Chapter 1

A Description of the Sun

The Sun is an object of great beauty and fascination that has been studied with interest for thousands of years. It was born from a contracting, rotating, interstellar cloud that spun up during the collapse. The protostar would have settled down into a state where gravity and a pressure gradient balance one another and where a continued slow contraction heats up the plasma and provides the luminosity. Eventually, the core temperature became high enough for fusion of hydrogen to helium to provide all the luminosity, so that the contraction ceased. The Sun then entered the main ten-billion year (10^{10} yr) phase of its life on the main sequence, during which essentially all of the hydrogen in the core is turned into helium. So far, the Sun is half-way through this stage. In about five billion years, when the core hydrogen is exhausted, fusion will continue to take place in a shell around the helium core, while the Sun will expand greatly into a red giant (see PROBLEM 1.1). (For PROBLEM 1.1 and all the other problems in this book, together with their solutions, see the web page at www.cambridge.org/9780521854719.) Eventually, the red giant will collapse to a white dwarf, containing most of its original mass in a size similar to the Earth.

During the twentieth century, it gradually became clear that much of the Sun's present observed structure and dynamic behaviour owe their existence to the magnetic field. This book is principally concerned with developments during the past few decades in our present understanding of the effect of the magnetic field on the solar atmosphere. To put them into perspective, however, we begin with a list of earlier developments, many of which took place before the solar magnetic field was discovered. Section 1.2 continues by describing the overall characteristics of the solar interior and atmosphere, and is followed by a more detailed account of them (Secs. 1.3–1.6) and of solar activity (Secs. 1.7–1.9).

1.1 Brief History

2000 BC	Eclipses are recorded by the Chinese and from 600 BC by the Greeks.
325 BC	Theophrastus of Athens refers to 'black spots' on the Sun.
280 BC	Aristarchus of Samos suggests that the Earth is travelling around the Sun and estimates the distance to the Sun as five million miles.
190–125 BC	Hipparchus explains the motion of the Moon and Sun in terms of epicycles, with the Earth as the centre of the universe.
165 BC	The Chinese make records of observations of sunspots with the naked eye.
23 BC	Sunspots begin to be observed systematically by the Chinese.
AD 140	Ptolemy maintains that the Sun moves around the Earth, a belief held for the next 1,400 years.
1530	Copernicus suggests that the six known planets revolve about the Sun in concentric circles.

1609	Kepler uses Tycho Brahe's observations to formulate his laws of planetary motion and gives the Sun–Earth distance as fourteen million miles (PROBLEM 1.2). He also suggests that the Sun might have a magnetic field (to keep planets moving in their orbits).
1610	In the West, sunspots have been forgotten until they are recorded by Thomas Harriot in England and later by Johannes Fabricius, Christoph Scheiner and Galileo Galilei through recently invented telescopes. Some who see sunspots regard them as planets, others as the slag of the burning Sun or opaque clouds of smoke!
1633	Galileo is sentenced to house arrest for heretical heliocentric views.
1645–1715	Very few sunspots are present (the *Maunder Minimum*).
1666	Newton formulates the law of universal gravitation and applies it to the motion of planets around the Sun.
1770	Euler gives the Sun–Earth distance correctly as 93 million miles.
1814	Fraunhofer discovers most of the first 547 lines in the solar spectrum, which show up when sunlight is passed through a prism. At this time it is commonly thought that the Sun is inhabited!
1842	At a solar eclipse, *prominences* are rediscovered, after having been mentioned previously in medieval Russian chronicles and observed by Vassenius in 1733. Also, the outer layers of the solar atmosphere (the *chromosphere* and *corona*) are clearly seen.
1843	Schwabe notices that the number of sunspots varies with an *eleven-year cycle*.
1851	The corona is photographed for the first time as a faint halo, visible around the Sun during an eclipse (PROBLEM 1.3).
1852	Sabine, Wolf and Gautier find that the sunspot cycle is related to geomagnetic storms.
1858	Carrington discovers that sunspots appear at lower and lower latitudes as the sunspot cycle progresses.
1859	Carrington and Hodgson observe a *solar flare*, possibly for the first time, although Stephen Gray may have seen one on 27 December 1705 (Hoyt and Schatten 1996).
1861	Spörer discovers his law for the distribution of sunspots.
1868	At an eclipse, Secchi detects the emission line of a new element, which is given the name helium (after Helios, the Greek Sun god).
1869	Another new emission line, in the corona, is ascribed (wrongly) to a hypothetical element called coronium.
1874	Langley gives a detailed description of the fine structure, called *granulation*, of the Sun's visible surface (the *photosphere*).
1875	Secchi describes the change in coronal form during a solar cycle.
1877	Secchi describes *spicules* as a burning prairie.
1889	Hale invents the spectroheliograph.
1908	Hale discovers that sunspots possess a strong magnetic field.
1909	The outward flow in sunspot penumbrae is observed by Evershed.
1919	Hale and Joy discover that pairs of sunspots tend to have opposite polarity and to be tilted with respect to latitude, and that leading sunspots have opposite polarity in the north and south hemispheres.

1920's	The dominance of hydrogen and helium in the atmosphere is realised by Celia Payne (1925) and Russell (1929) and in the interior by Eddington and Strömgren (1932).
1930	Lyot invents his coronagraph to view the corona without an eclipse.
1934	Cowling proposes a theory for sunspots and an anti-dynamo theorem.
1935	Hartmann conducts experiments on mercury flow in a magnetic field.
1938	The carbon-nitrogen and proton-proton chains are proposed by Bethe as an explanation for the source of solar energy.
1939	Grotrian (1939) and Edlén (1943) show that coronal emission lines arise from normal elements ionised by a very hot corona ($> 10^6$ K).
1941	Biermann realises sunspots are cool due to inhibition of convection.
1942	Alfvén sets up a theory for magnetic waves. Also, radio emission from the Sun is detected by radar.
1945	W. Roberts names and describes *spicules* in detail.
1948	Biermann and Schwarzschild propose that the outer atmosphere is heated by sound waves propagating up from the convection zone.
1951	Biermann suggests that the Sun is continuously emitting 'solar corpuscles' that make comet tails point away from it.
1951	Laboratory experiments begin on magnetic containment of a plasma; the ultimate goal is to contain plasma for only about a second at 10^6 K, so that light atoms can fuse together and release energy.
1952	Babcock and Babcock (1952, 1955) invent the magnetograph and discover properties of photospheric magnetic fields.
1955	Parker produces major works on the dynamo and magnetic buoyancy.
1956	The basic theory of magnetohydrodynamics (MHD) is summarised by Cowling in his book on the subject.
1957	The first satellite observations of interplanetary plasma are made. Kippenhahn and Schlüter propose a model for prominence support.
1958	Babcock and Livingston observe polar fields reverse at spot maximum.
1958	Parker predicts the existence of the *solar wind* and proposes a model.
1960	Leighton discovers *five-minute oscillations* in the photosphere.
1962	Leighton, Noyes and Simon find that the *network* (first seen by Hale and Deslandres in the 1890s) outlines *supergranule cells*.
1970	Ulrich suggests that global sound waves can be trapped below the surface of the Sun and proposes a relation between their frequency (ω) and horizontal wavenumber (k_h).
1972	Tousey and Koomen observe a coronal mass ejection from OSO 5.
1973	After early rocket flights, Skylab (1973–4) explores in detail the corona in soft X-rays with its holes, loops and X-ray bright points.
1973	Stenflo discovers kG magnetic fields in the network.
1975	Deubner first observes power in the ω-k_h ridges predicted by Ulrich.
1980	Hickey et al. (1980) discover that the solar irradiance is varying, that is, the *solar constant* is not constant.
1980s	Important advances occur in the MHD theory of equilibria, waves, instabilities and reconnection.
1990s	Yohkoh reveals the dynamic nature of the corona and the presence of magnetic reconnection in solar flares.

	SoHO produces a major increase in understanding, especially the rotation structure of the interior from the MDI instrument and the properties of coronal mass ejections from LASCO.
1998	TRACE revolutionises our view at high resolution of the corona.
2002	RHESSI transforms our knowledge of solar flares.
2000s	STEREO views coronal mass ejections in three dimensions.
	Hinode studies the link between photosphere and corona and changes our paradigm for photospheric magnetic fields.
2010	SDO begins a new revolution in understanding with super-TRACE and super-MDI instruments

In the past fifty years, our understanding of the Sun has been revolutionised by a combination of theoretical advances (both analytical and computational), ground-based observations (notably from helioseismology networks such as GONG and BiSON and from telescopes in La Palma, Tenerife, Sacramento Peak, Big Bear and Kitt Peak) and especially space observations. The satellites include (with their launch dates): Skylab (1973), Yohkoh (1991), Ulysses (1992), SoHO (1995), TRACE (1998), RHESSI (2002), STEREO (2006), Hinode (2006) and SDO (2010) (see APPENDIX 3), which is on the web site www.cambridge.org/9780521854719.

1.2 Overall Properties

Traditionally, solar phenomena have been divided into two classes: quiet and active. In this paradigm, the *quiet Sun* is viewed as a static, spherically symmetric ball of plasma, for which properties depend to a first approximation on radial distance from the centre and for which the magnetic field is negligible. The *active Sun* consists of transient phenomena, such as sunspots (Sec. 1.7), prominences (Sec. 1.8), flares and coronal mass ejections (Sec. 1.9), which are superimposed on the quiet atmosphere and owe their existence to the magnetic field. This division is hinted at here, by first describing aspects of the quiet Sun in Sections1.3–1.6 and then transient phenomena in Sections 1.7–1.9, even though the paradigm needs to be replaced by a much more dynamic, multi-scale view. For instance, the quiet atmosphere is influenced markedly by the magnetic field; it is structured by the magnetic network above and around evolving granule and supergranule cells (Sec. 1.4) and the normal heating of the outer atmosphere is due to the magnetic field (Chapter 10). Furthermore, in the atmosphere, the notion of a static, spherically symmetric structure is not even a first approximation to reality.

General descriptions of the Sun can be found in many books, but those I have found particularly readable and informative are Noyes (1982), Golub and Pasachoff (1997), Schrijver and Zwaan (2000), Stix (2002), Zirker (2002) and Lang (2009).

The Sun is a fairly ordinary star of spectral type G2 V and absolute stellar magnitude 4.8, but of course its proximity to the Earth makes it unique. It has profound effects on the Earth's climate and on space weather, and its study is of central importance for understanding the behaviour of stars and of cosmic plasma in general. Its overall physical properties are as follows:

Age:	$= 4.6\times10^9$ yr
Mass ($M_\odot$):	$= 1.99\times10^{30}$ kg
Radius ($R_\odot$):	$= 695.5$ Mm ($= 695{,}500$ km $= 6.955\times10^8$ m)
Mean density:	1.4×10^3 kg m^{-3}
Mean distance from Earth:	1 AU $= 1.496\times10^{11}$ m ($= 215$ R$_\odot$)
Surface gravity ($g_\odot$):	$= 274$ m s^{-2}
Escape velocity at surface:	$= 618$ km s^{-1}

Radiation (luminosity, $L_\odot$):	$= 3.86\times10^{26}$ W ($= 3.86\times10^{33}$ erg s^{-1})
Equatorial (synodic) rotation period:	$= 26.24$ days
Angular momentum:	$= 1.7\times10^{41}$ kg m^2 s^{-1}
Mass-loss rate:	$= 10^9$ kg s^{-1}
Effective temperature:	$= 5785$ K
1 arcsec ($\equiv 1''$):	≈ 726 km

Many of these numbers may be rather meaningless to the reader, so let us put them into perspective. The Earth has a mass of 6×10^{24} kg and a radius of 6 Mm (6×10^6 m); so the Sun is 330,000 times more massive than the Earth and is 109 times larger in radius, while its surface gravity is 27 times greater. The mean density of the Earth is 5.5×10^3 kg m^{-3}, roughly equal to that of the Sun, while the atmospheric density at the Earth's surface is 1 kg m^{-3}, which makes the Sun's surface pressure 0.2 times that of the Earth's sea-level atmosphere. The mean distance from the Earth to the Sun is 149.6 million km (or 93×10^6 miles), which takes light 8 minutes to travel. The radiation emitted by the Sun amounts to about 1 kW m^{-2} at the surface of the Earth. Furthermore, the Sun's equator is inclined at about 7^o to the plane of the Earth's orbit and the solar rotation gives the Sun an equatorial velocity of 2 km s^{-1}. An *arcsec* is a unit of angular measurement equal to 1/3,600 of a degree or $\pi/648{,}000$ radians. The 726 km refers to the corresponding distance at the Sun measured by an Earth-bound observer, but that does vary by ±12 km during the Earth's journey around the Sun in an eccentric orbit (PROBLEM 1.4) and could be quite different when measured from a spacecraft out of Earth's orbit.

1.2.1 The Structure of the Solar Interior

The Sun is such a massive plasma ball that it is held together and compressed by its own gravitational attraction. It comprises mainly H (92 per cent) and He (8 per cent) atoms by number, mostly ionised due to the high temperature; the remaining elements, such as C, N and O, total about 0.1 per cent and are present in roughly the same proportions as on Earth, suggesting a common origin such as the interiors of older stars.

The *interior* of the Sun is shielded from our view; only its surface layers can be seen directly. However, the field of *helioseismology* (Sec. 1.3.4) is now being used to infer many properties of the interior, which is divided into three regions, as sketched in Figure 1.1, namely the *core*, *radiative zone* and *convection zone*, where different physical processes are dominant.

In the nineteenth century, it was shown that, if the Sun's energy arose purely from gravitational contraction, it would last only 10^{15} sec $= 3\times10^7$ yr, the so-called *Kelvin-Helmholtz time*. This is calculated simply by dividing the Sun's gravitational potential energy ($M_\odot^2 G/R_\odot = 4\times10^{41}$ J, where G is the gravitational constant) by its present luminosity $L_\odot \approx 4\times10^{26}$ W. Eddington, however, concluded in 1925 that it would last much longer if the core of the Sun were a gigantic reactor converting nuclear energy. This energy (generated in the core) leaks continuously outwards in a very gentle manner across the *radiative zone* by radiative diffusion, as the photons are absorbed and emitted many times, taking many years to cross it.

By contrast, in the *convection zone*, convection is the dominant means of outward energy transport. Convection transports energy because an individual blob of plasma carries heat as it rises and then gives up some of it before falling and picking up more. At the lower boundary of the convection zone, there exists a strong shear layer [called the *tachocline* by Spiegel and Zahn (1992)], where much of the Sun's large-scale magnetic field is probably generated by a *dynamo* (Chapter 8).

The solar interior is so incredibly opaque that, whereas an unimpeded photon would take 2 sec (at the speed of light) to reach the surface from the centre, there are so many collisions (absorptions and

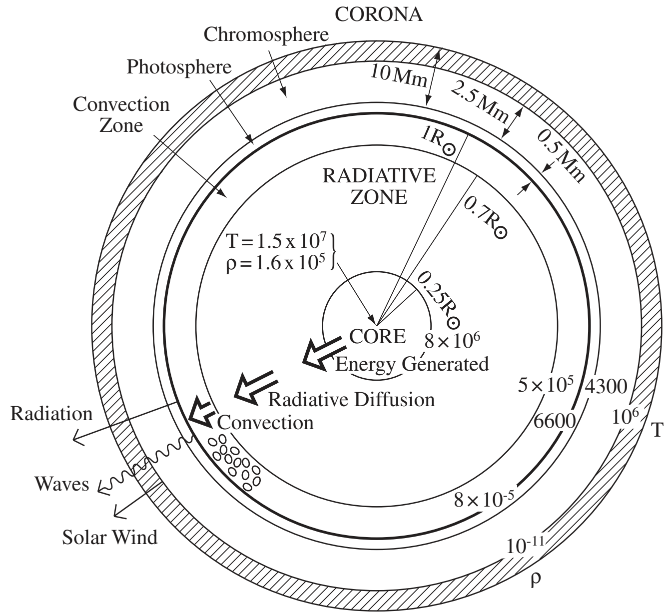


Figure 1.1. The overall structure of the Sun, indicating the sizes of the various regions and their temperatures (in K) and densities (in kg m^{-3}). The thicknesses of the various regions are not drawn to scale, and the boundary between chromosphere and corona is highly variable between 2.5 and 15 Mm, as indicated by the shaded region.

re-emissions) that photons in practice take 170,000 years for the journey (Mitalas and Sills 1992). The effect of these collisions is to increase the typical wavelength from that of high-energy gamma rays in the core to visible light at the solar surface, where most of the energy generated in the core is radiated into space.

The magnetic diffusion-time (Sec. 2.6.1) for the original magnetic flux that threaded the plasma cloud from which the Sun contracted is 10^{11} yr and so is somewhat longer than the Sun's age (4.6×10^9 yr). Since there are mechanisms for enhancing diffusion, it is unknown how much of this primordial magnetic field is still present in the solar interior. The other main global time-scales for the interior are: the viscous diffusion-time (10^{12} yr); the spin-down time (10^{10} yr) due to angular momentum loss in the solar wind, which is a little longer than the solar age; the thermal diffusion-time (1.2×10^7 yr) through the radiative envelope; the thermal relaxation-time of the convection zone (10^5 yr); the Alfvén travel-time (20 yr) across the radiative interior for a magnetic field of 2 kG; and the thermal or rotational equilibration-time (1 yr) of the convection zone. By comparison, the acoustic travel-time from the centre to the solar surface is only about one hour, which is why for most purposes the radiative envelope and core may be regarded as being in hydrostatic equilibrium.

1.2.2 The Structure of the Solar Atmosphere

The atmosphere is defined as the part of the Sun from which photons can escape directly into space. It consists of three regions with different physical properties, which are often conveniently but incorrectly pictured as a series of spherical shells. The lowest is an extremely thin layer of plasma only several hundred kilometers thick, called the *photosphere* (Sec. 1.4), which is relatively dense and opaque and emits most of the solar radiation. It has an optical thickness $\tau \lesssim 1$ in the near-ultraviolet, visible and near-infrared continua, but is optically thick in all except the weakest spectral lines. The *optical depth* or *thickness* (τ), is a measure of the transparency of a medium. It is defined by the equation

$$I = I_0 e^{-\tau},$$

where I_0 is the intensity of radiation at the source and I is the observed intensity after a given path, so that I/I_0 represents the fraction of radiation that remains (i.e., is not scattered or absorbed). Thus, $\tau = 1$ represents the location where the radiation has fallen by a factor e.

Above the photosphere lies the rarer and more transparent *chromosphere* (Sec. 1.5), which is optically thin in the near-ultraviolet, visible and near-infrared continua, but is optically thick in strong spectral lines. It can be glimpsed for a few seconds at the start and end of a solar eclipse as the red colour of the Balmer spectrum.

The *corona* (Sec. 1.6) extends from the top of a narrow *transition region* (Sec. 1.5) and out into the *solar wind*, which fills the *heliosphere*. The outer boundary of the corona may be defined as the Alfvén radius, where the solar-wind speed equals the Alfvén speed, so that Alfvén-wave communication with the Sun is no longer possible (Sec. 13.6.2). The corona is optically thin over the whole electromagnetic spectrum, except for radio waves and a few spectral lines.

The pressure of the solar transition region can be inferred from density-sensitive extreme ultra-violet (EUV) emission lines. Recent solar observations yield 4×10^{-3} N m^{-2} in the quiet Sun and 0.3 N m^{-2} in active region moss (Warren 2005; Warren et al. 2010), where 1 N m^{-2} = 1 0 dyne cm^{-2}. Hydrogen is almost wholly ionised in the upper chromosphere, but neutrals are important in the lower chromosphere and photosphere.

The density (n) decreases rather rapidly with height: typical values are 10^{23} m^{-3}, 10^{19} m^{-3} and 10^{15} m^{-3} in the photosphere, chromosphere and transition region, 10^{12} m^{-3} at a height of $1R_\odot$, 10^7 m^{-3} at 1 AU, and 10^6 m^{-3} in the interstellar medium. By comparison, the gas density at the Earth's surface is 10^{25} m^{-3}.

Before 1940, it was thought that the temperature decreases with height above the solar surface. But since then it has been realised that, for a steady, spherically symmetry atmosphere, after falling from about 6,600 K (at the bottom of the photosphere) to a minimum value of about 4,400 K at a height of 500 km, the temperature rises slowly through the lower chromosphere and then dramatically through the transition region (less than 100 km thick) to a few million degrees in the corona (Figure 1.2). Thereafter, it falls slowly in the outer corona, which is expanding outwards as the solar wind, to a value of 10^5 K at 1 AU. The photospheric temperature of a few thousand degrees may be compared with the temperature of red-hot iron (1,400 K) and that of the white-hot filament of an electric bulb (3,900 K).

Table 1.1 gives energy losses (after Withbroe and Noyes 1977) from different parts of the corona as conductive, radiative and solar-wind fluxes in W m^{-2}, where 1 W m^{-2} = 10^3 erg cm^{-2} s^{-1}. Lower estimates from coronal-hole models by Hansteen and Leer (1995) are shown in parentheses.

The *VAL model* of the solar atmosphere is a semi-empirical one-dimensional model that successfully fits a number of spectral lines from different regions and has been extremely useful. However, a representation such as Figure 1.2 of the atmospheric structure is a gross oversimplification, which indicates only the mean properties. In reality, the atmosphere does not consist of static plane-parallel layers but is a constantly seething mass of plasma with a far-from-uniform structure. The temperature and density at any location are continually changing as plasma heats and cools dynamically over tiny as-yet-unresolved length-scales and moves around in response to a variety of different physical processes. Indeed, along any line of sight there is an enormous range of temperatures. Thus, for instance, transition-region emission comes not in reality from a 100 km thick layer, but from plasma at many different heights that is heating up or cooling down through transition-region temperatures.

The bulk of the solar radiation comes from the photosphere (Figure 1.3a), which emits a continuous spectrum with superimposed dark *absorption lines*. Light of all wavelengths is emitted by the photosphere and most of it goes straight through the overlying atmosphere into space. However, at certain specific

Table 1.1. *Order-of-magnitude energy-loss fluxes in W m^{-2}*

	Coronal hole	Quiet Sun	Active region
Corona			
Conduction	60 (15)	200	10^3–10^4
Radiation	10 (15)	100	5000
Solar wind	700 (100)	$<$ 50	$<$ 100
TOTAL	800	300	10,000
Chromosphere			
Low	2000	2000	10,000
Middle	2000	2000	10,000
Upper	300	300	2000
TOTAL	4000	4000	20,000

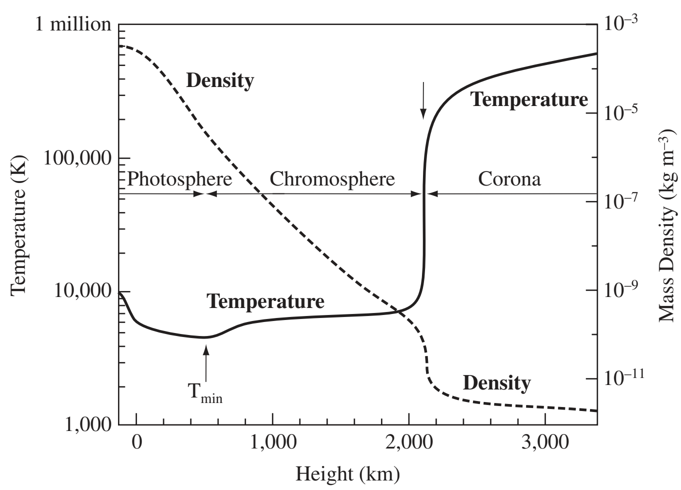
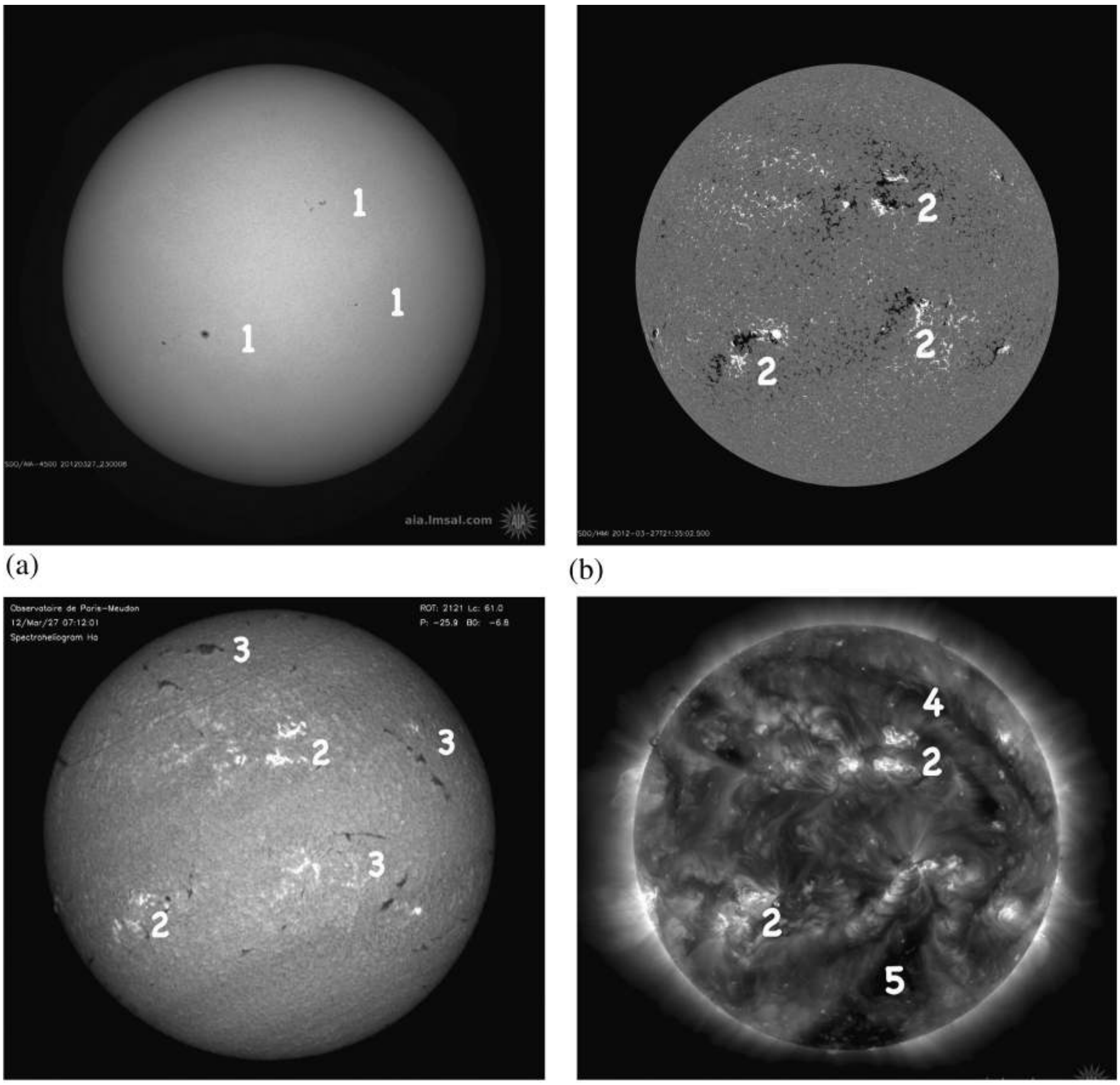


Figure 1.2. A schematic of the mean variation of temperature and density with height in the solar atmosphere according to the VAL (Vernazza-Avrett-Loeset) model (courtesy Eugene Avrett, see Sec. 1.4.3), although in practice the atmosphere is highly inhomogeneous, dynamic and time-varying.

wavelengths it is absorbed by particles in the overlying atmosphere, due to an increased opacity, which gives rise to the absorption lines. For example, the H Balmer line (Hα) is due to absorption of a photon making an H atom jump from its second to its third quantum level. Such lines give us much information on temperature and density (from intensity), magnetic field strength (from Zeeman splitting or Hanle effect) and local line-of-sight plasma motion (from Doppler shifts).

Most spectral lines are formed in the lower photosphere, but some (such as Hα) come from the chromosphere, and most lines in the transition region and corona are emission lines. The transition region emits mainly in UV wavelengths below 2,000 Å, which are strongly absorbed by the Earth's atmosphere. Due to its high temperature, the corona has increased UV, EUV and X-ray emission, but it also emits a pair of visible continua (the K and F coronae), as well as lines such as the green line (5,303 Å) and the red line (6,374 Å), which are due to forbidden transitions in highly ionised iron (Fe XIV and Fe XV, respectively).



(a) (b) (c) (d)

Figure 1.3. (a) The photosphere (SDO/AIA), (b) the line-of-sight photospheric magnetic field with positive polarity in white and negative in black (SDO/HMI), (c) the chromosphere in Hα (courtesy Jean-Marie Malherbe, Meudon Observatory) and (d) the corona (SDO/AIA) in 195 Å at 1.5 MK, all on 27 March 2012. Numbers indicate the presence of a (1) sunspot, (2) active region, (3) prominence or filament, (4) filament channel and (5) coronal hole.

When the Sun is observed through filters of different wavelengths, pictures can be obtained of the Sun's structure at a variety of levels (Figures 1.3a–d). For example, the *chromosphere* is revealed by using an Hα filter (Figure 1.3c), which is most important for following the evolution of *active regions* (Sec. 1.7.1) and *prominences* (Sec. 1.8) and for observing the low-temperature part of a *solar flare* (Sec. 1.9.2). Just at the start of an eclipse, you can see light that has originally come up from the photosphere and is then scattered towards you at the chromospheric level as well as the intrinsic chromospheric emission. This can be a very colourful effect, with reds from Hα and blue from Hβ, and it led Lockyer (1868) to give the chromosphere its name (from the Greek word for 'colour') (PROBLEM 1.5).

1.3 The Solar Interior

Models of the interior give a central temperature (15 million K) and density (1.6×10^5 kg m^{-3}) that are high enough for thermonuclear reactions to take place. The central temperature is so high that the material there remains in a gaseous (plasma) state under a pressure 230 billion times that of the Earth's sea-level air pressure; the central density is 13 times that of solid lead! Moving outwards, the temperature and density fall by 3.5 and 8.5 orders of magnitude to 6,000 K and 2×10^{-4} kg m^{-3} at the visible surface; thus, the temperature falls on average by only 2 degrees per 100 m.

The *core* has a radius of 150 Mm and contains half the mass of the Sun in only about one-fiftieth of its volume, but generates 99 per cent of the energy. The *convection zone* extends from a radius of 500 to 700 Mm and contains more than 60 per cent of the solar interior's volume but less than 2 per cent of its mass.

1.3.1 The Solar Core

For every kilogram of H that is fused to form He, 0.007 kg is converted into energy, so that the great furnace in the solar core uses up 5×10^6 tonne of H per second. In this process, He nuclei are built up from H nuclei mainly by the proton-proton (PP) chain but partly by the CNO cycle. At the end of these cycles, groups of four protons (^{1}H) have been fused into one helium nucleus (^{4}He) according to the reaction

$$4\,^1\mathrm{H} \rightarrow\ ^4\mathrm{He} + 2e^+ + 2\nu + 26.7\ \mathrm{MeV},$$

and other nuclei have just acted as catalysts. However, each He nucleus is smaller in mass by 3 per cent than the original protons, and this mass deficit appears as energy. According to Albert Einstein's equivalence of mass and energy ($E = mc^2$), each kilogram of mass is equivalent to 9×10^{16} J. In the above reaction, energy is released in the form of two high-frequency γ-rays (26.2 MeV) and two electron neutrinos (0.5 MeV), denoted by ν. The neutrinos are tiny subatomic particles with no electric charge that travel at nearly the speed of light and interact poorly with matter, escaping unimpeded from the core through the rest of the solar interior.

Neutrinos are our only direct diagnostic of core conditions and rain down on the Earth at a rate of about 70 billion per cm^2 per second. Davis, Jr and Evans (1978) observed the flux of solar electron neutrinos a mile underground in the Homestake Gold Mine in South Dakota, but it was lower by a factor of three than predicted from the standard solar model, a discrepancy known as the *solar neutrino problem*. Was it the solar model or the particle physics or the experiment that was wrong? After years of debate, it was established that, because of their tiny but non-zero mass, the electron neutrinos emitted by the Sun change to other forms (muon and tau neutrinos) during their journey to the Earth; this was discovered in 2002 at the Sudbury Neutrino Observatory, Canada, which detected all three types of neutrino and showed that one-third of the arriving solar neutrinos are electron neutrinos.

1.3.2 A Model for the Solar Interior

The *standard model* for the interior of the Sun assumes the pressure $[p(r)]$, density $[\rho(r)]$ and temperature $[T(r)]$ are functions only of radial distance (r) from the centre and that the interior consists of a set of spherical shells in hydrostatic and thermal equilibrium. The basic equations are: a *perfect gas law* (Eq. 2.24)

$$p = \frac{k_B}{m}\,\rho\,T, \tag{1.1}$$

where k_B is the Boltzmann constant and m the mean particle mass; a *hydrostatic force balance* (Sec. 3.1.2) between an outward pressure gradient and an inward force of gravity due to the mass $[M(r)]$ inside a