

## Subject index

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Any terms not found here may be in the list of notations and conventions p. xvi.

Left and right or upper and lower properties are usually not listed separately, thus 'left depth' and 'upper depth' are listed under 'depth'.

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