

Principles of Astrophysical Fluid Dynamics

Fluid dynamical forces drive most of the fundamental processes in the Universe and so play a crucial role in our understanding of astrophysics. This comprehensive textbook introduces the fluid dynamics necessary to understand a wide range of astronomical phenomena, from stellar structures to supernovae blast waves, to accretion discs.

The authors' approach is to introduce and derive the fundamental equations, supplemented by text that conveys a more intuitive understanding of the subject, and to emphasise the observable phenomena that rely on fluid dynamical processes. It has been developed for use by final year undergraduate and starting graduate students of astrophysics, based on the authors' many years of teaching their astrophysical fluid dynamics course at the University of Cambridge. The book contains over 50 exercises.

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Contents

<i>Preface</i>	<i>page ix</i>
1 Introduction to concepts	1
1.1 Fluids in the Universe	2
1.2 The concept of a ‘fluid element’	4
1.3 Formulation of the fluid equations	5
1.4 Relation between the Eulerian and Lagrangian descriptions	7
1.5 Kinematical concepts	8
2 The fluid equations	12
2.1 Conservation of mass	12
2.2 Pressure	14
2.3 Momentum equations	15
2.4 Momentum equation in conservative form: the stress tensor and concept of ram pressure	17
3 Gravitation	20
3.1 The gravitational potential	20
3.2 Poisson’s equation	22
3.3 Using Poisson’s equation	24
3.4 The potential associated with a spherical mass distribution	27
3.5 Gravitational potential energy	28
3.6 The virial theorem	30
4 The energy equation	32
4.1 Ideal gases	32
4.2 Barotropic equations of state: the isothermal and adiabatic cases	33
4.3 Energy equation	37
4.4 Energy transport	39
4.5 The form of \dot{Q}_{cool}	45
	v

vi Contents

5	Hydrostatic equilibrium	46
5.1	Basic equations	46
5.2	The isothermal slab	47
5.3	An isothermal atmosphere with constant g	49
5.4	Stars as self-gravitating polytropes	50
5.5	Solutions for the Lane–Emden equation	52
5.6	The case of $n = \infty$	55
5.7	Scaling relations	56
5.8	Examples of astrophysical interest	60
5.9	Summary: general method for scaling relations	62
6	Propagation of sound waves	63
6.1	Sound waves in a uniform medium	63
6.2	Propagation of sound waves in a stratified atmosphere	68
6.3	General approach to wave propagation problems	73
6.4	Transmission of sound waves at interfaces	74
7	Supersonic flows	77
7.1	Shocks	78
7.2	Isothermal shocks	85
8	Blast waves	89
8.1	Strong explosions in uniform atmospheres	89
8.2	Blast waves in astrophysics and elsewhere	96
8.3	Structure of the blast wave	98
8.4	Breakdown of the similarity solution	102
8.5	The effects of cooling and blowout from galactic disks	104
9	Bernoulli's equation	107
9.1	Basic equation	107
9.2	De Laval nozzle	113
9.3	Spherical accretion and winds	118
9.4	Stellar winds	123
9.5	General steady state solutions	126
10	Fluid instabilities	128
10.1	Convective instability	128
10.2	Rayleigh–Taylor and Kelvin–Helmholtz instabilities	133

10.3 Gravitational instability (Jeans instability)	139
10.4 Thermal instability	142
10.5 Method summary	149
11 Viscous flows	150
11.1 Linear shear and viscosity	150
11.2 Navier–Stokes equation	153
11.3 Evolution of vorticity in viscous flows	157
11.4 Energy dissipation in incompressible viscous flows	158
11.5 Viscous flow through a circular pipe and the transition to turbulence	159
12 Accretion discs in astrophysics	163
12.1 Derivation of viscous evolution equations for accretion discs	165
12.2 Viscous evolution equation with constant viscosity	167
12.3 Steady thin discs	173
12.4 Radiation from steady thin discs	176
13 Plasmas	179
13.1 Magnetohydrodynamic equations	180
13.2 Simplifying the magnetohydrodynamic equations	183
13.3 Charge neutrality	184
13.4 The induction equation and flux freezing approximation	186
13.5 The dynamical effects of magnetic fields	188
13.6 Summary	189
13.7 Waves in Plasmas	190
13.8 The Rayleigh–Taylor Instability revisited	194
Appendix Equations in curvilinear coordinates	200
Exercises	206
Books for background and further reading	222
<i>Index</i>	224

Preface

The material in this book is based on lecture notes of a course on astrophysical fluid dynamics which has been given for several years to third-year students at the University of Cambridge. There are several excellent books which cover fluid dynamics from a terrestrial standpoint, but very few provide a full introduction to the concepts and methods used to deal with the highly compressible flows which arise in astrophysical contexts. Our aim with this book is to provide just such an introduction, and we hope that it will also serve as a reference volume for advanced undergraduate and graduate students.

Several people have provided input at various stages of the preparation of this book. In particular we thank Jim Pringle, Donald Lynden-Bell and Giuseppe Lodato for their help. We are also grateful to the students who have taken the course at Cambridge for correcting typographical errors in the lecture notes, drawing our attention to parts where the description was less clear than it should have been, and helping us to develop the exercises.