AN INTRODUCTION TO THE STANDARD MODEL OF PARTICLE PHYSICS

Second Edition

The Standard Model of particle physics is the mathematical theory that describes the weak, electromagnetic and strong interactions between leptons and quarks, the basic particles of the Standard Model.

The new edition of this introductory graduate textbook provides a concise but accessible introduction to the Standard Model. It has been updated to account for the successes of the theory of strong interactions, and the observations on matter–antimatter asymmetry. It has become clear that neutrinos are not mass-less, and this book gives a coherent presentation of the phenomena and the theory that describes them. It includes an account of progress in the theory of strong interactions and of advances in neutrino physics. The book clearly develops the theoretical concepts from the electromagnetic and weak interactions of leptons and quarks to the strong interactions of quarks.

This textbook provides an up-to-date introduction to the Standard Model for graduate students in particle physics. Each chapter ends with problems, and hints to selected problems are provided at the end of the book. The mathematical treatments are suitable for graduates in physics, and more sophisticated mathematical ideas are developed in the text and appendices.

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Preface to the second edition

In the eight years since the first edition, the Standard Model has not been seriously discredited as a description of particle physics in the energy region (<2 TeV) so far explored. The principal discovery in particle physics since the first edition is that neutrinos carry mass. In this new edition we have added chapters that extend the formalism of the Standard Model to include neutrino fields with mass, and we consider also the possibility that neutrinos are Majorana particles rather than Dirac particles.

The Large Hadron Collider (LHC) is now under construction at CERN. It is expected that, at the energies that will become available for experiments at the LHC (∼20 TeV), the physics of the Higgs field will be elucidated, and we shall begin to see 'physics beyond the Standard Model'. Data from the 'B factories' will continue to accumulate and give greater understanding of CP violation. We are confident that interest in the Standard Model will be maintained for some time into the future.

Cambridge University Press have again been most helpful. We thank Miss V. K. Johnson for secretarial assistance. We are grateful to Professor Dr J. G. Körner for his corrections to the first edition, and to Professor C. Davies for her helpful correspondence.

Preface to the first edition

The 'Standard Model' of particle physics is the result of an immense experimental and inspired theoretical effort, spanning more than fifty years. This book is intended as a concise but accessible introduction to the elegant theoretical edifice of the Standard Model. With the planned construction of the Large Hadron Collider at CERN now agreed, the Standard Model will continue to be a vital and active subject.

The beauty and basic simplicity of the theory can be appreciated at a certain 'classical' level, treating the boson fields as true classical fields and the fermion fields as completely anticommuting. To make contact with experiment the theory must be quantised. Many of the calculations of the consequences of the theory are made in quantum perturbation theory. Those we present are for the most part to the lowest order of perturbation theory only, and do not have to be renormalised. Our account of renormalisation in Chapter 8 is descriptive, as is also our final Chapter 19 on the anomalies that are generated upon quantisation.

A full appreciation of the success and significance of the Standard Model requires an intimate knowledge of particle physics that goes far beyond what is usually taught in undergraduate courses, and cannot be conveyed in a short introduction. However, we attempt to give an overview of the intellectual achievement represented by the Model, and something of the excitement of its successes. In Chapter 1 we give a brief résumé of the physics of particles as it is qualitatively understood today. Later chapters developing the theory are interspersed with chapters on the experimental data. The amount of supporting data is immense and so we attempt to focus only on the most salient experimental results. Unless otherwise referenced, experimental values quoted are those recommended by the Particle Data Group (1996).

The mathematical background assumed is that usually acquired during an undergraduate physics course. In particular, a facility with the manipulations of matrix algebra is very necessary; Appendix A provides an *aide-mémoire*. Principles of symmetry play an important rôle in the construction of the model, and Appendix B is a self-contained account of the group theoretic ideas we use in describing these symmetries. The mathematics we require is not technically difficult, but the reader must accept a gradually more abstract formulation of physical theory than that presented at undergraduate level. Detailed derivations that would impair the flow of the text are often set as problems (and outline solutions to these are provided).

The book is based on lectures given to beginning graduate students at the University of Bristol, and is intended for use at this level and, perhaps, in part at least, at senior undergraduate level. It is not intended only for the dedicated particle physicist: we hope it may be read by physicists working in other fields who are interested in the present understanding of the ultimate constituents of matter.

We should like to thank the anonymous referees of Cambridge University Press for their useful comments on our proposals. The Department of Physics at Bristol has been generous in its encouragement of our work. Many colleagues, at Bristol and elsewhere, have contributed to our understanding of the subject. We are grateful to Mrs Victoria Parry for her careful and accurate work on the typescript, without which this book would never have appeared.

Notation

Position vectors in three-dimensional space are denoted by $\mathbf{r} = (x, y, z)$, or $\mathbf{x} = (x^1, x^2, x^3)$ where $x^1 = x, x^2 = y, x^3 = z$.

A general vector $\mathbf{a}$ has components (a^1, a^2, a^3), and $\hat{\mathbf{a}}$ denotes a unit vector in the direction of $\mathbf{a}$.

Volume elements in three-dimensional space are denoted by $\mathrm{d}^3\mathbf{x} = \mathrm{d}x\mathrm{d}y\mathrm{d}z = \mathrm{d}x^1\mathrm{d}x^2\mathrm{d}x^3$.

The coordinates of an event in four-dimensional time and space are denoted by $x = (x^0, x^1, x^2, x^3) = (x^0, \mathbf{x})$ where $x^0 = ct$.

Volume elements in four-dimensional time and space are denoted by $\mathrm{d}^4x = \mathrm{d}x^0\mathrm{d}x^1\mathrm{d}x^2\mathrm{d}x^3 = c\,\mathrm{d}t\,\mathrm{d}^3\mathbf{x}$.

Greek indices μ, ν, λ, ρ take on the values 0, 1, 2, 3.

Latin indices i, j, k, l take on the space values 1, 2, 3.

Pauli matrices

We denote by σ^μ the set $(\sigma^0, \sigma^1, \sigma^2, \sigma^3)$ and by $\tilde{\sigma}^\mu$ the set $(\sigma^0, -\sigma^1, -\sigma^2, -\sigma^3)$, where

$$\sigma^0 = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma^2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \ \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$(\sigma^1)^2 = (\sigma^2)^2 = (\sigma^3)^2 = \mathbf{I}; \quad \sigma^1\sigma^2 = \mathrm{i}\sigma^3 = -\sigma^2\sigma^1, \text{ etc.}$$

Chiral representation for γ-matrices

$$\gamma^0 = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{pmatrix}, \ \gamma^i = \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix},$$

$$\gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}.$$

Quantisation ($\hbar = c = 1$)

$$(E, \mathbf{p}) \to (\mathrm{i}\partial/\partial t, -\mathrm{i}\boldsymbol{\nabla}), \text{ or } p^{\mu} \to \mathrm{i}\partial^{\mu}.$$

For a particle carrying charge q in an external electromagnetic field,

$$(E, \mathbf{p}) \to (E - q\phi,\ \mathbf{p} - q\mathbf{A}), \text{ or } p^{\mu} \to p^{\mu} - qA^{\mu},$$
$$\mathrm{i}\partial^{\mu} \to (\mathrm{i}\partial^{\mu} - qA^{\mu}) = \mathrm{i}(\partial^{\mu} + \mathrm{i}qA^{\mu}).$$

Field definitions

$$Z_{\mu} = W_{\mu}{}^{3} \cos\theta_{\mathrm{w}} - B_{\mu} \sin\theta_{\mathrm{w}},$$
$$A_{\mu} = \mathrm{W}_{\mu}{}^{3} \sin\theta_{\mathrm{w}} + B_{\mu} \cos\theta_{\mathrm{w}},$$

where $\sin^2\theta_{\mathrm{w}} = 0.2315(4)$

$$g_2 \sin\theta_{\mathrm{w}} = g_1 \cos\theta_{\mathrm{w}} = e, \quad G_{\mathrm{F}} = g_2{}^2/(4\sqrt{2}M_{\mathrm{w}}{}^2).$$

Glossary of symbols

$\mathbf{A}$	electromagnetic vector potential Section 4.3
A^{μ}	electromagnetic four-vector potential
$A^{\mu\nu}$	field strength tensor Section 11.3
A_{FB}	forward–backward asymmetry Section 15.2
a	wave amplitude Section 3.5
$a, a^{\dagger}$	boson annihilation, creation operator
$\mathbf{B}$	magnetic field
B^{μ}	gauge field Section 11.1
$B^{\mu\nu}$	field strength tensor Section 11.2
$b, b^{\dagger}$	fermion annihilation, creation operator
$\mathbf{D}$	isospin doublet Section 16.6
$d, d^{\dagger}$	antifermion annihilation, creation operator
d_k	($k = 1,2,3$) down-type quark field
$\mathbf{E}$	electric field
E	energy
$e, e_{\mathrm{L}}, e_{\mathrm{R}}$	electron Dirac, two-component left-handed, right-handed field
$F^{\mu\nu}$	electromagnetic field strength tensor Section 4.1
f	radiative corrections factor Sections 15.1, 17.4
f_{abc}	structure constants of *SU(3)* Section B.7
G^{μ}	gluon matrix gauge field
$\mathrm{G}^{\mu\nu}$	gluon field strength tensor
G_{F}	Fermi constant Section 9.4

$g^{\mu\nu}$	metric tensor
g	strong coupling constant Section 16.1
g_1, g_2	electroweak coupling constants
H	Hamiltonian Section 3.1
$h(x)$	Higgs field
$\mathcal{H}$	Hamiltonian density Section 3.3
$\mathbf{I}$	isospin operator Sections 1.5, 16.6
$\mathbf{J}$	electric current density Section 4.1
$\mathbf{J}$	total angular momentum operator
J	Jarlskog constant Section 14.3
J^{μ}	lepton number current Section 12.4
$\mathbf{j}$	probability current Section 7.1
j^{μ}	lepton current Section 12.2
K	string tension Section 17.1
$\mathbf{k}$	wave vector
$\mathbf{L}$	lepton doublet Section 12.1
L	Lagrangian Section 3.1
$\mathcal{L}$	Lagrangian density Section 3.3
l^3	normalisation volume Section 3.5
$\mathbf{M}$	left-handed spinor transformation matrix Section B.6
M	proton mass Section D.1
m	mass
$\mathbf{N}$	right-handed spinor transformation matrix Section B.6
N	number operator Section C.1
$\hat{O}$	quantum operator
$\mathbf{P}$	total field momentum
$\mathbf{p}$	momentum
Q^2	$= -q_{\mu}q^{\mu}$
$\mathbf{q}$	quark colour triplet
q^{μ}	energy–momentum transfer
$\mathbf{R}$	rotation matrix Section B.2
$\mathbf{S}$	spin operator
S	action Section 3.1
s	square of centre of mass energy
T^{μ}_{ν}	energy–momentum tensor Section 3.6
$\mathbf{U}$	unitary matrix
u_k	$(k = 1, 2, 3)$ up-type quark field
u_L, u_R	two-component left-handed, right-handed spinors Section 6.1
u_+, u_-	Dirac spinors Section 6.3
$\mathbf{V}$	Kobayashi–Maskawa matrix Section 14.2

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V	normalisation volume
$\mathbf{v}$	velocity
v	$= \lvert\mathbf{v}\rvert$
$v_{\mathrm{L}}, v_{\mathrm{R}}$	two-component left-handed, right-handed spinors
v_+, v_-	Dirac spinors Section 6.4
$\mathbf{W}^{\mu}$	matrix of vector gauge field Section 11.1
$\mathrm{W}^{\mu\nu}$	field strength tensor Section 11.2
$W^1_\mu, W^2_\mu, W^+_\mu, W^-_\mu$	fields of W boson
Z_μ	field of Z boson
$\alpha(Q^2)$	effective fine structure constant Section 16.3
$\alpha_s(Q^2)$	effective strong coupling constant Section 16.3
α_{latt}	lattice coupling constant Section 17.1
α^i	Dirac matrix Section 5.1
β	Dirac matrix Section 5.1
β	$= v/c$
Γ	width of excited state, decay rate
γ^μ	Dirac matrix Section 5.5
γ	$= (1 - \beta^2)^{-1/2}$
δ	Kobayashi–Maskawa phase Section 14.3
$\boldsymbol{\varepsilon}$	polarisation unit vector Section 4.7
ε	helicity index
θ	boost parameter: $\tanh\theta = \beta,\ \cosh\theta = \gamma$ Section 2.1, phase angle, scattering angle, scalar potential Section 4.3, gauge parameter field Section 10.2
θ_{w}	Weinberg angle
Λ^{-1}	confinement length Section 16.3
Λ_{latt}	lattice parameter Section 17.1
λ_a	matrices associated with $SU(3)$ Section B.7
$\mu,\ \mu_{\mathrm{L}}, \mu_{\mathrm{R}}$	muon Dirac, two-component left-handed, right-handed field
$\nu_{e\mathrm{L}}, \nu_{\mu\mathrm{L}}, \nu_{\tau\mathrm{L}}$	electron neutrino, muon neutrino, tau neutrino field
Π	momentum density Section 3.3
ρ	electric charge density
$\rho(E)$	density of final states at energy E
Σ	spin operator acting on Dirac field Section 6.2
τ	mean life
$\tau,\ \tau_{\mathrm{L}}, \tau_{\mathrm{R}}$	tau Dirac, two-component left-handed, right-handed field
Φ	complex scalar field Section 3.7

ϕ	real scalar field Section 2.3, scalar potential Section 4.1, gauge parameter field Section 10.2
ϕ_0	vacuum expectation value of the Higgs field
χ	gauge parameter field Section 4.3, scalar field Section 10.3
ψ	four-component Dirac field
ψ_L, ψ_R	two-component left-handed, right-handed spinor field
$\bar{\psi}$	$\psi^\dagger\gamma^o$ Section 5.5
ω	frequency