

Cambridge University Press

978-0-521-85249-4 - An Introduction to the Standard Model of Particle Physics, Second Edition

W. N. Cottingham and D. A. Greenwood

Frontmatter

[More information](#)

AN INTRODUCTION TO THE STANDARD MODEL OF PARTICLE PHYSICS

Second Edition

The Standard Model of particle physics is the mathematical theory that describes the weak, electromagnetic and strong interactions between leptons and quarks, the basic particles of the Standard Model.

The new edition of this introductory graduate textbook provides a concise but accessible introduction to the Standard Model. It has been updated to account for the successes of the theory of strong interactions, and the observations on matter–antimatter asymmetry. It has become clear that neutrinos are not mass-less, and this book gives a coherent presentation of the phenomena and the theory that describes them. It includes an account of progress in the theory of strong interactions and of advances in neutrino physics. The book clearly develops the theoretical concepts from the electromagnetic and weak interactions of leptons and quarks to the strong interactions of quarks.

This textbook provides an up-to-date introduction to the Standard Model for graduate students in particle physics. Each chapter ends with problems, and hints to selected problems are provided at the end of the book. The mathematical treatments are suitable for graduates in physics, and more sophisticated mathematical ideas are developed in the text and appendices.

NOEL COTTINGHAM and DEREK GREENWOOD are theoreticians working in the H. H. Wills Physics Laboratory at the University of Bristol. They have published two undergraduate texts with Cambridge University Press, *Electricity and Magnetism* (1991) and *An Introduction to Nuclear Physics*, now in its second edition (2001).

Cambridge University Press

978-0-521-85249-4 - An Introduction to the Standard Model of Particle Physics, Second Edition

W. N. Cottingham and D. A. Greenwood

Frontmatter

[More information](#)

AN INTRODUCTION TO THE
STANDARD MODEL OF
PARTICLE PHYSICS
Second Edition

W. N. COTTINGHAM and D. A. GREENWOOD
University of Bristol, UK



Cambridge University Press

978-0-521-85249-4 - An Introduction to the Standard Model of Particle Physics, Second Edition

W. N. Cottingham and D. A. Greenwood

Frontmatter

[More information](#)

CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press

The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org

Information on this title: www.cambridge.org/9780521852494

© W. N. Cottingham and D. A. Greenwood 2007

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2007

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

ISBN-13 978-0-521-85249-4 hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Cambridge University Press

978-0-521-85249-4 - An Introduction to the Standard Model of Particle Physics, Second Edition

W. N. Cottingham and D. A. Greenwood

Frontmatter

[More information](#)

Contents

<i>Preface to the second edition</i>	<i>page</i> xi
<i>Preface to the first edition</i>	xiii
<i>Notation</i>	xv
1 The particle physicist's view of Nature	1
1.1 Introduction	1
1.2 The construction of the Standard Model	2
1.3 Leptons	3
1.4 Quarks and systems of quarks	4
1.5 Spectroscopy of systems of light quarks	5
1.6 More quarks	10
1.7 Quark colour	11
1.8 Electron scattering from nucleons	16
1.9 Particle accelerators	17
1.10 Units	18
2 Lorentz transformations	20
2.1 Rotations, boosts and proper Lorentz transformations	20
2.2 Scalars, contravariant and covariant four-vectors	22
2.3 Fields	23
2.4 The Levi–Civita tensor	24
2.5 Time reversal and space inversion	25
3 The Lagrangian formulation of mechanics	27
3.1 Hamilton's principle	27
3.2 Conservation of energy	29
3.3 Continuous systems	30
3.4 A Lorentz covariant field theory	32
3.5 The Klein–Gordon equation	33
3.6 The energy–momentum tensor	34
3.7 Complex scalar fields	36

4	Classical electromagnetism	38
4.1	Maxwell's equations	38
4.2	A Lagrangian density for electromagnetism	39
4.3	Gauge transformations	40
4.4	Solutions of Maxwell's equations	41
4.5	Space inversion	42
4.6	Charge conjugation	44
4.7	Intrinsic angular momentum of the photon	44
4.8	The energy density of the electromagnetic field	45
4.9	Massive vector fields	46
5	The Dirac equation and the Dirac field	49
5.1	The Dirac equation	49
5.2	Lorentz transformations and Lorentz invariance	51
5.3	The parity transformation	54
5.4	Spinors	54
5.5	The matrices γ^μ	55
5.6	Making the Lagrangian density real	56
6	Free space solutions of the Dirac equation	58
6.1	A Dirac particle at rest	58
6.2	The intrinsic spin of a Dirac particle	59
6.3	Plane waves and helicity	60
6.4	Negative energy solutions	62
6.5	The energy and momentum of the Dirac field	63
6.6	Dirac and Majorana fields	65
6.7	The $E \gg m$ limit, neutrinos	65
7	Electrodynamics	67
7.1	Probability density and probability current	67
7.2	The Dirac equation with an electromagnetic field	68
7.3	Gauge transformations and symmetry	70
7.4	Charge conjugation	71
7.5	The electrodynamics of a charged scalar field	73
7.6	Particles at low energies and the Dirac magnetic moment	73
8	Quantising fields: QED	77
8.1	Boson and fermion field quantisation	77
8.2	Time dependence	80
8.3	Perturbation theory	81
8.4	Renormalisation and renormalisable field theories	83
8.5	The magnetic moment of the electron	87
8.6	Quantisation in the Standard Model	89

Contents

vii

9	The weak interaction: low energy phenomenology	91
9.1	Nuclear beta decay	91
9.2	Pion decay	93
9.3	Conservation of lepton number	95
9.4	Muon decay	96
9.5	The interactions of muon neutrinos with electrons	98
10	Symmetry breaking in model theories	102
10.1	Global symmetry breaking and Goldstone bosons	102
10.2	Local symmetry breaking and the Higgs boson	104
11	Massive gauge fields	107
11.1	$SU(2)$ symmetry	107
11.2	The gauge fields	109
11.3	Breaking the $SU(2)$ symmetry	111
11.4	Identification of the fields	113
12	The Weinberg–Salam electroweak theory for leptons	117
12.1	Lepton doublets and the Weinberg–Salam theory	117
12.2	Lepton coupling to the W^\pm	120
12.3	Lepton coupling to the Z	121
12.4	Conservation of lepton number and conservation of charge	122
12.5	CP symmetry	123
12.6	Mass terms in \mathcal{L} : an attempted generalisation	125
13	Experimental tests of the Weinberg–Salam theory	128
13.1	The search for the gauge bosons	128
13.2	The W^\pm bosons	129
13.3	The Z boson	130
13.4	The number of lepton families	131
13.5	The measurement of partial widths	132
13.6	Left–right production cross-section asymmetry and lepton decay asymmetry of the Z boson	133
14	The electromagnetic and weak interactions of quarks	137
14.1	Construction of the Lagrangian density	137
14.2	Quark masses and the Kobayashi–Maskawa mixing matrix	139
14.3	The parameterisation of the KM matrix	142
14.4	CP symmetry and the KM matrix	143
14.5	The weak interaction in the low energy limit	144
15	The hadronic decays of the Z and W bosons	147
15.1	Hadronic decays of the Z	147
15.2	Asymmetry in quark production	149
15.3	Hadronic decays of the W^\pm	150

16	The theory of strong interactions: quantum chromodynamics	153
16.1	A local $SU(3)$ gauge theory	153
16.2	Colour gauge transformations on baryons and mesons	156
16.3	Lattice QCD and asymptotic freedom	158
16.4	The quark–antiquark interaction at short distances	161
16.5	The conservation of quarks	162
16.6	Isospin symmetry	162
16.7	Chiral symmetry	164
17	Quantum chromodynamics: calculations	166
17.1	Lattice QCD and confinement	166
17.2	Lattice QCD and hadrons	169
17.3	Perturbative QCD and deep inelastic scattering	171
17.4	Perturbative QCD and e^+e^- collider physics	173
18	The Kobayashi–Maskawa matrix	176
18.1	Leptonic weak decays of hadrons	176
18.2	$ V_{ud} $ and nuclear β decay	178
18.3	More leptonic decays	179
18.4	CP symmetry violation in neutral kaon decays	180
18.5	B meson decays and B^0 , \bar{B}^0 mixing	182
18.6	The CPT theorem	183
19	Neutrino masses and mixing	185
19.1	Neutrino masses	185
19.2	The weak currents	186
19.3	Neutrino oscillations	187
19.4	The MSW effect	190
19.5	Neutrino masses and the Standard Model	191
19.6	Parameterisation of U	191
19.7	Lepton number conservation	192
19.8	Sterile neutrinos	193
20	Neutrino masses and mixing: experimental results	194
20.1	Introduction	194
20.2	K2K	196
20.3	Chooz	198
20.4	KamLAND	198
20.5	Atmospheric neutrinos	200
20.6	Solar neutrinos	200
20.7	Solar MSW effects	203
20.8	Future prospects	204
21	Majorana neutrinos	206
21.1	Majorana neutrino fields	206

<i>Contents</i>		ix
21.2	Majorana Lagrangian density	207
21.3	Majorana field equations	208
21.4	Majorana neutrinos: mixing and oscillations	209
21.5	Parameterisation of \mathbf{U}	210
21.6	Majorana neutrinos in the Standard Model	210
21.7	The seesaw mechanism	211
21.8	Are neutrinos Dirac or Majorana?	212
22	Anomalies	215
22.1	The Adler–Bell–Jackiw anomaly	215
22.2	Cancellation of anomalies in electroweak currents	217
22.3	Lepton and baryon anomalies	217
22.4	Gauge transformations and the topological number	219
22.5	The instability of matter, and matter genesis	220
	Epilogue	221
	Reductionism complete?	221
	Appendix A An aide-mémoire on matrices	222
A.1	Definitions and notation	222
A.2	Properties of $n \times n$ matrices	223
A.3	Hermitian and unitary matrices	224
A.4	A Fierz transformation	225
	Appendix B The groups of the Standard Model	227
B.1	Definition of a group	227
B.2	Rotations of the coordinate axes, and the group $SO(3)$	228
B.3	The group $SU(2)$	229
B.4	The group $SL(2, \mathbb{C})$ and the proper Lorentz group	231
B.5	Transformations of the Pauli matrices	232
B.6	Spinors	232
B.7	The group $SU(3)$	233
	Appendix C Annihilation and creation operators	235
C.1	The simple harmonic oscillator	235
C.2	An assembly of bosons	236
C.3	An assembly of fermions	236
	Appendix D The parton model	238
D.1	Elastic electron scattering from nucleons	238
D.2	Inelastic electron scattering from nucleons: the parton model	239
D.3	Hadronic states	244
	Appendix E Mass matrices and mixing	245
E.1	K^0 and \bar{K}^0	245
E.2	B^0 and \bar{B}^0	246

Cambridge University Press

978-0-521-85249-4 - An Introduction to the Standard Model of Particle Physics, Second Edition

W. N. Cottingham and D. A. Greenwood

Frontmatter

[More information](#)

x

Contents

<i>References</i>	248
<i>Hints to selected problems</i>	250
<i>Index</i>	269

Cambridge University Press

978-0-521-85249-4 - An Introduction to the Standard Model of Particle Physics, Second Edition

W. N. Cottingham and D. A. Greenwood

Frontmatter

[More information](#)

Preface to the second edition

In the eight years since the first edition, the Standard Model has not been seriously discredited as a description of particle physics in the energy region (<2 TeV) so far explored. The principal discovery in particle physics since the first edition is that neutrinos carry mass. In this new edition we have added chapters that extend the formalism of the Standard Model to include neutrino fields with mass, and we consider also the possibility that neutrinos are Majorana particles rather than Dirac particles.

The Large Hadron Collider (LHC) is now under construction at CERN. It is expected that, at the energies that will become available for experiments at the LHC (~ 20 TeV), the physics of the Higgs field will be elucidated, and we shall begin to see ‘physics beyond the Standard Model’. Data from the ‘B factories’ will continue to accumulate and give greater understanding of CP violation. We are confident that interest in the Standard Model will be maintained for some time into the future.

Cambridge University Press have again been most helpful. We thank Miss V. K. Johnson for secretarial assistance. We are grateful to Professor Dr J. G. Körner for his corrections to the first edition, and to Professor C. Davies for her helpful correspondence.

Cambridge University Press

978-0-521-85249-4 - An Introduction to the Standard Model of Particle Physics, Second Edition

W. N. Cottingham and D. A. Greenwood

Frontmatter

[More information](#)

Preface to the first edition

The ‘Standard Model’ of particle physics is the result of an immense experimental and inspired theoretical effort, spanning more than fifty years. This book is intended as a concise but accessible introduction to the elegant theoretical edifice of the Standard Model. With the planned construction of the Large Hadron Collider at CERN now agreed, the Standard Model will continue to be a vital and active subject.

The beauty and basic simplicity of the theory can be appreciated at a certain ‘classical’ level, treating the boson fields as true classical fields and the fermion fields as completely anticommuting. To make contact with experiment the theory must be quantised. Many of the calculations of the consequences of the theory are made in quantum perturbation theory. Those we present are for the most part to the lowest order of perturbation theory only, and do not have to be renormalised. Our account of renormalisation in Chapter 8 is descriptive, as is also our final Chapter 19 on the anomalies that are generated upon quantisation.

A full appreciation of the success and significance of the Standard Model requires an intimate knowledge of particle physics that goes far beyond what is usually taught in undergraduate courses, and cannot be conveyed in a short introduction. However, we attempt to give an overview of the intellectual achievement represented by the Model, and something of the excitement of its successes. In Chapter 1 we give a brief résumé of the physics of particles as it is qualitatively understood today. Later chapters developing the theory are interspersed with chapters on the experimental data. The amount of supporting data is immense and so we attempt to focus only on the most salient experimental results. Unless otherwise referenced, experimental values quoted are those recommended by the Particle Data Group (1996).

The mathematical background assumed is that usually acquired during an undergraduate physics course. In particular, a facility with the manipulations of matrix algebra is very necessary; Appendix A provides an *aide-mémoire*. Principles of symmetry play an important rôle in the construction of the model, and Appendix B is a self-contained account of the group theoretic ideas we use in describing these

Cambridge University Press

978-0-521-85249-4 - An Introduction to the Standard Model of Particle Physics, Second Edition

W. N. Cottingham and D. A. Greenwood

Frontmatter

[More information](#)

xiv

Preface to the first edition

symmetries. The mathematics we require is not technically difficult, but the reader must accept a gradually more abstract formulation of physical theory than that presented at undergraduate level. Detailed derivations that would impair the flow of the text are often set as problems (and outline solutions to these are provided).

The book is based on lectures given to beginning graduate students at the University of Bristol, and is intended for use at this level and, perhaps, in part at least, at senior undergraduate level. It is not intended only for the dedicated particle physicist: we hope it may be read by physicists working in other fields who are interested in the present understanding of the ultimate constituents of matter.

We should like to thank the anonymous referees of Cambridge University Press for their useful comments on our proposals. The Department of Physics at Bristol has been generous in its encouragement of our work. Many colleagues, at Bristol and elsewhere, have contributed to our understanding of the subject. We are grateful to Mrs Victoria Parry for her careful and accurate work on the typescript, without which this book would never have appeared.

Notation

Position vectors in three-dimensional space are denoted by $\mathbf{r} = (x, y, z)$, or $\mathbf{x} = (x^1, x^2, x^3)$ where $x^1 = x, x^2 = y, x^3 = z$.

A general vector \mathbf{a} has components (a^1, a^2, a^3) , and $\hat{\mathbf{a}}$ denotes a unit vector in the direction of \mathbf{a} .

Volume elements in three-dimensional space are denoted by $d^3\mathbf{x} = dx dy dz = dx^1 dx^2 dx^3$.

The coordinates of an event in four-dimensional time and space are denoted by $x = (x^0, x^1, x^2, x^3) = (x^0, \mathbf{x})$ where $x^0 = ct$.

Volume elements in four-dimensional time and space are denoted by $d^4x = dx^0 dx^1 dx^2 dx^3 = c dt d^3\mathbf{x}$.

Greek indices μ, ν, λ, ρ take on the values 0, 1, 2, 3.

Latin indices i, j, k, l take on the space values 1, 2, 3.

Pauli matrices

We denote by σ^μ the set $(\sigma^0, \sigma^1, \sigma^2, \sigma^3)$ and by $\tilde{\sigma}^\mu$ the set $(\sigma^0, -\sigma^1, -\sigma^2, -\sigma^3)$, where

$$\sigma^0 = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$(\sigma^1)^2 = (\sigma^2)^2 = (\sigma^3)^2 = \mathbf{I}; \quad \sigma^1 \sigma^2 = i \sigma^3 = -\sigma^2 \sigma^1, \text{ etc.}$$

Chiral representation for γ -matrices

$$\gamma^0 = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix},$$

$$\gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}.$$

Quantisation ($\hbar = c = 1$)

$$(E, \mathbf{p}) \rightarrow (i\partial/\partial t, -i\nabla), \text{ or } p^\mu \rightarrow i\partial^\mu.$$

For a particle carrying charge q in an external electromagnetic field,

$$(E, \mathbf{p}) \rightarrow (E - q\phi, \mathbf{p} - q\mathbf{A}), \text{ or } p^\mu \rightarrow p^\mu - qA^\mu,$$

$$i\partial^\mu \rightarrow (i\partial^\mu - qA^\mu) = i(\partial^\mu + iqA^\mu).$$

Field definitions

$$Z_\mu = W_\mu^3 \cos \theta_w - B_\mu \sin \theta_w,$$

$$A_\mu = W_\mu^3 \sin \theta_w + B_\mu \cos \theta_w,$$

where $\sin^2 \theta_w = 0.2315(4)$

$$g_2 \sin \theta_w = g_1 \cos \theta_w = e, \quad G_F = g_2^2 / (4\sqrt{2}M_w^2).$$

Glossary of symbols

A	electromagnetic vector potential Section 4.3
A^μ	electromagnetic four-vector potential
$A^{\mu\nu}$	field strength tensor Section 11.3
A_{FB}	forward-backward asymmetry Section 15.2
a	wave amplitude Section 3.5
a, a^\dagger	boson annihilation, creation operator
B	magnetic field
B^μ	gauge field Section 11.1
$B^{\mu\nu}$	field strength tensor Section 11.2
b, b^\dagger	fermion annihilation, creation operator
D	isospin doublet Section 16.6
d, d^\dagger	antifermion annihilation, creation operator
d_k	($k = 1, 2, 3$) down-type quark field
E	electric field
E	energy
e, e_L, e_R	electron Dirac, two-component left-handed, right-handed field
$F^{\mu\nu}$	electromagnetic field strength tensor Section 4.1
f	radiative corrections factor Sections 15.1, 17.4
f_{abc}	structure constants of $SU(3)$ Section B.7
G^μ	gluon matrix gauge field
$G^{\mu\nu}$	gluon field strength tensor
G_F	Fermi constant Section 9.4

$g^{\mu\nu}$	metric tensor
g	strong coupling constant Section 16.1
g_1, g_2	electroweak coupling constants
H	Hamiltonian Section 3.1
$h(x)$	Higgs field
\mathcal{H}	Hamiltonian density Section 3.3
\mathbf{I}	isospin operator Sections 1.5, 16.6
\mathbf{J}	electric current density Section 4.1
\mathbf{J}	total angular momentum operator
J	Jarlskog constant Section 14.3
J^μ	lepton number current Section 12.4
\mathbf{j}	probability current Section 7.1
j^μ	lepton current Section 12.2
K	string tension Section 17.1
\mathbf{k}	wave vector
\mathbf{L}	lepton doublet Section 12.1
L	Lagrangian Section 3.1
\mathcal{L}	Lagrangian density Section 3.3
l^3	normalisation volume Section 3.5
\mathbf{M}	left-handed spinor transformation matrix Section B.6
M	proton mass Section D.1
m	mass
\mathbf{N}	right-handed spinor transformation matrix Section B.6
N	number operator Section C.1
$\hat{\mathcal{O}}$	quantum operator
\mathbf{P}	total field momentum
\mathbf{p}	momentum
Q^2	$= -q_\mu q^\mu$
\mathbf{q}	quark colour triplet
q^μ	energy–momentum transfer
\mathbf{R}	rotation matrix Section B.2
\mathbf{S}	spin operator
S	action Section 3.1
s	square of centre of mass energy
T_ν^μ	energy–momentum tensor Section 3.6
\mathbf{U}	unitary matrix
u_k	($k = 1, 2, 3$) up-type quark field
u_L, u_R	two-component left-handed, right-handed spinors Section 6.1
u_+, u_-	Dirac spinors Section 6.3
\mathbf{V}	Kobayashi–Maskawa matrix Section 14.2

V	normalisation volume
\mathbf{v}	velocity
v	$= \mathbf{v} $
ν_L, ν_R	two-component left-handed, right-handed spinors
ν_+, ν_-	Dirac spinors Section 6.4
\mathbf{W}^μ	matrix of vector gauge field Section 11.1
$W^{\mu\nu}$	field strength tensor Section 11.2
$W_\mu^1, W_\mu^2, W_\mu^+, W_\mu^-$	fields of W boson
Z_μ	field of Z boson
$\alpha(Q^2)$	effective fine structure constant Section 16.3
$\alpha_s(Q^2)$	effective strong coupling constant Section 16.3
α_{latt}	lattice coupling constant Section 17.1
α^i	Dirac matrix Section 5.1
β	Dirac matrix Section 5.1
β	$= v/c$
Γ	width of excited state, decay rate
γ^μ	Dirac matrix Section 5.5
γ	$= (1 - \beta^2)^{-1/2}$
δ	Kobayashi–Maskawa phase Section 14.3
ϵ	polarisation unit vector Section 4.7
ε	helicity index
θ	boost parameter: $\tanh \theta = \beta$, $\cosh \theta = \gamma$ Section 2.1, phase angle, scattering angle, scalar potential Section 4.3, gauge parameter field Section 10.2
θ_w	Weinberg angle
Λ^{-1}	confinement length Section 16.3
Λ_{latt}	lattice parameter Section 17.1
λ_a	matrices associated with $SU(3)$ Section B.7
μ, μ_L, μ_R	muon Dirac, two-component left-handed, right-handed field
$\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$	electron neutrino, muon neutrino, tau neutrino field
Π	momentum density Section 3.3
ρ	electric charge density
$\rho(E)$	density of final states at energy E
Σ	spin operator acting on Dirac field Section 6.2
τ	mean life
τ, τ_L, τ_R	tau Dirac, two-component left-handed, right-handed field
Φ	complex scalar field Section 3.7

Cambridge University Press

978-0-521-85249-4 - An Introduction to the Standard Model of Particle Physics, Second Edition

W. N. Cottingham and D. A. Greenwood

Frontmatter

[More information](#)*Notation*

xix

ϕ	real scalar field Section 2.3, scalar potential Section 4.1, gauge parameter field Section 10.2
ϕ_0	vacuum expectation value of the Higgs field
χ	gauge parameter field Section 4.3, scalar field Section 10.3
ψ	four-component Dirac field
ψ_L, ψ_R	two-component left-handed, right-handed spinor field
$\bar{\psi}$	$\psi^\dagger \gamma^0$ Section 5.5
ω	frequency