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978-0-521-85403-0 - Mathematics for Physics: A Guided Tour for Graduate Students

Michael Stone and Paul Goldbart

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