THEORY AND MODELING OF ROTATING FLUIDS

A systematic account of the theory and modeling of rotating fluids that highlights the remarkable advances in the area and brings researchers and postgraduate students in atmospheres, oceanography, geophysics, astrophysics and engineering to the frontiers of research. Sufficient mathematical and numerical detail is provided in a variety of geometries, such that the analysis and results can be readily reproduced, and many numerical tables are included to enable readers to compare or benchmark their own calculations. Traditionally, there are two disjointed topics in rotating fluids: convective fluid motion driven by buoyancy, discussed by Chandrasekhar (1961), and inertial waves and precession-driven flow, described by Greenspan (1968). Now, for the first time in book form, the authors present a unified theory for three topics – thermal convection, inertial waves, and precession-driven flow – to demonstrate that these seemingly complicated, and previously disconnected, problems become mathematically simple in the framework of an asymptotic approach that incorporates the essential characteristics of rotating fluids.

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Convection, Inertial Waves and Precession

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Preface

Rotation plays an essential role in the structure and variation of large-scale flows taking place in the interiors, atmospheres and oceans of planets. Knowledge of common hydrodynamical processes in rotating systems constitutes a major necessary component not only in oceanography, but also in planetary and astrophysical sciences. There have been few systematic accounts of the theory of rotating fluids in the more than quarter of a century since Chandrasekhar (1961) and Greenspan (1968) wrote their classic monographs. The second edition of Greenspan's book, Greenspan (1990), was not a major revision. Other volumes, such as the book edited by Roberts and Soward (1978), while containing some interesting articles, did not present the subject in a unified fashion. More recent books by Vanyo (1993) and Boubnov and Golitsyn (1995) mainly concentrate on experimental studies of general rotating flows. Many important developments have taken place in the study of rotating fluids and it has long been necessary to fill a significant gap in the existing literature.

Over the past several decades the subject of rotating fluids has blossomed remarkably and great strides have been made in our understanding of the topic. Not only have there been very many publications on the classic applications of the theory of rotating fluids to the dynamics of atmosphere and oceans, almost exclusively dealing with thin, nearly two dimensional spherical layers of fluids or infinitely unbounded fluid layers, but also considerable attention has been paid to rotating flows in fluid-filled containers such as cylinders, annuli and thick spherical layers or in complete spheres of fluid. Such studies are often considered to be relevant to the dynamics of planetary and stellar interiors and, more importantly, corresponding laboratory experiments in these fluid-filled rotating containers can be carried out, offering deep insight into the understanding of common processes at the heart of rotating flow phenomena.

It is clearly quite impossible today to cover the theory of rotating fluids to the same degree of completeness as Chandrasekhar (1961) or Greenspan (1968) could for the state of the subject in the 1960s. It is necessary to be highly selective. This monograph focuses primarily on the three areas we consider fundamental and central to geophysical and astrophysical applications: (i) inertial waves and oscillations in contained rotating systems caused solely by the effect of rotation, (ii) convective motions controlled by rotation but driven by the thermal instabilities of equilibria in which gravity is parallel to the basic temperature gradient and (iii) fluid motions forced by non-uniformly rotating systems due to precession or libration. These three topics are physically and mathematically closely related. All the flow phenomena considered in this monograph either take place only in rotating fluids or are critically affected by rotation. It is natural that the selection of topics reflects unavoidably some elements of our personal preferences. We regret that many important subjects, such as compressible flows, the effects of stable stratification and some complex nonlinear theories, are omitted. However, we do believe that the three chosen areas are at the heart of the subject since it is the effect of rotation that controls fluid dynamics in rapidly rotating systems.

Traditionally, there are two disjoint topics in rotating fluids: convective fluid motion driven by buoyancy, discussed by Chandrasekhar (1961), and inertial waves and precession-driven flow, described by Greenspan (1968). This monograph presents a unified theory for three seemingly mathematically and physically disconnected topics – thermal convection, inertial waves, and precession-driven flow – and demonstrates that these apparently complicated problems become mathematically tractable in the framework of an asymptotic approach that incorporates the essential characteristics of rotating fluids.

Our objectives in writing this monograph are threefold. We introduce several topics that have not received extensive attention in other monographs on rotating fluids. We attempt to present a systematic and unified account of the theory of rotating Boussinesq fluids in various geometries as it has developed over the last several decades, hoping that it will lead readers up to the frontiers of research on those topics. Consideration of some geometries such as a rotating annular channel or cylinder is motivated by either their mathematical simplicity and clarity or by possible realization in experimental studies, while studies of other geometries like spheres and spheroids are mainly motivated by their direct application to planetary and astrophysical bodies. Many parts of the monograph are based on our original research over the past decades. Finally, it is also our intention that this monograph should be particularly useful to postgraduate researchers in many different fields such as geophysics, astrophysics, planetary physics and engineering.

With regard to the presentation of the subjects, we follow the following policies on style, depth and details. A considerable effort is made to ensure that each chapter is largely self-contained, readable independently of other chapters, at the expense of often repeating basic equations, certain notations and definitions. In order to make the theoretical development accessible to postgraduate researchers, we have included the amount of mathematical detail that is sufficient for them readily to reproduce the mathematical analysis and results. We also include many numerical tables that enable readers to compare or benchmark their analysis and numerical results if needed. Though we have emphasized the analytical aspects of the theory of rotating fluids, wherever possible we compare the theory with the results of corresponding numerical and experimental studies. We have assumed that readers are familiar with fundamental fluid mechanics, the basic theories of partial and ordinary differential equations and vector analysis.

Much of the material presented in this monograph is based on our own original studies as well as the research of our PhD students and post doctors. KZ is grateful to his many colleagues, past and present collaborators, throughout the world. In particular, he is indebted to Professors F. H. Busse, D. Gubbins, A. Jackson, C. A. Jones, R. R. Kerswell, P. H. Roberts, G. Schubert and A. M. Soward; through numerous discussions with them over decades, they have significantly influenced his research on the subject. We are also grateful to Dr. S. J. Maskell who has carefully read the manuscript and provided numerous valuable comments. The blame for any defects and errors in this monograph rests squarely on us.

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