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978-0-521-85459-7 - Encyclopedia of Mathematics and Its Applications: Introduction to Radon Transforms: With Elements of Fractional Calculus and Harmonic Analysis

Boris Rubin

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