A MODERN APPROACH TO CRITICAL PHENOMENA

The study of critical phenomena is one of the most exciting areas of modern physics. This book provides a thorough but economic introduction into the principles and techniques of the theory of critical phenomena and the renormalization group, from the perspective of modern condensed matter physics. Assuming basic knowledge of quantum and statistical mechanics, the book discusses phase transitions in magnets, superfluids, superconductors, and gauge field theories. Particular attention is given to modern topics such as gauge field fluctuations in superconductors, the Kosterlitz-Thouless transition, duality transformations, and quantum phase transitions, all of which are at the forefront of today's physics research.

A Modern Approach to Critical Phenomena contains numerous problems of varying degrees of difficulty, with solutions. These problems provide readers with a wealth of material to test their understanding of the subject. It is ideal for graduate students and more experienced researchers in the fields of condensed matter physics, statistical physics, and many-body physics.

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Dedicated to my parents, Divna and Fedor Herbut

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Preface

It has been more than thirty years since the theory of universal behavior of matter near the points of continuous phase transitions was formulated. Since then the principles and the techniques of the theory of such "critical phenomena" have pervaded modern physics. The basic tenets of our understanding of phase transitions, the concepts of scaling and of the renormalization group, have been found to be useful well beyond their original domain, and today constitute some of our basic tools for thinking about systems with many interacting degrees of freedom. When applied to the original problem of continuous phase transitions in liquids, magnets, and superfluids, the theory is in remarkable agreement with measurements, and often even ahead of experiment in precision. For this reason alone the theory of critical phenomena would have to be considered a truly phenomenal physical theory, and ranked as one of the highest achievements of twentieth century physics.

The book before you originated in part from the courses on theory of phase transitions and renormalization group I taught to graduate students at Simon Fraser University. The students typically had a solid prior knowledge of statistical mechanics, and thus had some familiarity with the notions of phase transitions and of the mean-field theory, both being commonly taught nowadays as parts of a graduate course on the subject. In selecting the material and in gauging the technical level of the lectures I had in mind a student who not only wanted to become familiar with the basic concepts of the theory of critical phenomena, but also to learn how to actually use it to explain and compute. So I tried to provide the calculational details, particularly through solved problems, which would hopefully enable a motivated student to acquire what is today considered to be the standard working knowledge in the field, without having to take a separate course on field-theoretical techniques. The present book is an attempt to satisfy the perceived need for a graduate text that could accompany such a course.

The theme that runs through the book is the physics of the superfluid phase transition. There are several reasons for this. First, while historically it was the magnetic phase transitions for which the theory was first developed, the all-important notion of Ginzburg–Landau–Wilson theory is more naturally introduced for the system of interacting bosons. It is easy to then generalize the theory to other universality classes that include the more familiar Ising and Heisenberg magnetic phase transitions. Second, the superfluid order parameter allows the simplest topological defects, vortices, which are important in their own right and in fact play a crucial role at the superfluid phase transition. Finally, the superfluid critical point is experimentally the best quantitatively understood phase transition in nature, and as such provides the most stringent test for the theory.

A more experienced reader may notice the absence of so-called real-space renormalization on the pages that follow. While maybe more intuitive, the historically important method of real-space renormalization is much less systematic and general than Wilson's momentum-shell transformation, treated in detail here. If the reader is already familiar with real-space methods from a course on statistical mechanics, so much the better. But no such familiarity is in fact required. To draw an analogy with classical mechanics: while the concept of force is certainly important, one can almost completely dispose of it in favor of the Lagrangian or the Hamiltonian formulations. The general Ginzburg–Landau–Wilson field theory may be viewed as playing a somewhat similar role in the physics of continuous phase transitions.

The intended introductory level of the book notwithstanding, some of the chapters deal with a more advanced material. The selection criterion was that the subject, besides proven to be important and general, also had to be well established and relatively straightforward to discuss using the techniques already introduced elsewhere in the book. Chapter 4 deals with the issue of coupling of the order parameter to other soft modes, as exemplified by the Ginzburg–Landau theory of superconductors or the scalar electrodynamics. Chapter 7 deals with modern duality transformations which provide a precious non-perturbative perspective at some interesting phase transitions. These two chapters may be omitted at the first reading without consequences. Likewise, the sections in the remaining chapters marked with an asterisk represent more advanced material that, although in line with the rest of the book, may also be safely left for later times. On the other hand, some other important topics, like critical dynamics or phase transitions in disordered systems, have not been included. Although the selection of topics to some degree is certainly a matter of personal taste, the exclusion of these two may be partially justified by them not being on equally firm footing at the time of writing as the rest.

Conforming to my belief that physics is best learned by practising, the book contains numerous problems scattered throughout, all fully solved. Both the problems and their solutions either further illustrate some point in the main text, or provide complementary material interesting in its own right. Some problems are straightforward exercises, while others are more involved. Difficult, but often very instructive problems are again marked with an asterisk. The problem set represents an integral part of the book, and it is recommended that the reader goes through it as much as possible.

I am grateful to Matthew Case, Albert Curzon, Kamran Kaveh, Hidetoshi Nishimori, and Babak Seradjeh for reading parts of the manuscript and for their many useful suggestions for improvement. Of course, the responsibility for any remaining mistakes is solely mine. I am also grateful to Simon Fraser University for the sabbatical leave during which the manuscript was finalized, and to Masaki Oshikawa and the condensed matter theory group at the Tokyo Institute of Technology for their kind hospitality during that time. The last but not the least, I am thankful to my my wife Irena, and my children Leonard and Marlena, for tolerating long periods of my mental and physical absence.