leave Earth.) So there is some chemistry, and even biology, in GFD, insofar as they influence the motion and evolution of the reactive materials. Nevertheless, for the most part GFD is a branch of physics that includes relevant aspects of dynamics, energy transfer by radiation, and the atomic and molecular processes associated with phase changes.

Yet GFD is by no means the entirety of ocean–atmosphere physics, much less its biogeochemistry. Within its subject-matter boundaries, GFD is distinguished by its purpose and its methodology. It is not principally concerned with establishing the facts about Earth's natural fluids, but rather with providing them a mathematical representation and an interpretation. These, in my opinion, are its proper purposes.

Beyond the knowledge provided by basic physics and chemistry, the facts about Earth's fluids are established in several ways:

- in the laboratory, where the constitutive relations, radiative properties, and chemical reactions are established, and where some analog simulations of natural phenomena are made;
- in the field, where measurements are made of the motion fields, radiation, and material property distributions;

- by theory, where the fundamental laws of fluid dynamics are well known, although – primarily because of their nonlinearity – only a small fraction of the interesting problems can actually be solved analytically; and
- on the computer, where relatively recent experience has demonstrated that simulations, based upon the fundamental relations established in the laboratory and theory as well as parameterizations of influential but unresolved processes, can approach the reality of nature as represented by the field measurements, but with much more complete information than measurements can provide.

In physical oceanography most of the pioneering laboratory work (e.g., the equation of state for seawater) has already been done, and so it is easy to take it for granted. This is also true for physical meteorology, but to a lesser degree: there remain important mysteries about the physical properties of water droplets, aerosols, and ice crystals, especially in clouds since it is difficult to simulate cloud conditions in the laboratory. For many decades and still today, the primary activity in physical oceanography is making measurements in the field. Field measurements are also a major part of meteorology, although computer modeling has long been a large part as well, initially through the impetus of numerical weather forecasting. Field measurements are, of course, quite important as the "measurable reality" of nature. But anyone who does them comes to appreciate how difficult it is to make good measurements of the atmosphere and ocean, in particular the difficulty in obtaining a broad space-time sampling that matches the phenomena. Computer simulations – the "virtual reality" of nature – are still primitive in various aspects of their scope and skillfulness, though they are steadily improving. There are successful examples of synoptic weather forecasting and design of engineering fluid devices (such as an airplane) to encourage us in this. One can also do analog simulations of geophysical fluid motions under idealized conditions in laboratory experiments. Some valuable information has been obtained in this way, but for many problems it is limited both by the usually excessive influence of viscosity, compared to nature, and by instrumental sampling limitations. Looking ahead it seems likely that computer simulations will more often be fruitful than laboratory simulations.

The facts that come from laboratory experiments, field measurements, and computer simulations are usually not simple in their information content. There is nothing simple about the equation of state for seawater, for example. As another example, a typical time series of velocity at a fixed location usually has a broad-band spectrum with at most a few identifiable frequency lines that are rarely sharp (tides are an exception). Associated with this will be a generally decaying temporal lag correlation function, hence a finite time horizon of predictability. Furthermore, most geophysical time series are more appropriately called chaotic rather than deterministic, even though one can defend the use of governing dynamical equations that are deterministic in a mathematical sense but have the property of *sensitive dependence*, where any small differences amplify rapidly in time (Chapter 3). The complexity of geophysical motions is, in a generic way, a consequence of fluid turbulence. Even the tides, arising from spatially smooth, temporally periodic astronomical forcing, can be quite complex in their spatial response patterns. There is no reason to expect the relevant simulations to be appreciably simpler than the observations; indeed, their claim to credibility requires that they not be. An illustration of fluid dynamical complexity is the accompanying satellite image of sea surface temperature off the West Coast of the United States where coastal upwelling frequently occurs (Fig. 1.1). Figure 1.2 illustrates the comparable complexity of a computational simulation of this regime.

Arthur Eddington, the British astrophysicist, remarked, "Never trust an observation without a supporting theory." Facts about nature can be either important or trivial (i.e., generic or incidental) and can be grouped with other facts either aptly or misleadingly (i.e., causal or coincidental). Only a theory can tell you how to make these distinctions. For complex geophysical fluid motions, I think there is little hope of obtaining a fundamental theory that can be applied directly to most observations. Perhaps the Navier–Stokes equation (Chapter 2) is the only fundamental theory for fluid dynamics, albeit only in a highly implicit form. Since it cannot be solved in any general way, nor can it even be generally proven that unique, non-singular solutions exist, this theory is often opaque to any observational comparison except through some simulation that may be no easier to understand than the observations. Therefore, for geophysics I prefer a rephrasing of the remark to the more modest, "Never trust a fact, or a simulation, without a supporting interpretation."

It is the purpose of GFD to provide interpretations, and its methodology is idealization and abstraction, i.e., the removal of unnecessary geographic detail and contributing dynamical processes. Insofar as an observed or simulated fact can be identified as a phenomenon that, in turn, can be reproduced in the solution of a simple model, then the claim can be made (or, to be more cautious, the hypothesis advanced) that the essential nature of the phenomenon, including the essential ingredients for its occurrence, is understood. And this degree of understanding is possibly as good as can be hoped for, pending uncertain future insights. The proper practice of GFD, therefore, is to identify generic phenomena, and devise and solve simple models for them. The scientist who comes up with the simplest, relevant model wins the prize! Occam's Razor ("given two theories consistent with the known facts, prefer the one that is simpler or involves fewer assumptions") is an important criterion for judging GFD.

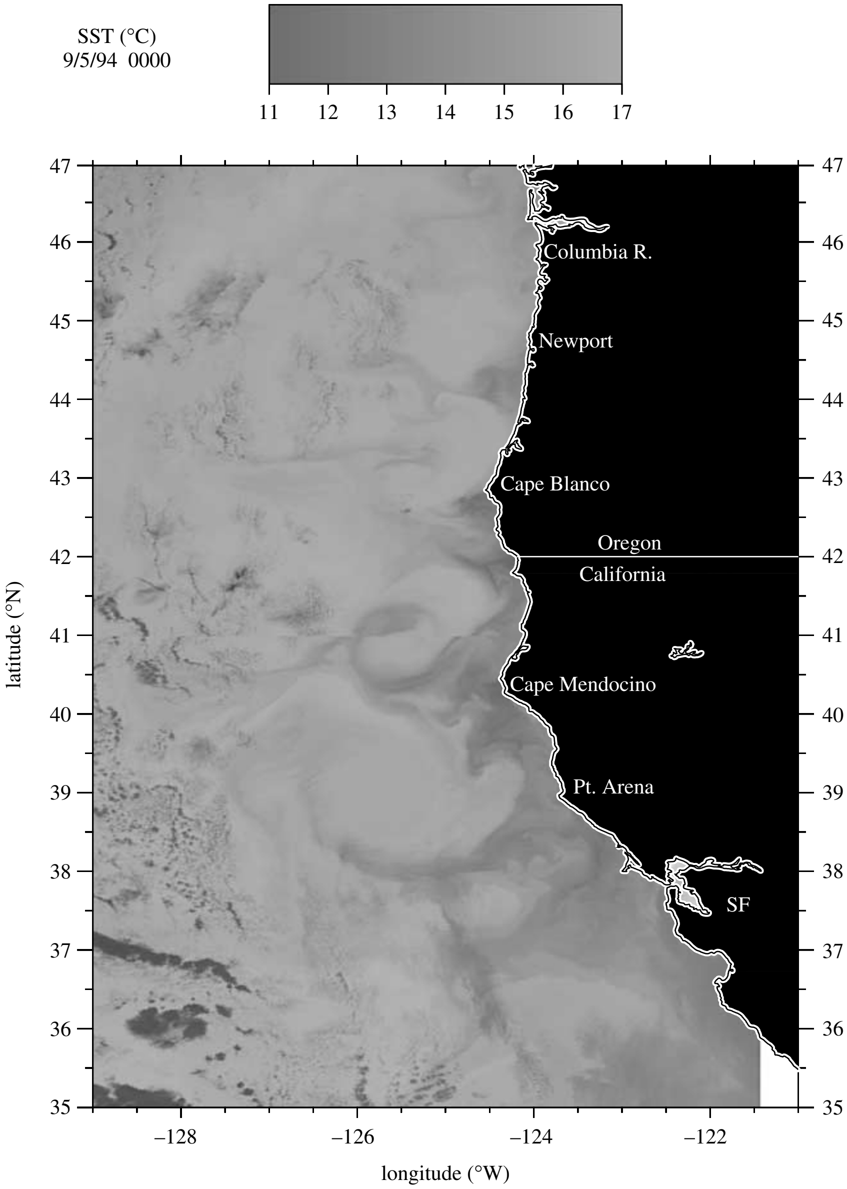

Fig. 1.1. Sea surface temperature (SST) off the US West Coast on 5 September 1994, measured with a satellite radiometer. The water near the coastline is much colder due to upwelling of cold sub-surface water. The upwelling is caused by an equatorward along-shore wind stress in association with a horizontally divergent, off-shore Ekman flow in the upper ocean (Chapter 6) as well as an along-shore surface geostrophic current (Chapter 2). The along-shore current is baroclinically unstable (Chapter 5) and generates mesoscale vortices (Chapter 3) and cold filaments advected away from the boundary, both with characteristic horizontal scales of 10–100 km. The light patches to the left are obscuring clouds. (Courtesy of Jack Barth and Ted Strub, Oregon State University.)

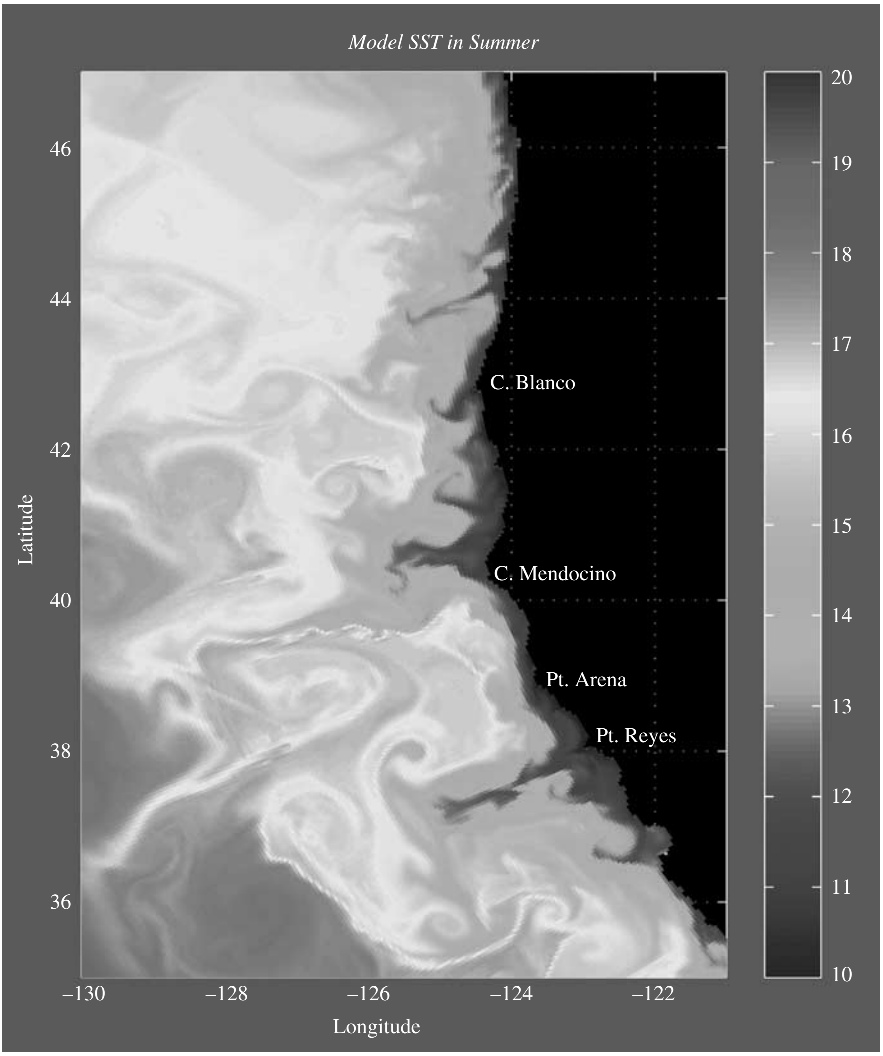

Fig. 1.2. Color plate 1. Sea surface temperature off the US West Coast in late summer, from a numerical oceanic model. Note the general pattern similarity with Fig. 1.1 for cold upwelled water near the coastline, mesoscale vortices, and cold filaments advected away from the boundary. However, the measured and simulated patterns should not be expected to agree in their individual features because of the sensitive dependence of advective dynamics. (Marchesiello *et al.*, 2003.)

An objection might be raised that since computers will always be smaller than the universe, or even the atmosphere and the ocean, then any foreseeable simulated virtual reality can itself only be an abstraction and an idealization of nature, and thus no different, in principle, from a GFD model. While literally this is true, there is such an enormous and growing gap in complexity between the most accurate simulation models and simple GFD models of idealized situations that I believe this objection can be disregarded in practice. Nevertheless, the finite scope of geophysical simulation models must be conceded, and in doing so another important purpose for GFD must be recognized: to provide simple models for the effects of physically necessary, but computationally unresolved, processes in a simulation model. This is often called *parameterization*. The most common reason for parameterization is that something essential happens on a spatial or temporal scale smaller than the computational grid of the simulation model. Two examples of necessary parameterizations are (1) the *transport* (i.e., systematic spatial movement of material and dynamical properties by the flow) by turbulent eddies in a planetary boundary layer near the surface of the land or ocean and (2) the radiative energy transfer associated with cloud water droplets in the context of a global simulation model. Each of these micro-scale phenomena could be made simulation subjects in their own right, but not simultaneously with the planetary- or macro-scale *general circulation*, because together they would comprise too large a calculation for current or foreseeable computers. Micro-scale simulations can provide facts for GFD to interpret and summarily represent, specifically in the form of a useful parameterization.

Dynamical theory and its associated mathematics are a particular scientific practice that is not to everyone's taste, nor one for which every good scientist has a strong aptitude. Nevertheless, even for those who prefer working closer to the discovery and testing of facts about the ocean and atmosphere, it is important to learn at least some GFD since it provides one of the primary languages for communicating and judging the facts. Nature's facts are infinite in number. But which facts are the interesting ones? And how does one decide whether different putative facts are mutually consistent or not (and thus unlikely both to be true)? The answer usually is found in GFD.

Since this book is drawn from a course that lasts only three months, it helps to take some short cuts. One important short cut is to focus, where possible, on dynamical equations that have only zero (e.g., a fluid parcel), one, or two spatial dimensions, although nature has three. The lower-dimensional equations are more easily analyzed, and many of their solutions are strongly analogous to the solutions of three-dimensional dynamical equations that are more literally relevant to natural phenomena. Another short cut is to focus substantially on linear and/or steady solutions since they too are more easily analyzed, even though most oceanic and atmospheric behaviors are essentially transient and appreciably influenced by nonlinear dynamics (turbulence). In particular, pattern complexity and chaos (illustrated in Fig. 1.1 for a coastal sea surface temperature pattern) are widespread and essentially the result of nonlinearity in the governing equations. Nevertheless, the study of GFD properly starts with simpler reduced-dimensional, linear, and steady solutions that provide relevant, albeit incomplete, paradigms.

A list of symbols, exercises, and an index are included to help make this book a useful learning tool.

2

Fundamental dynamics

This chapter establishes, but does not fully derive, the basic equations of geophysical fluid dynamics and several of their most commonly used approximate forms, such as incompressible, Boussinesq, hydrostatic, and geostrophic equations. It also includes some particular solutions of these equations in highly idealized circumstances. Many more solutions will be examined in later chapters.

2.1 Fluid dynamics

2.1.1 Representations

For the most part the governing equations of fluid dynamics are partial differential equations in space ($\mathbf{x}$) and time (t). Any field (i.e., a property of the fluid), q, has an *Eulerian expression* as $q(\mathbf{x}, t)$. Bold face symbols denote vectors. Alternatively, any field also has an equivalent *Lagrangian expression* as $q(\mathbf{a}, t)$, where $\mathbf{a}$ is the $\mathbf{x}$ value at $t = 0$ of an infinitesimal fluid element (or *material parcel*) and $\mathbf{r}(\mathbf{a}, t)$ is its subsequent $\mathbf{x}$ value moving with the local fluid velocity, $\mathbf{u}$

$$\frac{d\mathbf{r}(t)}{dt} = \frac{\partial \mathbf{r}(\mathbf{a}, t)}{\partial t} = \mathbf{u}(\mathbf{x}, t)\Big|_{\mathbf{x}=\mathbf{r}}, \quad \mathbf{r}(\mathbf{a}, 0) = \mathbf{a}. \tag{2.1}$$

$\mathbf{r}$ is the *trajectory* of the parcel initially at $\mathbf{a}$ (Fig. 2.1). A line tangent to $\mathbf{u}$ everywhere at a fixed time, $t = t_0$, is a *streamline*, $\mathbf{X}(s, t_0)$, where s is the spatial coordinate along the streamline. Thus,

$$\frac{d\mathbf{X}}{ds} \times \mathbf{u} = 0.$$

If

$$\frac{d\mathbf{X}}{ds} = \mathbf{u},$$

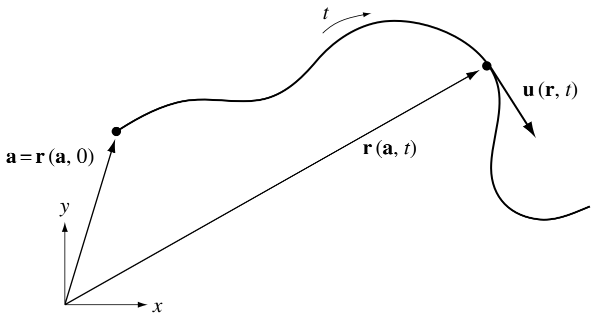

Fig. 2.1. The geometry of a trajectory, $\mathbf{r}(\mathbf{a}, t)$, projected onto the (x, y) plane. $\mathbf{a}$ is the position of the fluid parcel at time, $t = 0$, and the parcel moves along the trajectory with velocity, $\mathbf{u}(\mathbf{r}, t)(\mathbf{a})$, as indicated in (2.1).

then s has a normalization as a pseudo-time of movement along the streamline that would be equivalent to real time if the flow were stationary (i.e., $\partial_t \mathbf{u} = 0$). Alternatively, a *streakline* is the line traced in space of particles released continuously in time from a single point (which is experimentally much easier to determine by dye release and photography than a streamline). In a stationary flow streamlines, streaklines, and trajectories are all equivalent.

2.1.2 Governing equations

The starting point is the fundamental dynamical equations for a compressible fluid in a Cartesian coordinate frame – transformations can always be made to alternative frames such as a rotating spherical coordinate frame for planetary flows – with a general equation of state and variable material composition. For further discussion of basic fluid dynamics, refer to Batchelor (1967).

In GFD it is customary to associate the coordinate z with the vertical direction, parallel to the gravitational force and directed outward from Earth's center; x with the eastward direction; and y with the northward direction. It is also common usage to refer to the (x, y) directions as *zonal* and *meridional*, in association with longitude and latitude. The associated directional vectors with unit magnitude are denoted by $\hat{\mathbf{z}}$, $\hat{\mathbf{x}}$, and $\hat{\mathbf{y}}$, respectively, and the accompanying velocity components are by w, u, and v.

Momentum

A balance of acceleration and forces (i.e., Newton's law, $F = ma$, where F is force, m is mass, a is acceleration, and $m \times a$ is momentum) is expressed by the following equation involving the time derivative of velocity (i.e., the acceleration vector):

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p + \nabla\Phi + \mathbf{F}. \tag{2.2}$$

This is referred to as the Navier–Stokes equation. Here **u** is the velocity [with units, $\mathrm{m\,s^{-1}}$]; ρ is the density [$\mathrm{kg\,m^{-3}}$]; p is the pressure [$\mathrm{kg\,m^{-1}\,s^{-2}}$ or, equivalently, 1 Pa (for pascal)]; Φ is the force potential [$\mathrm{m^2\,s^{-2}}$] (e.g., for gravity, $\Phi = -gz$, with $g = 9.81\,\mathrm{m\,s^{-2}}$); and $\mathbf{F}\,[\mathrm{m\,s^{-2}}]$ is all the *non-conservative forces* that do not appear in Φ (e.g., molecular viscous diffusion with $\mathbf{F} = \nu\nabla^2\mathbf{u}$ and viscosity, ν). ∇ is the spatial gradient operator. The *substantial time derivative* is the acceleration of a fluid parcel in a reference frame moving with the flow,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla. \tag{2.3}$$

The second term is called the advective operator, or more briefly *advection*; it represents the movement of material with the fluid. (For notational compactness we sometimes abbreviate these and other derivatives by D_t, ∂_t, etc.)

The Eulerian counterpart of the trajectory equation (2.1) is

$$\frac{D\mathbf{x}}{Dt} = \mathbf{u}, \tag{2.4}$$

which is a tautology given the definition (2.3). It means that the velocity is the rate of change with time of the coordinate as it moves with the fluid.

The essential nonlinearity of fluid dynamics – the source of instability, chaos, and turbulence – appears in the quadratic product of velocities that is the advection of momentum. Advection also is a prevalent influence on the evolution of material tracer distributions (in (2.7) below) that necessarily move with the flow. This leads to three common statements about fluid dynamics, in general, and geophysical fluid dynamics, in particular. The first statement is that the effect of advection usually dominates over molecular diffusion. In a scale estimation analysis, if V is a characteristic velocity scale and L is a characteristic length scale for flow variation, then advective dominance is expressed as the largeness of the *Reynolds number*,

$$Re = \frac{VL}{\nu} \gg 1. \tag{2.5}$$

Since typical values for ν are $10^{-5}\,\mathrm{m^2\,s^{-1}}$ (air) and $10^{-6}\,\mathrm{m^2\,s^{-1}}$ (seawater), then even a modest velocity difference of $V = 1\,\mathrm{m\,s^{-1}}$ (air) or $0.1\,\mathrm{m\,s^{-1}}$ (seawater) over a distance of $L = 100\,\mathrm{m}$, has $Re = 10^7$, and even larger Re values occur for