1 Introduction

1.1 Scope of the Book

Turbulence is well known to be one of the most complex and exciting fields of research that raises many theoretical issues and that is a key feature in a large number of application fields, ranging from engineering to geophysics and astrophysics. It is still a dominant research topic in fluid mechanics, and several conceptual tools developed within the framework of turbulence analysis have been applied in other fields dealing with nonlinear, chaotic phenomena (e.g., nonlinear optics, nonlinear acoustics, econophysics, etc.).

Despite more than a century of work and a number of important insights, a complete understanding of turbulence remains elusive, as witnessed by the lack of fully satisfactory theories of such basic aspects as transition and the Kolmogorov $k^{-5/3}$ spectrum. Nevertheless, quantitative predictions of turbulence have been developed. They are often based on theories and models that combine "true" dynamical equations and closure assumptions and are supported by physical and – more and more – numerical experiments.

Homogeneous turbulence remains a timely subject, even half a century after the publication of Batchelor's book in 1953, and this framework is pivotal in the present book. Homogeneous isotropic turbulence (HIT) is the best known canonical case; it is very well documented – even if not completely understood – from experiments and simple models to recent 4096^3 full direct numerical simulation (DNS). Of course, this case is addressed (in Chapter 3), but more generally emphasis is put on homogeneous anisotropic turbulence (HAT) in the presence of mean (velocity, temperature, etc.) gradients, body forces, or both. This context is illustrated by several physical and numerical experiments (the latter being easy to perform by slight modification of pseudo-spectral numerical methods designed for DNS of isotropic turbulence following the method introduced by Rogallo in the late 1970s), but its interest for developing fundamental understanding and improving theories and models is largely underestimated regarding the existing literature. Depending on the strength of the distortion (by mean gradients and/or body forces) and its time of application, it is possible to move from pure linear approaches, such as the *rapid distortion theory* (RDT), to fully nonlinear statistical theories, with the important intermediate step of "weak" turbulence theories, such as the *wave-turbulence theory*. As far as possible, it is proposed to pass from "weak" to "strong" turbulence by

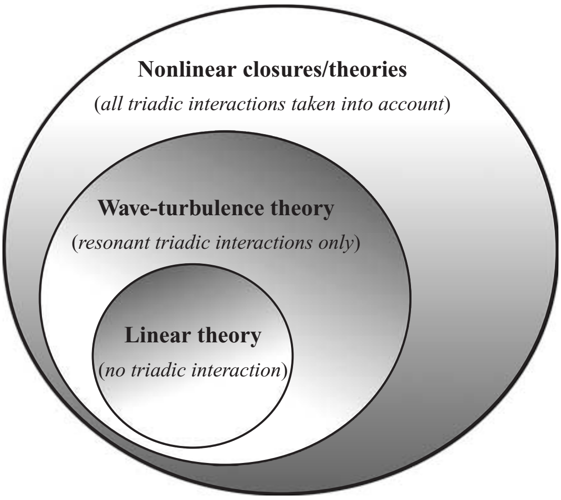

Figure 1.1. Sketch of the hierarchy of embedded turbulence theories and closures.

following a strict hierarchy of embedded models and theories, which is illustrated in Fig. 1.1.

This strategy was introduced by the second author in his contribution to the recent book *Theories of Turbulence* (Oberlack and Busse, 2002). Even if the most original part of the present book deals with two-point statistics, the Reynolds stress budget is very informative and therefore Reynolds stress equations are discussed before more complex approaches are addressed. Limits or failures of single-point closures are highlighted in each case.

A discussion of the physical relevance of the HAT cannot be avoided, and we show that homogeneous turbulence in the presence of space-uniform mean gradients is not so ideal and restrictive. In addition to physical and numerical experiments that are capable of reproducing HAT, some typical equations (e.g., Townsend or Craya equations) are shown to remain relevant for analyzing flows with nonuniform mean gradients [e.g. short-wave stability analyses, Wentzel–Kramers–Brillouin (WKB) RDT]. In some cases, pedagogical explanations for “pure” homogeneous turbulence can be extended toward inhomogeneous turbulence (e.g., near-wall turbulent shear flow). Another important point is that homogeneous sheared turbulence exhibits self-sustained cycles, which are key features of turbulence dynamics in near-wall regions.

A large number of books devoted to turbulence are available that put the emphasis on three aspects: statistical properties of isotropic, incompressible turbulence (e.g., Batchelor, 1953; Frisch, 1995; Tsinober, 2001, Davidson, 2004), descriptions of global dynamics and statistical properties of some academic flows (boundary layer, mixing layer, jet, wake, etc.; e.g., Townsend, 1976; Smits and Dussauge, 2006), and modeling of turbulent motion for engineering purpose (among others, Durbin and Petersson Reif, 2001; Wilcox, 2004). Only little information on the *dynamics* of turbulent scales is usually provided, and most authors put the emphasis on a particular feature. One should of course mention general-purpose textbooks (see Pope, 2000; Tennekes and Lumley, 1994; Bailly and Comte-Bellot, 2003), which provide the reader with a general survey of different issues related to turbulence research. Therefore, recent results dealing with dynamics of turbulent motion obtained from DNSs, advanced statistical models (linear theories and models, nonlinear triadic closures, etc.), and experiments are not available to the reader in a single book. Results are disseminated among a huge number of journal articles, technical reports, and conference papers that do not always use the same terminology.

The present book aims at providing a state-of-the-art sum of results and theories dealing with homogeneous turbulence, including anisotropic effects and compressiblity effects. The underlying idea is to gather the most recent results dealing with the dynamics of homogeneous turbulence when it interacts with external forcing (strain, rotation, etc.) and when compressibility effects are in play. Each chapter will be devoted to a given type of interaction and will present and compare experimental data, DNS/LES (large-eddy simulation) results, analysis of the Reynolds stress budget equations, and advanced linear and nonlinear theoretical models. The roles of both linear and nonlinear mechanisms are emphasized. The link between the statistical properties and the dynamics of coherent structures is also addressed. Despite its being restricted to homogeneous turbulence, this book will be of interest to all people involved in turbulence studies, as it will highlight basic physical mechanisms that are present in all turbulent flows.

Another interest of this book is the possiblity for the reader to find a unified presentation of the results and also a clear presentation of existing controversies and shortcomings in the theoretical background. Special attention is paid to bridging gaps among the results obtained in different research communities. This last point is developed concerning both results dealing with turbulence dynamics and the tools used to investigate it.

1.2 Structure and Contents of the Book

The presentation of the results is carried out in such a way that it allows for two levels of reading: a first level for readers interested in the results but who do not want to enter into the details of the tools (i.e., linear and nonlinear theoretical models) employed to get them, and a second level for readers interested in these details.

The book is organized in 15 chapters, with turbulent-flow cases ranging from HIT (without distortion, Chapter 3) to HAT subjected to various distorting processes (rotation, strain, shear, stratification) in Chapters 4–7. Flows subjected to coupled forcing effects are collected in Chapter 8, whereas compressible turbulence is addressed in Chapters 9–11. Chapter 2 presents the basis of dynamical (conservation equations) and statistical analyses of turbulence.

Technical details about theroretical tools and theories used in Chapters 3–11 are gathered in dedicated chapters whose reading is not mandatory. The linear-interaction theory for shock–turbulence interaction is presented in Chapter 12. Linear theories such as the RDT are detailed in Chapter 13, and two-point nonlinear closure theories [e.g., eddy-damped quasi-normal Markovian (EDQNM) theory] are addressed in Chapter 14. Some concluding comments are presented Chapter 15.

Constraints for ensuring consistency of statistical homogeneity – for turbulence – with the distorting processes are given in the most general way, for both incompressible (particularly in Chapter 2) and compressible (Chapter 10) flows. The physical relevance of this framework is also discussed.

Every typical flow case is revisited under different angles of attack, from observations and simulations, models, to theories, combining dynamical, statistical, and structural aspects, as follows:

1. observations, physical, and numerical experiments
2. analysis through *Reynolds stress tensor* (RST) equations, and balance and coupling terms
3. refined analysis using linear theory
4. refined analyses through full nonlinear theories and models for two-point statistics (if available)
5. phenomenological (and possibly dynamical) approach to structures, evolution, coupling.

It is worth noting that two classes of flows are discussed in this book. The first one is the class of flows without turbulence-production mechanisms (e.g., decaying isotropic turbulence, rotating homogeneous turbulence, stably stratified homogeneous turbulence, etc.) and flows with turbulent-kinetic-energy-production mechanisms (e.g. homogeneous sheared turbulence). In the former case, nonlinear dynamics and its modification by mean-flow effects are the sole features of the flow, whereas in the latter case linear mechanisms are the main dynamical characteristics. Therefore nonlinear models are the tools of choice in the first case (but eigenfunctions of the linear theories can provide an optimal basis to write them), whereas they are only briefly discussed in flows with productions for which linear theories are very powerful.

The most complete illustration of the hierarchy of models embedded in each other is the case of pure rotation (Chapter 4). Common models, such as RST closure models, are shown to present definite flaws in this case, and some limited attempts to improve single-point closure techniques are only briefly reviewed. As an important related point, linear theories such as the RDT were only briefly reviewed for irrotational mean flows only in other recent monographs about turbulence (e.g., Pope, 2000; Durbin and Petersson Reif, 2001), with the only exception of pure shear in the book by Townsend (1976), written a long time ago. In contrast, linear theory for HAT subjected to more general rotational mean flows is a very important part of the present book. In addition, our extended linear theory is a building block that may be useful for a wider community (e.g., elliptical-flow instability from the viewpoint of stability analysis, rotating and/or stratified shear flow, in Chapter 8).

The application domain of two-point nonlinear closures is even more restricted in existing monographs (e.g., Monin and Yaglom, 1975; Leslie, 1973; Lesieur, 1997; Frisch, 1995). Only isotropic turbulence is treated in a straightforward way, and only a few attempts to deal with small anisotropy are offered, whereas the linkage to linear models and wave turbulence is ignored.

The last item about "structures" deserves some clarification. On the one hand, it is recognized that typical structures can be evidenced by snapshots, or random realizations, of statistically homogeneous flows. The first example is the appearence of vortex tubes in isotropic turbulence. Other well-known structures are streaklike (in shear flows), cigar (in flows with dominant rotation), or pancake (flows with dominant stable stratification) structures. On the other hand, the relevance of low-order statistics to identify and quantify these structures is controversial. Second-order statistics, if they include fully anisotropic two-point correlations, can give real insight into these structures, with quantitative information (elongation parameters, aspect ratios). An objection can be made that phase coherence is lost in homogeneous statistics – at least for single-time second order – so that some aspects of coherent structures are not accounted for. Accordingly, we will speak of structures, or structuring effects, avoiding "coherent," when we identify them by using anisotropic statistics and not only using visualizations of snapshots.

The advanced models and theories selected here systematically incorporate dynamical operators that are really based on Navier–Stokes equations, even if they deal with "weak" turbulence only (e.g., linearized models, wave turbulence), not to mention exact triadic equations and conventional two-point closures based on them. Three-dimensional (3D) Fourier space is an unavoidable tool in HAT analysis; it is first considered here as a mathematical convenience to account for solenoidal properties (in isovolume turbulence) and to simplify related *modal decompositions*. Special use is made of decomposition of the fluctuating velocity in Fourier space, often referred to as the Craya–Herring decomposition, which amounts to a general Helmholtz decomposition, in terms of two solenoidal (toroidal–poloidal type), or vortical, modes and one dilatational (or divergent) mode. In incompressible turbulence, a Poisson equation is immediately recovered by projecting momentum equations onto the dilatational mode, the dilatational velocity mode being zero, so that dynamical equations deal with only the two solenoidal modes. This decomposition readily generates the helical-mode decomposition, and various "vortex-wave" decompositions when buoyancy fluctuation is accounted for (Chapters 7 and 8). The dilatational mode recovers its dynamic role, together with the pressure mode, when compressibility is introduced. The increase of the complexity of the system can be presented as follows:

1. Two-mode turbulence, in which the two independent unknowns are $u^{(1)}$, $u^{(2)}$ using the toroidal–poloidal decomposition, or $u^{(2)} \pm \imath u^{(1)}$ considering the helical-mode variant. The dilatational mode $u^{(3)}$ is strictly zero so that the pressure mode $u^{(4)}$ is completely solved in terms of the two solenoidal ones, and therefore removed from consideration (Chapters 2–6).
2. Three-mode turbulence. Same situation as before, but an additional buoyancy term is incorporated as a pseudo-dilatational mode. The physical problem with five components (three for velocity fluctuations, one for pressure fluctuations, one for buoyancy fluctuations) is turned into a three-mode one thanks to the Boussinesq approximation (divergence-free velocity field and the related

Poisson equation for pressure hold again, even if buoyancy exists), used in Chapters 7 and 8.

3. Four-mode turbulence $u^{(1)}, \ldots, u^{(4)}$, as in the quasi-isentropic flow cases addressed in Chapters 9 and 10. If the acoustic-equilibrium hypothesis holds, $u^{(3)}$ and $u^{(4)}$ can be combined as $[u^{(4)} \pm \imath u^{(3)}]$, where $u^{(3)}$ corresponds to the kinetic energy of acoustic waves and $u^{(4)}$ gives its potential counterpart.
4. Five-mode-turbulence, in which the last, fifth, entropy mode is added. In practice, decomposition in terms of five modes is possible, but not completely universal (discussed in Chapter 9). Introduction of a realistic entropy mode can be puzzling in homogeneous turbulence, not to mention the question of using density-weighted variables (velocity, momentum, or intermediate mixed quantity). Nevertheless, a decomposition very close to the $u^{(1)}$–$u^{(5)}$ one (toroidal–poloidal–dilatational–pressure–entropy) is used in Chapter 11 to describe upcoming perturbations passing through an idealized shock wave. Here, the Chu–Kovasznay decomposition is a preferential tool, as it makes it possible to split the incoming fluctuations into vortical, acoustic, and entropy modes. It is sufficient, however, to take the solenoidal (vortical) mode as one component only, so that four-mode turbulence is eventually used because upstream- and downstream-traveling acoustic perturbations must be treated in separate ways.

Isotropy is generally broken by the dynamical operators, so that a complete anisotropic description is needed, consistent with the symmetries of background equations, both in physical (two-point correlations) and in Fourier space (spectral tensors). It is worthwhile stressing that our detailed anisotropic description includes *dimensionality*, with a possibility of quantifying a 3D to two-dimensional (2D) [or to one-dimensional (1D)] transition. For instance, the structure-based modeling by Kassinos and Reynolds, which allows us to distinguish dimensionality and componentality, becomes a by-product of our general description, at least for homogeneous turbulence.

This viewpoint allows us to classify the theoretical approaches to turbulence as follows:

1. Theoretical "spectral-shell models" (as used by physicists to work on intermittency) are not considered in the present book, and empirical (spherically averaged) spectral models are only very briefly discussed (in Chapters 13 and 14), as solenoidal properties, and related exact pressure terms, cannot be preserved by spherically averaged transport equations in Fourier space. Consequently, equations that are exact and closed in the linear – rapid distortion – limit are no longer closed after spherical averaging.
2. "Modern" phenomenological theories about scaling and intermittency, from the legacy of Kolmogorov, are touched on, but in a minimal way, as they retain very little from Navier–Stokes equations. Only the Kolmogorov equation for the third-order structure function is partly based on Navier–Stokes, but it also relies on additional assumptions like local isotropy and quasi-steadiness. In

addition, a strong departure of the "anomalous exponents"* from the original Kolmogorov theory (which leads to $\zeta_n = n/3$) is interpreted as intermittency by physicists (see Bohr et al., 1998; Frisch, 1995). In contrast, wave turbulence based on (even weakly nonlinear) Navier–Stokes dynamics can radically question this viewpoint. Typical anomalous exponents can be found in the case of rapid rotation, even for low-order structure functions ($n = 2$ and $n = 3$), with no connection to intermittency (see Fig. 4.18 in Chapter 4 and the corresponding discussion). The "anomality" of exponents reflects the strong anisotropy linked to a partial transition from 3D to 2D structures and has probably nothing to do with intermittency in this example. Generally, the pure statistical description based on anomalous exponents, or extended self-similarity (ESS) laws, mixes anisotropy, inhomogeneity, and intermittency in an intricate way.

3. "Old-fashioned" statistical two-point "triadic" closures, the simplest one being EDQNM, are reconciled with linear models and wave-turbulence theory, and finally are shown to be still useful and relevant (especially with respect to the modern phenomenological theories quoted just before).
4. Low-order two-point (or more) moments are shown to be very informative: second-order moments for energy distribution, third-order moments for energy transfers (cascades), and fourth-order ones for typical closure, especially in connection with associated dynamical equations. Higher-order moments, by means of n-structure functions and full probability density functions (pdf's) are very briefly discussed.

Finally, Lagrangian statistics and passive scalar transport are not addressed, but it is worth noting that linear theories and two-point closures have relevant applications in these domains.

Let us go back to Chapters 9–11, dealing with dynamics of compressible turbulence. This issue is almost absent in most previous books dealing with turbulence fundamentals. Chapter 9 is devoted to presentation of state-of-the-art knowledge about the dynamics of compressible isotropic turbulence. The Chu–Kovazsnay modal decomposition of turbulent fluctuations is first introduced to provide the reader with a physical insight into coupling among acoustics, entropy, and vorticity. Then the different regimes observed in numerical simulations and theoretical analyses are described: the pseudo-acoustic regime, the subsonic regime (both pseudo-acoustic and thermal regimes are considered) and the supersonic regime. In each case, details of the interactions and transfers among scales and modes are discussed, and the link with the dynamics of coherent events (vortical structures, acoustic waves, shocklets, etc.) is made. Some low-Mach triadic-interaction-theory results are included, together with simplified models. Chapter 10 presents the coupling of compressible turbulence with mean-gradient effects. In this chapter, the emphasis is put on linear theory and DNS results because they are well suited to describe

* Often denoted ζ_n in the literature, n being the order of the structure function that is supposed to decay as $r^{-\zeta_n}$.

dominant dynamical mechanisms in such strongly anisotropic flows. The theory of compressible RDT is highlighted. Chapter 11 is dedicated to the shock–turbulence interaction, which has been proved to be very accurately predicted by the *linear interaction approximation* (LIA) for a large class of flows. The LIA is presented in Chapter 12 in its most achieved version, and it is used to illustrate the physics of the interaction of a shock with different kinds of fluctuations corresponding to the Chu–Kovazsnay modes. A comparison with DNS and experimental results is also made. Despite its being restricted to simple flow configurations, the basic physical mechanisms emphasized in this part are the building blocks for the interpretation and understanding of the properties of compressible turbulent flows in complex configurations.

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2 Statistical Analysis of Homogeneous Turbulent Flows: Reminders

2.1 Background Deterministic Equations

2.1.1 Mass Conservation

The equation of mass conservation is well known and does not need a long explanation to be derived. Both Eulerian and Lagrangian forms are subsequently given. The latter is less common in fluid dynamics but it deserves some attention, as it brings in some fundamental *Lagrangian* concepts and relationships.

Let us begin by addressing the Eulerian description. To this end, we consider a fixed arbitrary control volume $\mathcal{V}$, delineated by a surface S. The total mass of the fluid is governed by the following integral balance equation:

$$\underbrace{\frac{d}{dt}\iiint_{\mathcal{V}} \rho(\boldsymbol{x}, t) d^3\boldsymbol{x}}_{\text{variation}} = \underbrace{-\iint_S \rho(\boldsymbol{x}, t)\boldsymbol{u}(\boldsymbol{x}, t) \cdot \boldsymbol{n} d\sigma}_{\text{flux}} + \underbrace{\iiint_{\mathcal{V}} m(\boldsymbol{x}, t) d^3\boldsymbol{x}}_{\text{production}}, \tag{2.1}$$

in which ρ, $\boldsymbol{u}$, and m are the density, the velocity, and the rate of mass production, respectively. All these fields are assumed to be continuous fields in terms of time t and Eulerian and Cartesian coordinates $\boldsymbol{x}$. In this equation, $d^3\boldsymbol{x}$ is the elementary volume of a fluid particle, $d\sigma$ is the elementary surface with outward normal, and $\boldsymbol{n}$ is the unit vector. The classical Ostrogradsky formula yields $\iint_S \rho\boldsymbol{u} \cdot \boldsymbol{n} d\sigma = \iiint_{\mathcal{V}} \nabla \cdot (\rho\boldsymbol{u}) d^3\boldsymbol{x}$, so that the previous equation is rewritten as

$$\iiint_{\mathcal{V}} \left[\frac{\partial\rho}{\partial t} + \nabla \cdot (\rho\boldsymbol{u}) - m\right] d^3\boldsymbol{x}.$$

For the sake of clarity, the divergence of a vector $\boldsymbol{V}$ is denoted as $\nabla \cdot (\boldsymbol{V})$ or, alternatively, $\frac{\partial V_i}{\partial x_i}$ in the following. The classical local and instantaneous counterpart of the preceding equation is the continuity equation,

$$\frac{\partial\rho}{\partial t} + \nabla \cdot (\rho\boldsymbol{u}) = m. \tag{2.2}$$

In the Lagrangian description, fluid particles follow trajectories, which are given by the relationship

$$x_i = x_i^L(\boldsymbol{X}, t, t_0), \tag{2.3}$$