

Quantitative Modeling of Earth Surface Processes

Geomorphology is undergoing a renaissance made possible by new techniques in numerical modeling, geochronology and remote sensing. Earth surface processes are complex and richly varied, but analytical and numerical modeling techniques are powerful tools for interpreting these systems and the landforms they create.

This textbook describes some of the most effective and straightforward quantitative techniques for modeling earth surface processes. By emphasizing a core set of equations and solution techniques, the book presents state-of-the-art models currently employed in earth surface process research, as well as a set of simple but practical tools that can be used to tackle unsolved research problems. Detailed case studies demonstrate application of the methods to a wide variety of processes including hillslope, fluvial, eolian, glacial, tectonic, and climatic systems. The computer programming codes used in the case studies are also presented in a set of appendices so that readers can readily utilize these methods in their own work. Additional references are also provided for readers who wish to fine-tune their models or pursue more sophisticated techniques.

Assuming some knowledge of calculus and basic programming experience, this quantitative textbook is designed for advanced geomorphology courses and as a reference book for professional researchers in Earth and planetary sciences looking for a quantitative approach to earth surface processes. Exercises at the end of each chapter begin with simple calculations and then progress to more sophisticated problems that require computer programming. All the necessary computer codes are available online at www.cambridge.org/9780521855976.

Jon Pelletier was awarded a Ph.D. in geological sciences from Cornell University in 1997. Following two years at the California Institute of Technology as the O.K. Earl Prize Postdoctoral Scholar, he was made an associate professor of geosciences at the University of Arizona where he teaches geomorphology. Dr Pelletier's research involves mathematical modeling of a wide range of surface processes on Earth and other planets, including the evolution of mountain belts, the transport and deposition of dust in arid environments, and fluvial and glacial processes on Mars.

Cambridge University Press

978-0-521-85597-6 - Quantitative Modeling of Earth Surface Processes

Jon D. Pelletier

Frontmatter

[More information](#)

Quantitative Modeling of Earth Surface Processes

Jon D. Pelletier

University of Arizona



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
 978-0-521-85597-6 - Quantitative Modeling of Earth Surface Processes
 Jon D. Pelletier
 Frontmatter
[More information](#)

CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press

The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org

Information on this title: www.cambridge.org/9780521855976

© J. D. Pelletier 2008

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2008

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data

Pelletier, Jon D.

Quantitative modeling of earth surface processes / Jon D. Pelletier.

p. cm.

Includes bibliographical references and index.

ISBN 978-0-521-85597-6 (hardback)

1. Geomorphology—Mathematical models. 2. Geomorphology—Data processing.

3. Earth sciences—Mathematical models. 4. Earth sciences—Data processing.

I. Title.

GB400.42.M33P45 2008

551.4101'5118—dc22

2008003225

ISBN 978-0-521-85597-6 hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Contents

Preface page ix

Chapter 1	Introduction	1
1.1	A tour of the fluvial system	1
1.2	A tour of the eolian system	12
1.3	A tour of the glacial system	19
1.4	Conclusions	29
Chapter 2	The diffusion equation	30
2.1	Introduction	30
2.2	Analytic methods and applications	34
2.3	Numerical techniques and applications	57
	Exercises	63
Chapter 3	Flow routing	66
3.1	Introduction	66
3.2	Algorithms	66
3.3	“Cleaning up” US Geological Survey DEMs	70
3.4	Application of flow-routing algorithms to estimate flood hazards	72
3.5	Contaminant transport in channel bed sediments	74
	Exercises	85
Chapter 4	The advection/wave equation	87
4.1	Introduction	87
4.2	Analytic methods	88
4.3	Numerical methods	90
4.4	Modeling the fluvial-geomorphic response of the southern Sierra Nevada to uplift	93
4.5	The erosional decay of ancient orogens	101
	Exercises	107
Chapter 5	Flexural isostasy	109
5.1	Introduction	109
5.2	Methods for 1D problems	111
5.3	Methods for 2D problems	113
5.4	Modeling of foreland basin geometry	116
5.5	Flexural-isostatic response to glacial erosion in the western US	120
	Exercises	123

Chapter 6	Non-Newtonian flow equations	125
6.1	Introduction	125
6.2	Modeling non-Newtonian and perfectly plastic flows	125
6.3	Modeling flows with temperature-dependent viscosity	130
6.4	Modeling of threshold-sliding ice sheets and glaciers over complex 3D topography	132
6.5	Thrust sheet mechanics	147
6.6	Glacial erosion beneath ice sheets	149
	Exercises	160
Chapter 7	Instabilities	161
7.1	Introduction	161
7.2	An introductory example: the Rayleigh–Taylor instability	162
7.3	A simple model for river meandering	164
7.4	Werner’s model for eolian dunes	166
7.5	Oscillations in arid alluvial channels	169
7.6	How are drumlins formed?	174
7.7	Spiral troughs on the Martian polar ice caps	183
	Exercise	187
Chapter 8	Stochastic processes	188
8.1	Introduction	188
8.2	Time series analysis and fractional Gaussian noises	188
8.3	Langevin equations	191
8.4	Random walks	193
8.5	Unsteady erosion and deposition in eolian environments	194
8.6	Stochastic trees and diffusion-limited aggregation	196
8.7	Estimating total flux based on a statistical distribution of events: dust emission from playas	199
8.8	The frequency-size distribution of landslides	205
8.9	Coherence resonance and the timing of ice ages	210
	Exercises	221
Appendix 1	Codes for solving the diffusion equation	222
Appendix 2	Codes for flow routing	235
Appendix 3	Codes for solving the advection equation	242
Appendix 4	Codes for solving the flexure equation	256

Appendix 5	Codes for modeling non-Newtonian flows	263
Appendix 6	Codes for modeling instabilities	267
Appendix 7	Codes for modeling stochastic processes	274
	References	278
	Index	290

The colour plates are to be found between pages 108 and 109.

Preface

Geomorphology is undergoing a renaissance made possible by new techniques in numerical modeling, geochronology, and remote sensing. Advances in numerical modeling make it possible to model surface processes and their feedbacks with climate and tectonics over a wide range of spatial and temporal scales. The Shuttle Radar Topography Mission (SRTM) has mapped most of Earth's topography at much higher spatial resolution and accuracy than ever before. Cosmogenic dating and other geochronologic techniques have provided vast new data on surface-process rates and landform ages. Modeling, geochronology, and remote sensing are also revolutionizing natural-hazard assessment and mitigation, enabling society to assess the hazards posed by floods, landslides, windblown dust, soil erosion, and other geomorphic hazards.

The complexity of geomorphic systems poses several challenges, however. First, the relationship between process and form is often difficult to determine uniquely. Many geomorphic processes cannot be readily quantified, and it is often unclear which processes are most important in controlling a particular geomorphic system, and how those processes interact to form the geomorphic and sedimentary records we see today. Terraces and sedimentary deposits on alluvial fans, for example, are controlled by climate, tectonics, and internal drainage adjustments in a way that geomorphologists have not been able to fully unravel. Second, surface processes are strongly influenced by fluid motions, and most classic geomorphic techniques (e.g. field mapping) are not well suited to quantifying fluid dynamics and their interactions with the surface. Third, the geomorphic community must bridge the gap between process-based geomorphology and Quaternary geology. Process-based geomorphologists have made great strides in quantifying transport and erosion laws for geomorphic systems, but this approach has not yet led to major advances in big geologic questions, such as how quickly the Grand Canyon was carved, or how mountain belts respond to glacial erosion, for example. Quaternary geolo-

gists, on the other hand, are adept at reading the geomorphic and sedimentary records to address big questions, but those records often cannot be fully interpreted using field observations and geochronology alone. A fourth challenge is geomorphic prediction. In order for applied geomorphology to realize its full potential, geomorphologists must be able to predict where geomorphic hazards are most likely to occur, taking into account the full complexity of processes, feedbacks, and the multi-scale heterogeneity of Earth's surface.

Analytical and numerical modeling are powerful tools for addressing these challenges. First, modeling is useful for establishing relationships between process and form. Using quantitative models for different processes, modeling allows us to determine the signatures of those processes in the landscape. In some cases, the histories of external forcing mechanisms (e.g. climate and tectonics) can also be inferred. These linkages are important because there is generally no direct observation of landforms that enables us to understand how they evolved. Modeling has played a significant role in recent contributions to our understanding of many classic landform types. In this book, I will present analytic and numerical models for many different landform types, including drumlins, sand dunes, alluvial fans, and bedrock drainage networks, just to name a few. Modeling is particularly useful for exploring the feedbacks between different components of geomorphic systems (i.e. hillslopes and channel networks). Channel aggradation and incision, for example, controls the base level of hillslopes, and hillslopes supply primary sediment flux to channels. Yet many geomorphic textbooks treat these as essentially independent systems. In some cases, the boundary conditions posed by one aspect of a geomorphic system can be considered to be fixed. In many of the most interesting unsolved problems, however, they cannot. An understanding of fluvial-system response to climate change, for example, cannot be achieved without a quantitative understanding of the coupled

evolution and feedbacks between hillslopes and channels.

Numerical modeling is also useful in geomorphology because of the central role that fluid dynamics play in landform evolution. Geomorphology can be roughly defined as the evolution of landforms by the fluid flow of wind, liquid water, and ice above the surface. Yet, fluid mechanics generally plays a minor role in many aspects of geomorphic research. Many landform types evolve primarily by a dynamic feedback mechanism in which the topography influences the fluid flow (and hence the shear stresses) above the topography, which, in turn, controls how the topography evolves by erosion and deposition. Numerical modeling is one of the most successful ways to quantify fluid flow in a complex environment, and hence it can and should play a central role in nearly all geomorphic research. In this book, numerical models of fluid flow serve as the basis for many of the book's numerical landform evolution models. Despite the power of numerical models, they cannot be used in a vacuum. All numerical modeling studies must integrate field observations, digital geospatial data, geochronology, and small-scale experiments to motivate, constrain, and validate the numerical work.

Earth surface processes are complex and richly varied. Most books approach the inherent variety of geomorphic systems by serially cataloging the processes and landforms characteristic of each environment (hillslope, fluvial, eolian, glacial, etc.). The disadvantage of this approach is that each geomorphic environment is presented as being essentially unique, and common dynamical behaviors are not emphasized. This book follows a different path, taking advantage of a common mathematical framework to emphasize universal concepts. This framework focuses on linear and nonlinear diffusion, advection, and boundary-value problems that quantify the stresses in the atmosphere and lithosphere. The diffusion equation, for example, can be used to describe hillslope evolution, channel-bed evolution, delta progradation, hydrodynamic dispersion in groundwater aquifers, turbulent dispersion in the atmosphere, and heat conduction in soils and the Earth's crust. By first providing the reader with a solid foundation in the

behavior and solution methods for diffusion, the reader will be poised to understand diffusive phenomena as they arise in different contexts. Similarly, boundary-layer and non-Newtonian fluid flows are also examples of equation sets that have broad applicability in Earth surface processes. Non-Newtonian fluid flows are the basis for understanding and modeling lava flows, debris flows, and alpine glaciers. In the course of developing my quantitative skills and applying them to a wide range of research problems, I have been continually amazed at how often a concept or technique from one area can provide the missing piece required to solve a long-standing problem in another field. It is this cross-fertilization of ideas and methods that I want to foster and share with readers. By emphasizing a core set of equations and solution techniques, readers will come away with powerful tools that can be used to tackle unsolved problems, as well as specific knowledge of state-of-the-art models currently used in Earth surface process research. The book is designed for use as a textbook for an advanced geomorphology course and as a reference book for professional geomorphologists. Mathematically, I assume that readers have mastered multivariable calculus and have had some experience with partial differential equations. The exercises at the end of each chapter begin with simple problems that require only the main concepts and a few calculations, then progress to more sophisticated problems that require computer programming.

The purpose of this book is not to provide an exhaustive survey of all analytic or numerical methods for a given problem, but rather to focus on the most powerful and straightforward methods. For example, advective equations can be solved using the upwind-differencing, Lax-Wendroff method, staggered-leapfrog, pseudo-spectral, and semi-Lagrangian methods, just to name a few. Rather than cover all available methods, this book focuses primarily on the simpler methods for readers who want to get started quickly or who need to solve problems of modest computational size. In this sense, the book will follow the model of *Numerical Recipes* (Press *et al.*, 1992) by providing tools that work for most applications, with additional references for

Cambridge University Press

978-0-521-85597-6 - Quantitative Modeling of Earth Surface Processes

Jon D. Pelletier

Frontmatter

[More information](#)

readers who want to fine-tune their models. The appendices provide the reader with computer code to illustrate technique application in real-world research problems. Hence, many of the applications I cover in this book necessarily come from my own research. In focusing on my own work, I don't mean to imply that my work is the only or best approach to a given problem. The book is not intended to be a complete survey of geomorphology, and I knowingly have left out many important contributions in favor of a more focused, case-study approach.

Modelers are not always consistent in the way that they use the terms one- (1D), two- (2D), and three-dimensional (3D) when referring to a model. Usually when physicists and mathematicians use the term 1D they are referring to a model that has one independent spatial variable. This can be confusing when applied to geomorphic problems, however, because a model for the evolution of a topographic profile, $h(x, t)$, for example, would be called 1D even though it represents a 2D profile. Similarly, a model for the evolution of a topographic surface $h(x, y, t)$ would be described as 2D even though the surface itself is three-dimensional. In this book we will use the convention that the dimensionality of the problem refers to the number of independent spatial variables. Therefore, when we model 2D topographic profiles we will classify the model as

1D and when we model the evolution of surfaces we will refer to the model as 2D. This convention may seem strange at first, but it makes the book more consistent overall. For example, if we model heat flow in a thin layer using the diffusion equation, everyone would agree that we are solving a 2D problem for $T(x, y, t)$. If we use the diffusion equation to model the evolution of a hillslope described by $h(x, y, t)$, that, too, is *mathematically* a 2D problem even though it represents a 3D landform.

I wish to acknowledge my colleagues at the University of Arizona for taking a chance on an unconventional geomorphologist and for creating such a collegial working environment. I also wish to thank my graduate students Leslie Hsu, Jason Barnes, James Morrison, Steve DeLong, Michael Cline, Joe Cook, Maria Banks, Joan Blainey, Jennifer Boerner, and Amy Rice for helpful conversations and collaborations. I hope they have learned as much from me as I have from them. Funding for my research has come from the National Science Foundation, the Army Research Office, the US Geological Survey, the NASA Office of Space Science, the Department of Energy, and the State of Arizona's Water Sustainability Program. I gratefully acknowledge support from these agencies and institutions. Finally, I wish to thank my wife Pamela for her patience and support.