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COMPLEX TOPOLOGICAL *K*-THEORY

Topological *K*-theory is a key tool in topology, differential geometry, and index theory, yet this is the first contemporary introduction for graduate students new to the subject. No background in algebraic topology is assumed; the reader need only have taken the standard first courses in real analysis, abstract algebra, and point-set topology.

The book begins with a detailed discussion of vector bundles and related algebraic notions, followed by the definition of *K*-theory and proofs of the most important theorems in the subject, such as the Bott periodicity theorem and the Thom isomorphism theorem. The multiplicative structure of *K*-theory and the Adams operations are also discussed and the final chapter details the construction and computation of characteristic classes.

With every important aspect of the topic covered, and exercises at the end of each chapter, this is the definitive book for a first course in topological *K*-theory.

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Complex Topological *K*-Theory

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Efton Park
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More Information

To Alex, Connor, Nolan, and Rhonda

Contents

Preface

Topological K-theory first appeared in a 1961 paper by Atiyah and Hirzebruch; their paper adapted the work of Grothendieck on algebraic varieties to a topological setting. Since that time, topological K-theory (which we will henceforth simply call K-theory) has become a powerful and indespensible tool in topology, differential geometry, and index theory. The goal of this book is to provide a self-contained introduction to the subject.

This book is primarily aimed at beginning graduate students, but also for working mathematicians who know little or nothing about the subject and would like to learn something about it. The material in this book is suitable for a one semester course on K-theory; for this reason, I have included exercises at the end of each chapter. I have tried to keep the prerequisites for reading this book to a minimum; I will assume that the reader knows the following:

- Linear Algebra: Vector spaces, bases, linear transformations, similarity, trace, determinant.
- Abstract Algebra: Groups, rings, homomorphisms and isomorphisms, quotients, products.
- Topology: Metric spaces, completeness, compactness and connectedness, local compactness, continuous functions, quotient topology, subspace topology, partitions of unity.

To appreciate many of the motivating ideas and examples in K-theory, it is helpful, but not essential, for the reader to know the rudiments of differential topology, such as smooth manifolds, tangent bundles, differential forms, and de Rham cohomology. In Chapter 4, the theory of characteristic classes is developed in terms of differential forms and de

Rham cohomology; for readers not familiar with these topics, I give a quick introduction at the beginning of that chapter. I do not assume that the reader has any familiarity with homological algebra; the necessary ideas from this subject are developed at the end of Chapter 1.

To keep this book short and as easy to read as possible (especially for readers early in their mathematical careers), I have kept the scope of this book very limited. Only complex K-theory is discussed, and I do not say anything about equivariant K-theory. I hope the reader of this book will be inspired to learn about other versions of K-theory; see the bibliography for suggestions for further reading.

It is perhaps helpful to say a little bit about the philosophy of this book, and how this book differs from other books on K-theory. The fundamental objects of study in K-theory are vector bundles over topological spaces (in the case of K^0) and automorphisms of vector bundles (in the case of K^1). These concepts are discussed at great length in this book, but most of the proofs are formulated in terms of the equivalent notions of idempotents and invertible matrices over Banach algebras of continuous complex-valued functions. This more algebraic approach to K-theory makes the presentation "cleaner"(in my opinion), and also allows readers to see how K-theory can be extended to matrices over general Banach algebras. Because commutativity of the Banach algebras is not necessary to develop K-theory, this generalization falls into an area of mathematics that is often referred to as *noncommutative topology*. On the other hand, there are important aspects of K-theory, such as the existence of operations and multiplicative structures, that do not carry over to the noncommutative setting, and so we will restrict our attention to the K-theory of topological spaces.

I thank my colleagues, friends, and family for their encouragement while I was writing this book, and I especially thank Scott Nollet and Greg Friedman for reading portions of the manuscript and giving me many helpful and constructive suggestions.