

## Contents

<i>Preface</i>	<i>page xi</i>
<b>Part 1 Fundamentals of Rotating Fluids</b>	<b>1</b>
1 Basic Concepts and Equations for Rotating Fluids	3
1.1 Introduction	3
1.2 Equations of Motion in Rotating Systems	4
1.3 The Heat Equation	6
1.4 The Boussinesq Equations	7
1.5 The Kinetic Energy Equation	10
1.6 Taylor–Proudman Theorem and Thermal Wind Equation	11
1.7 A Unified Approach	13
<b>Part 2 Inertial Waves in Uniformly Rotating Systems</b>	<b>15</b>
2 Introduction	17
2.1 Formulation	17
2.2 Frequency Bound $ \sigma  \leq 1$	19
2.3 Special Cases: $\sigma = 0$ and $\sigma = \pm 1$	21
2.4 Orthogonality	23
2.5 The Poincaré Equation	25
3 Inertial Modes in Rotating Narrow-gap Annuli	27
3.1 Formulation	27
3.2 Axisymmetric Inertial Oscillations	29
3.3 Geostrophic Mode	31
3.4 Non-axisymmetric Inertial Waves	32
4 Inertial Modes in Rotating Cylinders	35
4.1 Formulation	35
4.2 Axisymmetric Inertial Oscillations	36
4.3 Geostrophic Mode	41
4.4 Non-axisymmetric Inertial Waves	43
5 Inertial Modes in Rotating Spheres	50
5.1 Formulation	50

vi	<i>Contents</i>	
	5.2 Geostrophic Mode	52
	5.3 Equatorially Symmetric Modes: $m = 0$	54
	5.4 Equatorially Symmetric Modes: $m \geq 1$	60
	5.5 Equatorially Antisymmetric Modes: $m = 0$	71
	5.6 Equatorially Antisymmetric Modes: $m \geq 1$	75
	5.7 An Exact Nonlinear Solution in Rotating Spheres	81
6	Inertial Modes in Rotating Oblate Spheroids	83
	6.1 Formulation	83
	6.2 Geostrophic Mode	91
	6.3 Equatorially Symmetric Modes: $m = 0$	92
	6.4 Equatorially Symmetric Modes: $m \geq 1$	94
	6.5 Equatorially Antisymmetric Modes: $m = 0$	96
	6.6 Equatorially Antisymmetric Modes: $m \geq 1$	99
	6.7 An Exact Nonlinear Solution in Rotating Spheroids	102
7	A Proof of Completeness of Inertial Modes in Rotating Channels	105
	7.1 Significance of the Completeness of Inertial Modes	105
	7.2 Bessel's Inequality and Parseval's Equality	107
	7.3 A Proof of the Completeness Relation	109
8	Indications of Completeness of Inertial Modes in Rotating Spheres	118
	8.1 Seeking Signs of Completeness	118
	8.2 A Proof of the Vanishing Dissipation-type Integral	119
<b>Part 3</b>	<b>Precession and Libration in Non-uniformly Rotating Systems</b>	127
9	Introduction	129
	9.1 Non-uniform Rotation: Precession and Libration	129
	9.2 Precession/Libration in Different Geometries	130
	9.3 Key Parameters and Reference Frames	134
	9.4 Asymptotic Expansion Without Using $\sqrt{Ek}$	135
10	Fluid Motion in Precessing Narrow-gap Annuli	138
	10.1 Formulation	138
	10.2 Conditions for Resonance	141
	10.3 Asymptotic Solution at Resonance with $\Gamma = \sqrt{3}$	142
	10.4 Asymptotic Solution at Resonance with $\Gamma = 1/\sqrt{3}$	152
	10.5 Linear Numerical Analysis	155
	10.6 Nonlinear Direct Numerical Simulation	157
	10.7 Comparison: Analytical vs. Numerical	159
	10.8 A Byproduct: The Viscous Decay Factor	160
11	Fluid Motion in Precessing Circular Cylinders	164
	11.1 Formulation	164
	11.2 Conditions for Resonance	166
	11.3 Divergence of the Inviscid Precessing Solution	168
	11.4 General Asymptotic Solution for $0 < Ek \ll 1$	172

<i>Contents</i>		vii
11.5	Asymptotic Solution at Primary Resonances	180
11.6	Linear Numerical Analysis Using Spectral Methods	187
11.7	Nonlinear Properties of Weakly Precessing Flow	190
11.8	Numerical Simulation Using Finite Element Methods	193
11.9	Nonlinear Precessing Flow at Primary Resonances	195
11.9.1	Decomposition of Nonlinear Flow into Inertial Modes	195
11.9.2	The Structure of Nonlinear Precessing Flow	199
11.9.3	Search for Triadic Resonance	205
11.10	A Byproduct: The Viscous Decay Factor	209
12	Fluid Motion in Precessing Spheres	213
12.1	Formulation	213
12.2	Asymptotic Expansion and Resonance	215
12.3	Asymptotic Solution	217
12.4	Nonlinear Direct Numerical Simulation	224
12.5	Comparison: Analytical vs. Numerical	226
12.6	Nonlinear Effects: Mean Azimuthal Flow	227
12.7	A Byproduct: The Viscous Decay Factor	229
13	Fluid Motion in Longitudinally Librating Spheres	231
13.1	Formulation	231
13.2	Asymptotic Solutions	232
13.2.1	Why Resonance Cannot Occur	232
13.2.2	Asymptotic Analysis	233
13.2.3	Three Fundamental Modes Excited	239
13.3	Linear Numerical Solution	244
13.4	Nonlinear Direct Numerical Simulation	246
14	Fluid Motion in Precessing Oblate Spheroids	250
14.1	Formulation	250
14.2	Inviscid Solution	252
14.3	Exact Nonlinear Solution	258
14.4	Viscous Solution	260
14.5	Properties of Nonlinear Precessing Flow	268
14.6	A Byproduct: The Viscous Decay Factor	273
15	Fluid Motion in Latitudinally Librating Spheroids	276
15.1	Formulation	276
15.2	Analytical Solution: Non-resonant Librating Flow	279
15.3	Analytical Solution: Resonant Librating Flow	283
15.4	Nonlinear Direct Numerical Simulation	293
15.5	Comparison: Analytical vs. Numerical	293
<b>Part 4</b>	<b>Convection in Uniformly Rotating Systems</b>	<b>297</b>
16	Introduction	299
16.1	Rotating Convection vs. Precession/Libration	299

viii	<i>Contents</i>	
16.2	Key Parameters for Rotating Convection	300
16.3	Rotational Constraint on Convection	302
16.4	Types of Rotating Convection	303
16.4.1	Viscous Convection Mode	303
16.4.2	Inertial Convection Mode	305
16.4.3	Transitional Convection Mode	306
16.5	Convection in Various Rotating Geometries	307
16.5.1	Rotating Annular Channels	307
16.5.2	Rotating Circular Cylinders	308
16.5.3	Rotating Spheres or Spherical Shells	309
17	Convection in Rotating Narrow-gap Annuli	313
17.1	Formulation	313
17.2	A Finite-difference Method for Nonlinear Convection	316
17.3	Stationary Viscous Convection	318
17.3.1	Governing Equations	318
17.3.2	Asymptotic Solution for $\Gamma(Ta)^{1/6} \ll O(1)$	320
17.3.3	Asymptotic Solution for $\Gamma(Ta)^{1/6} = O(1)$	325
17.3.4	Numerical Solution Using a Galerkin-tau Method	327
17.3.5	Comparison: Analytical vs. Numerical	329
17.3.6	Nonlinear Properties of Stationary Convection	330
17.4	Oscillatory Viscous Convection	332
17.4.1	Governing Equations	332
17.4.2	Symmetry between Two Different Oscillatory Solutions	334
17.4.3	Asymptotic Solutions Satisfying the Boundary Condition	335
17.4.4	Comparison: Analytical vs. Numerical	343
17.4.5	Comparison with an Unbounded Rotating Layer	348
17.4.6	Nonlinear Properties with $\Gamma = O(Ta^{-1/6})$	352
17.4.7	Nonlinear Properties with $\Gamma \gg O(Ta^{-1/6})$	354
17.5	Viscous Convection with Curvature Effects	356
17.5.1	Onset of Viscous Convection	356
17.5.2	Nonlinear Properties of Viscous Convection	359
17.6	Inertial Convection: Non-axisymmetric Solutions	366
17.6.1	Asymptotic Expansion	366
17.6.2	Non-dissipative Thermal Inertial Wave	367
17.6.3	Asymptotic Solution with Stress-free Condition	369
17.6.4	Asymptotic Solution with No-slip Condition	373
17.6.5	Numerical Solution Using a Galerkin Spectral Method	383
17.6.6	Comparison: Analytical vs. Numerical	385
17.6.7	Nonlinear Properties of Inertial Convection	386
17.7	Inertial Convection: Axisymmetric Torsional Oscillation	393

18	Convection in Rotating Cylinders	396
18.1	Formulation	396
18.2	Convection with Stress-free Condition	399
18.2.1	Asymptotic Solution for Inertial Convection	399
18.2.2	Asymptotic Solution for Viscous Convection	405
18.2.3	Numerical Solution Using a Chebyshev-tau Method	408
18.2.4	Comparison: Analytical vs. Numerical	410
18.3	Convection with No-slip Condition	412
18.3.1	Asymptotic Solution for Inertial Convection	412
18.3.2	Asymptotic Solution for Viscous Convection	418
18.3.3	Numerical Solution Using a Galerkin-type Method	419
18.3.4	Comparison: Analytical vs. Numerical	421
18.3.5	Effect of Thermal Boundary Condition	424
18.3.6	Axisymmetric Inertial Convection	426
18.4	Transition to Weakly Turbulent Convection	430
18.4.1	A Finite Element Method for Nonlinear Convection	430
18.4.2	Inertial Convection: From Single Inertial Mode to Weak Turbulence	431
18.4.3	Viscous Convection: From Sidewall-localized Mode to Weak Turbulence	435
19	Convection in Rotating Spheres or Spherical Shells	439
19.1	Formulation	439
19.2	Numerical Solution using Toroidal/Poloidal Decomposition	442
19.2.1	Governing Equations under Toroidal/Poloidal Decomposition	442
19.2.2	Numerical Analysis for Stress-free or No-slip Condition	444
19.2.3	Several Numerical Solutions for $0 < Ek \ll 1$	447
19.2.4	Nonlinear Effects: Differential Rotation	452
19.3	Local Asymptotic Solution: A Small-gap Annular Model	459
19.3.1	The Local and Quasi-geostrophic Approximation	459
19.3.2	Asymptotic Relation for $0 < Ek \ll 1$	461
19.3.3	Comparison: Asymptotic vs. Numerical	463
19.4	Global Asymptotic Solution with Stress-free Condition	464
19.4.1	Hypotheses for Asymptotic Analysis	464
19.4.2	Asymptotic Analysis for Inertial Convection	465
19.4.3	Several Analytical Solutions for Inertial Convection	470
19.4.4	Differential Rotation Cannot be Sustained by Inertial Convection	474
19.4.5	Asymptotic Analysis for Viscous Convection	476
19.4.6	Typical Asymptotic Solutions for Viscous Convection	479
19.4.7	Nonlinear Effects: Differential Rotation in Viscous Convection	481
19.5	Global Asymptotic Solution with No-slip Condition	484
19.5.1	Hypotheses for Asymptotic Analysis	484
19.5.2	Asymptotic Analysis for Inertial Convection	485

19.5.3	Several Analytical Solutions for Inertial Convection	490
19.5.4	Asymptotic Analysis for Viscous Convection	493
19.5.5	Several Asymptotic Solutions for Viscous Convection	496
19.5.6	Nonlinear Effects: Differential Rotation in Viscous Convection	497
19.6	Transition to Weakly Turbulent Convection	503
19.6.1	A Finite-Element Method for Rotating Spheres	503
19.6.2	Transition to Weak Turbulence in Rotating Spheres	504
19.6.3	A Finite Difference Method for Rotating Spherical Shells	508
19.6.4	Multiple Stable Nonlinear Equilibria in Slowly Rotating Thin Spherical Shells	509
	Appendix A Vector Identities and Theorems	513
	Appendix B Vector Definitions	514
	<i>References</i>	516
	<i>Index</i>	523