1 Introduction

1.1 Overview

The aim of this book is to introduce readers to modern methods of modeling and analyzing rotating machines to determine their dynamic behavior. This is usually referred to as *rotordynamics*. The text is suitable for final-year undergraduates, postgraduates, and practicing engineers who require both an understanding of modern techniques used to model and analyze rotating systems and an ability to interpret the results of such analyses.

Before presenting a text on the dynamics of rotating machines, it is appropriate to consider why one would wish to study this subject. Apart from academic interest, it is an important practical subject in industry, despite the forbidding appearance of some of the mathematics used. There are two important application areas for the techniques found in the following pages. First, when designing the rotating parts of a machine, it is clearly necessary to consider their dynamic characteristics. It is crucial that the design of a machine is such that while running up to and functioning at its operating speed(s), vibration does not exceed safe and acceptable levels. An unacceptably high level of rotor vibration can cause excessive wear on bearings and may cause seals to fail. Blades on a rotor may come into contact with the stationary housing with disastrous results. An unacceptable level of vibration might be transmitted to the supporting structure and high levels of vibration could generate an excessive noise level. The second aspect of importance is the understanding of a machine's behavior when circumstances change, implying that a fault has occurred in the rotating parts of the machine. This understanding is needed for the diagnosis of the fault and for the formulation of repair strategies involving important decisions, such as "Is it safe to run?" and "How long can it be run?" These questions concern personnel and machine safety, legal issues, and, in many cases, very large sums of money.

After accepting that the dynamics of rotors is an important subject worthy of study, it is necessary to explain why a book on this topic is required. Are not the dynamics of rotors simply particular cases of the more general dynamics of structures? In fact, whereas in many respects, a rotor system behaves dynamically like a fixed structure, there are some important differences because of the rotation. The most fundamental of these differences is that fixed structures do not have inherent forcing, whereas rotating machines do. The following is a list of some, but by no means all, of the phenomena unique to rotating systems:

- When a rotor spins, lateral forces and moments may be generated. These so-called unbalance forces and moments are always present due to limitations in machining and assembly accuracy. These forces and moments give rise to vibration at the same frequency as the rotational speed.
- Gyroscopic moments also act on the spinning rotor and cause its natural frequencies to change with rotational speed.
- The stiffness and damping properties of some types of bearings vary with rotor speed; these changes also influence the system's natural frequencies.
- Centrifugal forces acting on a blade attached to a spinning rotor cause the blade stiffness to increase with rotational speed.
- Errors in gear profiles generate forces on the rotor, which are generated by the imposed motion introduced by geometric errors.
- Not all rotors are perfectly symmetric, and even minor asymmetries can have significant effects. When an asymmetric rotor spins, its stiffness changes periodically at the rotational speed or a multiple thereof, when viewed in a fixed set of coordinates. This can cause instability.
- The damping in rotating machines, in some circumstances, may be relatively high compared with that found in most fixed structures. In other circumstances, the effective damping may be negative, causing instability.

Thus, there is a variety of respects in which the analysis of rotating machines differs from that of a normal fixed structure. Study of the behavior of such systems is the science of rotordynamics and, whereas great reliance is placed on techniques of analysis from structural dynamics, rotordynamic analysis represents a considerable extension in scope and complexity. Given the wide application of rotating machinery, the field is one of great practical importance.

1.2 Rotating Machine Components

Any attempt to describe a typical rotating machine is inevitably fraught with difficulty in view of the wide range of machine sizes, duties, and speeds. Nevertheless, the techniques of modeling and analysis described in the following chapters can be applied to a wide variety of machines. The methods are equally applicable to small motors with rotors of, for instance, 40 mm in length with a mass of 0.04 kg (or smaller), or to turbo-generators, in which a typical large 660 MW machine has a total length of 50 m and a mass of 250,000 kg.

Every rotating machine consists of three principal components – namely, the rotor, the bearings, and their supporting structure. Some type of rationalization may be reached by considering the features that are pertinent to each component, which has its own properties that range enormously in complexity. Despite this, however, there are some generic methods to aid analysis that can be applied to gain insight into the behavior of a machine. It is important to emphasize this concept of insight: It is frequently more important to gain an understanding of a machine's behavior than it is to calculate precise numerical values of its response. Very often, the level of forces in service are unknown or known only approximately; therefore, detailed response calculations cannot be performed. Nevertheless, an understanding of the behavior in such circumstances can lead an analyst to propose appropriate courses of action to ensure the safe operation of a machine. We now consider each component in turn.

1.2.1 Features of Rotors

The rotor is at the heart of any rotating machine; therefore, the first part of any analysis must address the dynamic properties of the rotating element. In most cases, the rotor is relatively simple insofar as it often can be represented as a single beam or as a series of beam elements and rigid disks. The rotor, of course, may be complicated by numerous changes of section and the need for some treatment of effects, such as the shrink-fitting of disks. However, because the rotor is conceptually simple, it is a straightforward matter to check the accuracy of the rotor model by comparing the predicted behavior of the rotor alone with measured data, for example, by performing an impulse test on the rotor freely suspended.

The rotor of a small machine is usually rigid but, with increasing machine size, it generally becomes more flexible, and this must be accounted for in the analysis. However, there are exceptions. For example, the large electrical motors that drive rolling mills have rotors that are essentially rigid.

In many machines, the lateral stiffness of the rotor is the same in all planes containing the axis of rotation; in other cases, this is not so. For example, the rotors of many two-pole electrical machines have a lateral stiffness in one plane that is lower than in a perpendicular plane because of groups of slots that are cut into the rotor to carry the electrical conductors. Another feature of rotors is that they sometimes carry components or subassemblies with dynamic characteristics of their own. For example, a helicopter rotor consists of a relatively short, rigid vertical shaft carrying three very flexible blades. Finally, a rotor may have internal damping; contrary to intuition, internal damping can cause instability.

1.2.2 Features of Bearings and Rotor–Stator Interactions

The rotor is connected to the supporting structure by means of bearings, which may be of several types. For small machines with light loads, the bearing may take the form of a simple bush in which the rotor runs. However, as bearing loads increase, such a simple arrangement becomes inadequate; rotors are then mounted on ball or rolling-element bearings. These provide greater load capacity and stiffness but because they have internal moving components, they can make significant contributions to the dynamics of the overall machine. For large, heavy machines, such as turbo-generators, the bearings are almost invariably of the journal type, of which there are several forms. In a journal, or hydrodynamic, bearing, there is a film of oil in a small clearance between the static and rotating elements. The rotor creates a hydrodynamic pressure distribution within the oil film, which supports the weight and unbalance forces of the rotor.

In the past two decades, magnetic bearings have been introduced for some machine types. In these bearings, the rotor and stator are held apart by a magnetic field. The advantage of this system is the complete elimination of contact and the consequent very low values of effective friction; the disadvantage is the need to generate the magnetic field necessary to support the rotor.

Although the discussion so far has focused implicitly on the bearings that locate the rotor laterally, similar considerations are made in the selection of thrust bearings, which locate the rotor axially. Many machines – including jet engines, pumps, and turbines – generate considerable axial thrust; indeed, in the case of jet engines, this is their principal purpose. The function of the thrust bearing, therefore, is to react this force to the casing structure and to maintain the relative position of the casing and the rotor. This is clearly crucial in determining the axial dynamics of the system. It can also have substantial influence on the lateral vibrations by contributing rotational stiffness terms. The design and use of each bearing type is a substantial topic in its own right and is described extensively in the literature. In this book, bearings are treated simply as a local rotor–stator interaction with their own damping and stiffness properties; however, in some cases, these properties are speed-dependent. Using these properties, methods are provided to predict the behavior of a rotor under a variety of conditions.

The bearing is the most obvious device that couples the moving part of a machine (i.e., the rotor) to the stationary parts (i.e., the foundations, or the stator). However, there are other forms of interaction. Seals and glands also cause forces between the rotor and the stator. In steam and water turbines, the steam and water act on both the moving and fixed parts of the machine, thereby coupling them. Finally, in electrical machines, magnetic forces between the rotor and stator play a similar role in coupling the stator and rotor.

1.2.3 Stators and Foundations

The last component to consider is the structure supporting the bearings. In general, this structure consists of two major components: the *stator* and the *foundation*. For the purposes of this book, the stator is an integral part of the machine and does not rotate. In contrast, the foundation is the supporting structure and serves only to hold the machine in place. When considering the dynamics of the rotor, there is no distinction between the stator and the foundation. However, we make the distinction because we may want to know the forces transmitted by the machine to the foundations. In situations in which these forces are not of interest, we may refer to the combined stator and foundation as the foundation.

Although it is commonly thought that the modeling of the bearings represents the greatest challenge in the description of machinery, in many cases, there is an even more difficult problem. Although small machines may be supported on simple structures, which are relatively stiff compared to the properties of the machine, many large turbo-machines are mounted on complex structures that are relatively flexible. In such cases, the properties of the foundation have a significant influence on the dynamics of the complete machine and, therefore, should be represented within the model of the complete unit. This may be difficult due to problems in determining the parameters of the supporting structure model. In the case of a large turbine, the supporting foundation may be a complex structure consisting of many components and welded joints, to which a number of items of ancillary equipment are connected. It has been found that due to build tolerances, even machines and structures built to the same design show significant differences in vibration behavior. In practice, the approach frequently used is simply to select parameters that give adequate agreement with experimental data. In addition, the foundations can couple the horizontal and vertical motion, and they might introduce coupling between the bearing supports.

1.3 Aspects of Rotating Machine Behavior

In discussing the dynamic behavior of rotating machinery throughout this book, considerable reliance is placed on mathematical descriptions of the physical processes involved. Before embarking on this discussion, however, it is worth reflecting on the motion of a machine rotor. A rotor can vibrate in three ways: axially, torsionally, and laterally. *Axial vibrations* occur along the axis of the rotor, and *torsional vibrations* cause the rotor to twist about its axis of rotation. *Lateral vibrations* cause displacements of the rotor in both the horizontal and vertical directions (for a horizontal rotor). These motions combine to produce an orbital motion of the rotor in a plane perpendicular to its axis of rotation. In some systems, the three types of vibration are independent of one another, and they can be analyzed separately. However, in other systems, there is coupling among all three forms of vibration (see Chapter 10).

The analysis and, indeed, the manner in which a machine performs in service are determined largely by whether the rotors are rigid or flexible, the type of bearings used, and the nature of the supporting structure. This is true whatever the size or complexity of the machine. In all machines, there are features that must be modeled; however, in individual instances, it may be possible to demonstrate that they have a small or negligible influence on the dynamics of the overall system. In these instances, detailed modeling is not required; however, this cannot be assumed without investigation.

1.3.1 Lateral Vibrations

Rotor lateral vibration (sometimes called *transverse* or *flexural vibration*) is perpendicular to the axis of the rotor and is the largest vibration component in most high-speed machinery. Understanding and controlling this lateral vibration is important because excessive lateral vibration leads to bearing wear and, ultimately, failure. In extreme cases, lateral vibration also can cause the rotating parts of a machine to come into contact with stationary parts, with potentially disastrous consequences.

Lateral vibration is generally caused by lateral forces, the most common of which are unbalance forces that are present in all rotating machines, despite efforts to minimize or eliminate them. In subsequent chapters, we discuss the effects of rotor unbalance and the methods for balancing real machines, but the balance will never be perfect.

As in all elastic systems, a machine has natural frequencies of lateral vibration determined by the lateral stiffness and mass distribution of the rotor–bearing–foundation system. When the rotational speed – and, hence, the frequency of the out-of-balance forces – is equal to any of these natural frequencies, the vibration response becomes large and the rotor is considered to be rotating at a critical speed. When a machine is accelerated from rest to its operating speed, it might have to pass through one or more of these critical speeds. For most classes of machine, it is important that it is not permitted to operate at or close to a critical speed for any length of time.

Because the rotor can vibrate laterally in two mutually perpendicular directions, the vibrations combine to create an orbit for the rotor motion. If the supporting structure of the bearings of a horizontal rotor has identical stiffness and damping properties in both the horizontal and vertical directions, then this orbit is circular and the bending stresses in the rotor are constant. In many instances, however, the structure supporting the bearings is stiffer in the vertical than in the horizontal direction. In such a situation, the rotor orbit is elliptical and the bending stress in the rotor varies at twice the rotational speed.

In the discussion thus far, the role of dissipative or damping forces on the motion has not been mentioned. As in structural dynamics, damping has a major influence close to the resonant frequencies. Although it might be anticipated that damping always tends to reduce vibration, this is not always the case. If the damping forces arise in the supporting structure, then the effects are invariably beneficial and may be treated in much the same way as damping in any structural system. Problems arise, however, when there is damping in the rotor itself. Far from being beneficial, this type of damping can be destabilizing.

1.3.2 Axial Vibrations

The ultimate function of a jet engine is to produce thrust in the axial direction. A thrust bearing must be fitted to transmit this thrust to the housing and, hence, the aircraft to which it is attached. Without this thrust bearing, the rotor would simply be propelled away from the engine housing and, therefore, would be ineffective! Of course, there is some time-varying fluctuation about the mean level of thrust, which gives rise to axial vibrations of the rotor, with this motion having its own set of resonance frequencies. In contrast to the lateral motion of the rotor, stresses arising from axial vibration are uniform across a complete cross-section of the rotor. There may be cross-coupling between axial and lateral vibrations – for example, in helical and bevel gear meshes.

1.3.3 Torsional Vibrations

The third type of vibration is torsional vibration, or a twisting motion of the rotor about its own axis. In some respects, this is relatively straight-forward to model because bearings and supporting structures have little influence on the natural frequencies. There is also a practical problem: lateral and, to a lesser extent, axial vibrations become obvious by their effects on the machine and its surroundings, enabling the deployment of appropriate effort to resolve developing problems. In complete contrast, torsional problems can go unnoticed without special instrumentation. Furthermore, because little motion is transmitted to components other than the rotor, torsional modes often have low damping. During this undetected phase, however, considerable damage may be caused to a machine.

Figure 1.1. The quietrevolution QR5 vertical-axis wind turbine. This photograph is reproduced with the permission of quietrevolution ltd, copyright ©quietrevolution ltd 2009.

1.4 Examples of Rotating Machines

Rotating machines can vary enormously in size, complexity, and general configuration. The basic rotor–bearing–foundation system can be found in numerous products and systems ranging from small electrical motors in refrigerators and washing machines to turbo-generator sets. The list of examples is almost endless but includes centrifuges and vacuum pumps running at up to 90,000 rev/min, gyroscopes, machine spindles, helicopters, reciprocating gasoline and diesel engines, rotary and reciprocating compressors, and gas and steam turbines of all sizes. Figure 1.1 shows a typical vertical-axis wind turbine and Figure 1.2 shows an example of a turbocharger. In the following sections, we provide a short description of five types of rotating machines, including two of the largest and most powerful: namely, turbo-generators and aircraft gas turbines.

1.4.1 Electrical Machines

Rotating electrical machines convert mechanical power to electrical power or vice versa. These are the most numerous of all rotating machines, with several tens of them being included in every modern car (hundreds in luxury cars) and similar numbers in most modern households. Depending on which way the power conversion is made, the machines are described as motors or generators. The flexibility and control offered by electrical power transmission is such that most prime-movers drive electrical machines and most loads are driven by electrical machines. In many cases, electrical motors are integrated with a load to form more complex machines, such as the integrated motor and compressor shown in Figure 1.3; it is meaningless in

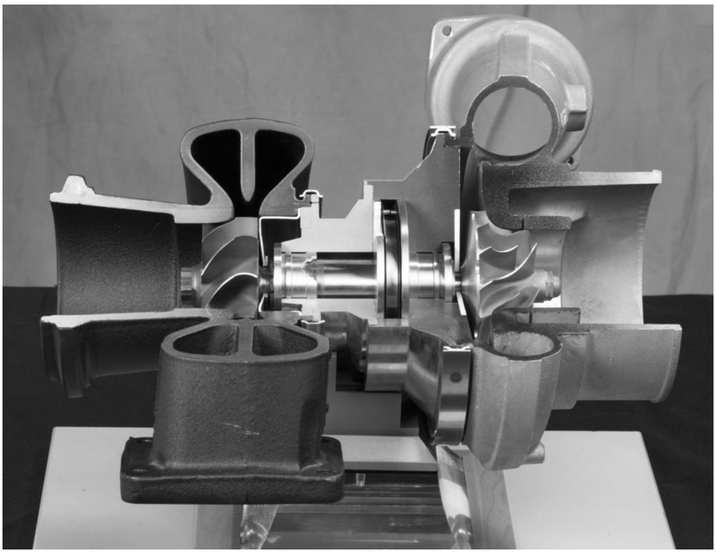

Figure 1.2. A cutaway view of a turbocharger. Photograph courtesy of NASA.

these cases to discuss the rotordynamics of the electrical motor in its own right. This integration is most common at small scales, and one example (i.e., vacuum pumps) is discussed in Section 1.4.4. At larger scales, electrical machines tend to be separate items coupled to the load or prime-mover only through a shaft-coupling and, indirectly, through the stator mountings. One example concerns turbo-generators, which are discussed in Section 1.4.2. When the electrical machine rotor is coupled rigidly to a load or prime-mover, the rotordynamics of the combination must be considered as a whole. Often, however, flexible couplings are provided between the

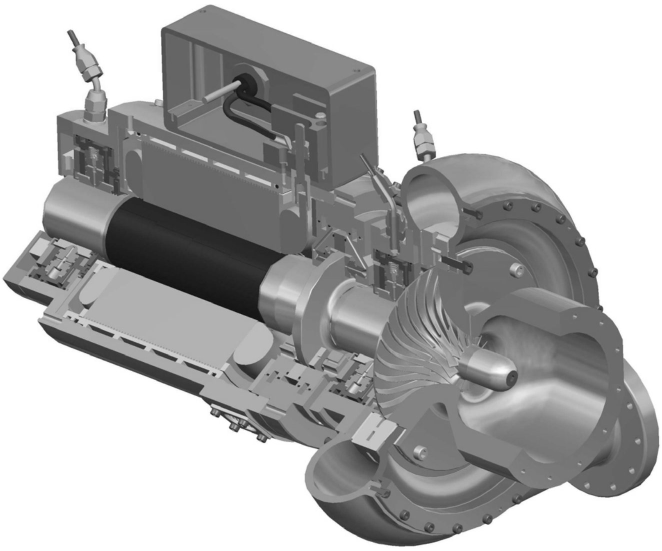

Figure 1.3. Schematic of an integrated motor and compressor. This image is reproduced with the permission of Corac Group plc, copyright ©Corac Group plc 2009.

electrical machine and its load or prime-mover; the lateral and axial dynamic behavior of the rotating machine then can be considered reasonably accurately in isolation. Rotating electrical machines present at least two specific interesting features with regard to their dynamics: (1) the rotors are often laminated and/or contain insulated conductors, which gives rise to uncertainty about the rotor stiffness and to rotor damping; and (2) electrical machines experience unbalanced magnetic pull (UMP), which can significantly influence the rotordynamics (see Chapters 5 and 7). Certain electrical machines can themselves produce strong torsional forcing and, in the presence of significant static eccentricity, most alternating current (AC) motors produce strong transverse forcing at twice the frequency of the electrical supply.

Electrical machines come in a vast range of sizes and types spanning submillimeter-scale machines delivering fractions of a Watt to machines delivering hundreds of MW. Torques vary from a few mNm in true micro scale machines (usually electrostatic rather than electromagnetic) to several MNm (e.g., for ship-propulsion machines). The types of electrical machine commonly used at present include the following:

- *Direct current (DC) machines*. These machines, which still dominate automotive applications, have brushes to conduct current onto the rotating part. The most efficient and most expensive DC machines include permanent magnets to provide the main magnetic field.
- *Universal machines*. These are essentially DC motors that also can work from AC electricity at any frequency. Universal machines dominate the white-goods market.
- *Induction motors*. These are inexpensive and robust and the most common industrial electrical machine (approximately 35 percent of all electricity generated in Europe in 2000 was consumed in induction machines).
- *Permanent-magnet synchronous machines*. Permanent-magnet synchronous machines often are used as servo-motors when very accurate and high-bandwidth control of rotor angular position is critical. Applications include packaging and printing machinery; specialist manufacturing machines for folding, cutting, and stitching; and so forth. Permanent-magnet machines are also commonly used in cordless power tools because of their high efficiency. They are usually used with a power-electronic drive that requires a sensor to establish rotor angular position.
- *Wound-rotor synchronous machines*. Large generators are almost invariably of this construction because it enables separate control over the phase relationships between voltage and current.
- *Stepper-motors*. These are used in many lower-grade servo-control applications where open-loop control is acceptable. Stepper-motors are particularly common in printers and photocopiers because they can achieve position control with acceptably small error and low cost.
- *Switched-reluctance machines*. These machines have no single major established niche at present but they compete with induction and permanent-magnet machines in numerous applications. Switched-reluctance machines are often more efficient than their induction-machine counterparts because there are no currents on the rotor; however, they are reputed to be much noisier. Like

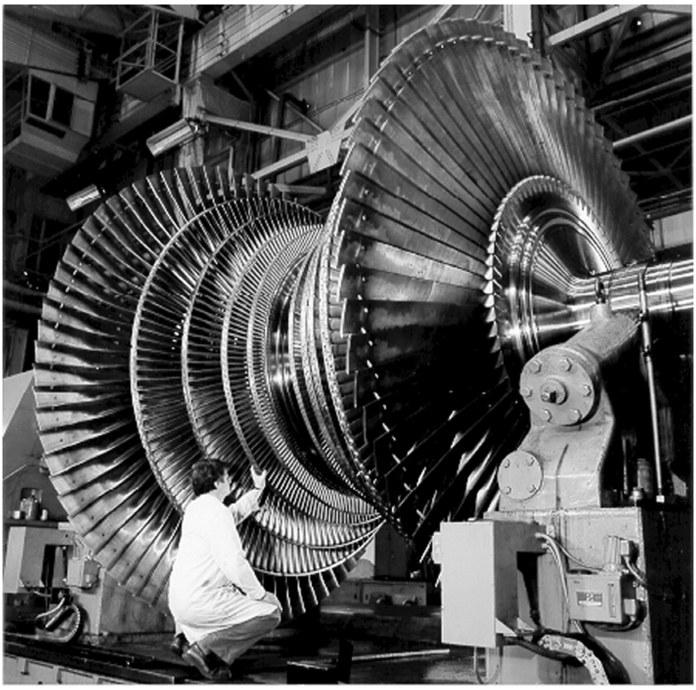

Figure 1.4. A typical steam-turbine rotor. This photograph is reproduced with the permission of Alstom, copyright ©Alstom 2009.

permanent-magnet synchronous machines, they require some sensing provision to detect rotor angular position. However, they offer an advantage over permanent-magnet machines in that they do not generate high voltages if the machine rotates quickly and if, for any reason, the stator circuits are open-circuit.

1.4.2 Turbo-Generator Sets

Turbo-generator sets, used in the generation of electrical power, are examples of large and complex rotating machines. Figure 1.4 shows a typical rotor of one of these machines. The total system typically consists of up to eight individual rotors coupled together, resulting in a typical total rotor length of 50 m, and supported on 16 oil-film journal bearings. The rotational speed is 3,000 rev/min (3,600 rev/min in the United States) and the power output is typically 660 MW, but there are machines of 1,300 MW. It is worth noting that for a 660 MW machine, the mass of the composite rotor is about 250 tonnes. This implies that if the mass axis center of gravity is displaced from the geometric center by 25 μm, then the forces exerted at each bearing are of the order of 44 kN. The bearings are supported on a foundation consisting of a steel or concrete structure, which – although appearing massive and rigid – is, in fact, often relatively flexible. In addition to unbalance, many different forces act on these machines. Steam forces act on the turbine rotors and an unbalanced magnetic force acts on the generator. If the rotor suffers any type of deterioration in service (e.g., a bend or rubbing against its housing), then new forces arise that influence the dynamic behavior of the machine. Nonuniform heating can result from