

Fundamentals of Signals and Systems

This innovative textbook provides a solid foundation in both signal processing and systems modeling using a building block approach. The authors show how to construct signals from fundamental building blocks (or basis functions), and demonstrate a range of powerful design and simulation techniques in MATLAB®, recognizing that signal data are usually received in discrete samples, regardless of whether the underlying system is discrete or continuous in nature.

The book begins with key concepts such as the orthogonality principle and the discrete Fourier transform. Using the building block approach as a unifying principle, the modeling, analysis and design of electrical and mechanical systems are then covered, using various real-world examples. The design of finite impulse response filters is also described in detail.

Containing many worked examples, homework exercises, and a range of MATLAB laboratory exercises, this is an ideal textbook for undergraduate students of engineering, computer science, physics, and other disciplines.

Accompanying the text is a CD containing the MATLAB m-files for MATLAB generated figures, examples, useful utilities and some interesting demonstrations. The files can be easily modified to solve other related problems.

Further resources for this title including solutions, lecture slides, additional problems and labs, are available at www.cambridge.org/9780521849661.

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Cambridge University Press

0521849667 - Fundamentals of Signals and Systems: A Building Block Approach

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Frontmatter

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Fundamentals of Signals and Systems

A building block approach

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CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press

The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org

Information on this title: www.cambridge.org/9780521849661

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First published 2006

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

ISBN-13 978-0-521-84966-1 hardback

ISBN-10 0-521-84966-7 hardback

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Contents

<i>List of figures</i>	<i>page x</i>
<i>List of tables</i>	xxi
<i>Preface</i>	xxiii
<i>Acknowledgments</i>	xxvii
1 Introduction to signals and systems	1
1.1 Signals and systems	1
1.2 Examples of signals	4
1.3 Mathematical foundations	7
1.4 Phasors	9
1.5 Time-varying frequency and instantaneous frequency	12
1.6 Transformations	14
1.7 Discrete-time signals	18
1.8 Sampling	22
1.9 Downsampling and upsampling	23
1.10 Problems	24
2 Constructing signals from building blocks	29
2.1 Basic building blocks	29
2.2 The orthogonality principle	33
2.3 Orthogonal basis functions	35
2.4 Fourier series	42
2.5 Alternative forms of the Fourier series	43
2.6 Approximating signals numerically	47
2.7 The spectrum of a signal	49
2.8 The discrete Fourier transform	56
2.9 Variations on the DFT and IDFT	59

2.10	Relationship between $X[k]$ and c_k	60
2.11	Examples	62
2.12	Proof of the continuous-time orthogonality principle	69
2.13	A note on vector spaces	72
2.14	Problems	78
3	Sampling and data acquisition	85
3.1	Sampling theorem	87
3.2	Discrete-time spectra	88
3.3	Aliasing, folding and reconstruction	89
3.4	Continuous- and discrete-time spectra	96
3.5	Aliasing and folding (time domain perspective)	97
3.6	Windowing	104
3.7	Aliasing and folding (frequency domain perspective)	106
3.8	Handling data with the FFT	110
3.9	Problems	112
4	Lumped element modeling of mechanical systems	118
4.1	Introduction	118
4.2	Building blocks for lumped mechanical systems	121
4.3	Inputs to mechanical systems	131
4.4	Governing equations	132
4.5	Parallel combination	141
4.6	Series combination	144
4.7	Combination of masses	146
4.8	Examples of parallel and series combinations	146
4.9	Division of force in parallel combination	147
4.10	Division of displacement in series combination	148
4.11	Problems	150
5	Lumped element modeling of electrical systems	158
5.1	Building blocks for lumped electrical systems	158
5.2	Summary	165
5.3	Inputs to electrical systems	166
5.4	Governing equations	167
5.5	Parallel combination	172
5.6	Series combination	175
5.7	Division of current in parallel combination	177

5.8	Division of voltage in series combination	177
5.9	Problems	178
6	Solution to differential equations	183
6.1	First-order ordinary differential equations	184
6.2	Second-order ordinary differential equations	185
6.3	Transient response	189
6.4	Transient specifications	196
6.5	State space formulation	199
6.6	Problems	211
7	Input–output relationship using frequency response	217
7.1	Frequency response of linear, time-invariant systems	219
7.2	Frequency response to a periodic input and any arbitrary input	221
7.3	Bode plots	222
7.4	Impedance	238
7.5	Combination and division rules using impedance	241
7.6	Problems	249
8	Digital signal processing	266
8.1	More frequency response	268
8.2	Notation clarification	270
8.3	Utilities	271
8.4	DSP example and discrete-time FRF	272
8.5	Frequency response of discrete-time systems	281
8.6	Relating continuous-time and discrete-time frequency response	291
8.7	Finite impulse response filters	299
8.8	The mixer	305
8.9	Problems	307
9	Applications	310
9.1	Communication systems	310
9.2	Modulation	310
9.3	AM radio	311
9.4	Vibration measuring instruments	317
9.5	Undamped vibration absorbers	323
9.6	JPEG compression	325
9.7	Problems	335

10 Summary	341
10.1 Continuous-time signals	343
10.2 Discrete-time signals	345
10.3 Lumped element modeling of mechanical and electrical systems	347
10.4 Transient response	350
10.5 Frequency response	351
10.6 Impedance	353
10.7 Digital signal processing	354
10.8 Transition to more advanced texts (or, what's next?)	356
 Laboratory exercises	 363
Laboratory exercise 1 Introduction to MATLAB	365
L1.1 Objective	365
L1.2 Guided introduction to MATLAB	365
L1.3 Vector and matrix manipulation	366
L1.4 Variables	370
L1.5 Plotting	371
L1.6 M-files	373
L1.7 Housekeeping	377
L1.8 Summary of MATLAB commands	378
L1.9 Exercises	379
 Laboratory exercise 2 Synthesize music	 382
L2.1 Objective	382
L2.2 Playing sinusoids	382
L2.3 Generating musical notes	383
L2.4 <i>Für Elise</i> project	385
L2.5 Extra credit	386
L2.6 Exercises	387
 Laboratory exercise 3 DFT and IDFT	 388
L3.1 Objective	388
L3.2 The discrete Fourier transform	388
L3.3 The inverse discrete Fourier transform	391
L3.4 The fast Fourier transform	392
L3.5 Exercises	393
 Laboratory exercise 4 FFT and IFFT	 394
L4.1 Objective	394
L4.2 Frequency response of a parallel <i>RLC</i> circuit	395

L4.3	Time response of a parallel <i>RLC</i> circuit to a sweep input	396
L4.4	Exercises	399
	Laboratory exercise 5 Frequency response	400
L5.1	Objective	400
L5.2	Automobile suspension	400
L5.3	Frequency response	400
L5.4	Time response to sinusoidal input	401
L5.5	Numerical solution with the Fourier transform	402
L5.6	Time response to step input	403
L5.7	Optimizing the suspension	404
L5.8	Exercises	404
	Laboratory exercise 6 DTMF	405
L6.1	Objective	405
L6.2	DTMF dialing	405
L6.3	<i>f</i> domain and <i>t</i> domain	406
L6.4	Band-pass filters	407
L6.5	DTMF decoding	407
L6.6	Forensic engineering	408
L6.7	Exercises	408
	Laboratory exercise 7 AM radio	409
L7.1	Objective	409
L7.2	Amplitude modulation	409
L7.3	Demodulation	410
L7.4	Pirate radio	411
L7.5	Exercises	412
	<i>Appendix A</i> Complex arithmetic	413
	<i>Appendix B</i> Constructing discrete-time signals from building blocks – least squares	416
	<i>Appendix C</i> Discrete-time upsampling, sampling and downsampling	419
	<i>Index</i>	425

Figures

1.1	A 200 Hz sine wave, $x(t) = \cos[2\pi(200t)]$.	<i>page</i> 4
1.2	The spectrum of a 200 Hz sine wave.	4
1.3	A 200 Hz square wave.	5
1.4	The spectrum of a 200 Hz square wave.	5
1.5	A 200 Hz triangle wave.	5
1.6	The spectrum of a 200 Hz triangle wave.	6
1.7	A graph of digitized signal.	6
1.8	Complex exponential plotted on a complex plane.	7
1.9	Graphical representation of (a) cosine and (b) sine on a complex plane.	8
1.10	Graphical representation of sinusoid on a complex plane.	8
1.11	The real part of a rotating phasor.	9
1.12	Graphical addition of sinusoids with phasors.	11
1.13	Plot of a sweep.	14
1.14	A sawtooth signal.	14
1.15	The signal of Figure 1.14 time shifted by (a) -2 and (b) $+1$.	15
1.16	The signal of Figure 1.14 vertically shifted by (a) -2 and (b) $+1$.	15
1.17	The reverse of the signal of Figure 1.14.	16
1.18	The signal of Figure 1.14 (a) compressed by a factor of 2 and (b) expanded by a factor of 2.	16
1.19	The signal of Figure 1.14 (a) amplified by a factor of 2 and (b) attenuated by a factor of 2.	17
1.20	The successive transformations of Eq. (1.44).	18
1.21	The lollipop diagram for the discrete-time signal of Eq. (1.45).	19
1.22	The discrete-time unit step function.	20
1.23	The discrete-time unit impulse function.	20
1.24	A continuous-time unit step function.	20

Cambridge University Press

0521849667 - Fundamentals of Signals and Systems: A Building Block Approach

Philip D. Cha and John I. Molinder

Frontmatter

[More information](#)**xi****Figures**

1.25	Sampling of an arbitrary signal.	22
1.26	(a) Down-sampling and (b) upsampling of an arbitrary discrete-time signal.	24
1.27	Figure for Problem 1-8.	26
1.28	Figure for Problem 1-10.	26
1.29	Figure for Problem 1-11.	27
1.30	Figure for Problem 1-12.	27
2.1	A rectangular pulse.	30
2.2	Approximating a continuous-time signal with a series of pulses.	30
2.3	A better approximation for the initial part of a continuous-time signal.	30
2.4	Approximating a sawtooth wave with a single pulse.	32
2.5	A sawtooth signal.	34
2.6	Basis functions for the example in Figure 2.5.	34
2.7	Product of $x(t)$ and $\phi_0(t)$.	35
2.8	Approximation to the sawtooth signal of Figure 2.5 using the basis functions of Figure 2.6.	35
2.9	Third basis function for the example in Figure 2.5.	35
2.10	Example of a pair of orthogonal basis functions.	37
2.11	A third orthogonal basis function.	37
2.12	Multiplication of $\phi_0(t)$ and $\phi_2(t)$.	37
2.13	Approximation of a pulse function using sinusoids.	38
2.14	Sinusoidal basis functions (Fourier series).	38
2.15	Approximation of sinusoid using Walsh functions.	39
2.16	Walsh basis functions.	40
2.17	Approximation of sinusoid using Haar Wavelet basis functions.	41
2.18	Haar Wavelet basis functions.	41
2.19	A periodic pulse train.	45
2.20	A periodic square wave.	46
2.21	Square wave approximation showing Gibbs' phenomenon.	48
2.22	Square wave approximation using windowed coefficients.	48
2.23	Sawtooth approximation showing Gibbs' phenomenon.	49
2.24	Spectrum of a sinusoid.	50
2.25	(a) Magnitude and (b) phase spectrum of a sinusoid.	51
2.26	A periodic pulse train.	52
2.27	The spectrum of a periodic pulse train.	52
2.28	A periodic square wave.	53
2.29	The spectrum of a periodic square wave.	53

Cambridge University Press

0521849667 - Fundamentals of Signals and Systems: A Building Block Approach

Philip D. Cha and John I. Molinder

Frontmatter

[More information](#)

xii

Figures

2.30	A periodic sawtooth.	54
2.31	The spectrum of a periodic sawtooth.	54
2.32	Curious signals.	55
2.33	Magnitude spectra for the signals of Figure 2.32.	56
2.34	An arbitrary signal $x(t)$.	57
2.35	Approximating the integral with a series of rectangular pulses.	57
2.36	Sampled 1 Hz cosine for $N = 10$.	63
2.37	Sample 3 Hz cosine for $N = 10$.	65
2.38	(a) Pulse train and (b) Fourier coefficients for $N = 1000$.	66
2.39	(a) Pulse train and (b) Fourier coefficients for $N = 50$.	67
2.40	Exponential and Fourier coefficient magnitude and phase.	68
2.41	$X[k]$ and the spectra of a 3 Hz cosine.	70
2.42	Figure for Problem 2-1.	79
2.43	Figure for Problem 2-3.	80
2.44	Figure for Problem 2-4.	81
2.45	Figure for Problem 2-6.	81
2.46	Figure for Problem 2-10.	83
2.47	Figure for Problem 2-11.	83
2.48	Figure for Problem 2-12.	83
3.1	Continuous to discrete (C/D) converter.	85
3.2	Switch symbol for C/D converter.	86
3.3	Continuous-time spectrum of a complex exponential.	88
3.4	Discrete-time spectrum of a complex exponential.	89
3.5	(a) Continuous- and (b) discrete-time spectra of a cosine signal ($f_s = 4f_0$).	90
3.6	(a) Continuous- and (b) discrete-time spectra of a cosine signal ($f_s = 4f_0/5$).	91
3.7	Discrete to continuous (D/C) converter.	91
3.8	Continuous-time spectra of the reconstructed signals ($f_s = 4f_0$ and $f_s = 4f_0/5$).	92
3.9	Continuous- and discrete-time spectra of a cosine signal ($f_s = 4f_0/3$).	93
3.10	Continuous-time spectra of $x(t)$.	97
3.11	Discrete-time spectra of $x[n]$.	97
3.12	A 10 Hz sinusoid sampled at 25 Hz.	98
3.13	Aliasing: a 52 Hz sinusoid sampled at 50 Hz.	98
3.14	Folding: a 48 Hz sinusoid sampled at 50 Hz.	99
3.15	Spectrum for $T = 0.1$ s and $N = 3$.	101

Cambridge University Press

0521849667 - Fundamentals of Signals and Systems: A Building Block Approach

Philip D. Cha and John I. Molinder

Frontmatter

[More information](#)**xiii****Figures**

3.16	Spectrum for $T = 0.5$ s and $N = 11$.	102
3.17	Spectrum for $T = 0.5$ s and $N = 32$.	103
3.18	Spectrum for $T = 0.55$ s and $N = 32$.	103
3.19	Sinusoid repeating after 5.5 periods.	104
3.20	Tapering sinusoid with a triangular window.	105
3.21	Spectrum of a windowed sinusoid for $T = 0.55$ s and $N = 32$.	105
3.22	Sinusoid sampled at $2f_0$.	106
3.23	Spectrum of a 52 Hz sinusoid sampled at 50 Hz.	107
3.24	Spectrum of a 48 Hz sinusoid sampled at 50 Hz.	107
3.25	Dial tone (entire signal and zoomed section).	108
3.26	Spectrum of dial tone (entire section and zoomed section).	109
3.27	Spectrum of dial tone truncated before silence.	109
3.28	Noisy signal.	110
3.29	Spectrum of noisy signal.	111
3.30	Data acquisition system.	111
3.31	Figure for Problem 3-1.	112
3.32	Figure for Problem 3-5.	115
3.33	Figure for Problem 3-10.	117
4.1	The input–output relationship for a general system.	119
4.2	(a) A cantilever or fixed-free beam and (b) its model.	122
4.3	(a) A fixed-free rod and (b) its model.	123
4.4	(a) A fixed-free shaft and (b) its model.	124
4.5	Schematic of a spring attached to a fixed wall.	125
4.6	Free body diagram for the free end of the spring of Figure 4.5.	125
4.7	Restoring force on the right end of a spring.	126
4.8	Restoring force on the left end of a spring.	126
4.9	Force–displacement relationship for a linear spring.	128
4.10	A spring with a force applied at both ends.	128
4.11	(a) A viscous damper and (b) its symbolic representation.	129
4.12	A spring–mass system.	133
4.13	Free body diagram for the spring–mass system of Figure 4.12.	133
4.14	A spring–mass system suspended from a ceiling.	134
4.15	Static equilibrium and the displacement from static equilibrium.	135
4.16	Free body diagram for the system of Figure 4.14.	135
4.17	A base excitation system.	136
4.18	Free body diagram for the system of Figure 4.17.	136
4.19	Propeller of a ship.	137
4.20	A simple mathematical model for the propeller of Figure 4.19.	137

Cambridge University Press

0521849667 - Fundamentals of Signals and Systems: A Building Block Approach

Philip D. Cha and John I. Molinder

Frontmatter

[More information](#)**xiv****Figures**

4.21	Free body diagram for the system of Figure 4.20.	138
4.22	A pendulum system.	138
4.23	Free body diagram for the system of Figure 4.22.	139
4.24	Parallel combination of two springs.	142
4.25	Free body diagram for the system of Figure 4.24.	142
4.26	Equivalent spring for the spring system of Figure 4.24.	143
4.27	Free body diagram for the system of Figure 4.26.	143
4.28	Parallel combination of N springs.	143
4.29	Series combination of two springs.	144
4.30	Equivalent spring for the spring system of Figure 4.29.	145
4.31	Springs k_1 and k_2 in series.	146
4.32	Springs k_1 and k_2 in parallel.	146
4.33	k and k_{beam} in parallel.	146
4.34	k and k_{beam} in series.	147
4.35	k_1 and k_{beam} in parallel, and the result in series with k_2 .	147
4.36	Parallel combination of N springs.	148
4.37	Series combination of N springs.	149
4.38	Figure for Problem 4-1.	150
4.39	Figure for Problem 4-2.	150
4.40	Figure for Problem 4-3.	151
4.41	Figure for Problem 4-4.	151
4.42	Figure for Problem 4-5.	151
4.43	Figure for Problem 4-6.	152
4.44	Figure for Problem 4-7.	152
4.45	Figure for Problem 4-8.	153
4.46	Figure for Problem 4-9.	154
4.47	Figure for Problem 4-10.	154
4.48	Figure for Problem 4-11.	154
4.49	Figure for Problem 4-12.	155
4.50	Figure for Problem 4-13.	156
5.1	Current flow through an element.	159
5.2	Voltage drop and node voltages across an element.	159
5.3	Ground symbols.	160
5.4	Schematic of an inductor.	161
5.5	Circuit symbol for an inductor.	161
5.6	Voltage source and a wire.	162
5.7	Circuit symbol for a resistor.	163
5.8	Schematic of a capacitor.	164

Cambridge University Press

0521849667 - Fundamentals of Signals and Systems: A Building Block Approach

Philip D. Cha and John I. Molinder

Frontmatter

[More information](#)**xv****Figures**

5.9	Circuit symbol for a capacitor.	165
5.10	Voltage and current sources.	166
5.11	Voltage drop across elements in series.	168
5.12	Parallel LC circuit with a current input.	169
5.13	Direction of currents for the circuit of Figure 5.12.	169
5.14	A spring–mass system with a force input.	170
5.15	Electrical circuit with current and voltage inputs.	171
5.16	A base excitation system.	172
5.17	N resistors in parallel.	173
5.18	N resistors in series.	175
5.19	Figure for Problem 5-1.	179
5.20	Figure for Problem 5-2.	179
5.21	Figure for Problem 5-3.	180
5.22	Figure for Problem 5-4.	180
5.23	Figure for Problem 5-5.	181
5.24	Figure for Problem 5-6.	181
6.1	A series RC circuit.	184
6.2	Mechanical system consisting of a disk attached to a rotational damper.	184
6.3	A spring–mass system.	186
6.4	A parallel RLC circuit.	186
6.5	Step responses of a second-order system as a function of ζ with $\omega_n = 1$ rad/s.	194
6.6	Transient response characteristics.	196
6.7	A spring–mass–damper system.	198
6.8	An electrical circuit with a switch.	208
6.9	Figure for Problem 6-2.	212
6.10	Figure for Problem 6-3.	212
6.11	Figure for Problem 6-4.	213
6.12	Figure for Problem 6-5.	213
6.13	Figure for Problem 6-6.	214
6.14	Figure for Problem 6-7.	215
6.15	Figure for Problem 6-8.	215
6.16	Figure for Problem 6-13.	216
7.1	Input–output relationship using the frequency response function.	218
7.2	A spring–mass–damper system under base excitation.	220
7.3	Input–output relationship using the frequency response function.	221
7.4	Bode plot axes.	223

Cambridge University Press

0521849667 - Fundamentals of Signals and Systems: A Building Block Approach

Philip D. Cha and John I. Molinder

Frontmatter

[More information](#)**xvi****Figures**

7.5	Bode plot of a positive constant factor.	225
7.6	Log magnitude plot of a first-order factor in the denominator.	226
7.7	Bode plot of a first-order factor in the denominator.	228
7.8	Bode plot of a first-order factor in the numerator.	229
7.9	Log magnitude plot of the product of two first-order factors in the denominator (see Eq. (7.38)), for $\tau_1 = 1$ s and $\tau_2 = 0.1$ s.	230
7.10	Phase plot of the product of two first-order factors in the denominator (see Eq. (7.38)) for $\tau_1 = 1$ s and $\tau_2 = 0.1$ s.	231
7.11	Straight-line approximation versus the exact Bode plot for the system of Eq. (7.38) for $\tau_1 = 1$ s and $\tau_2 = 0.1$ s.	232
7.12	Bode plot of an undamped or underdamped second-order factor in the denominator.	237
7.13	Exact Bode plot of an underdamped second-order factor in the denominator.	237
7.14	(a) Parallel and (b) series impedance combinations.	242
7.15	Electric circuit for Example 1.	243
7.16	Circuit of Figure 7.15 in terms of impedances.	244
7.17	General frequency response function.	245
7.18	Electric circuit for Example 2.	246
7.19	Circuit of Figure 7.18 in terms of impedances.	246
7.20	Simplified circuit of Figure 7.19.	247
7.21	Electric circuit for Example 3.	248
7.22	Circuit of Figure 7.21 in terms of impedances.	248
7.23	Figure for Problem 7-1.	249
7.24	Figure for Problem 7-3.	250
7.25	Figure for Problem 7-4.	251
7.26	Figure for Problem 7-5.	251
7.27	Figure for Problem 7-6.	252
7.28	Figure for Problem 7-7.	253
7.29	Figure for Problem 7-10.	254
7.30	Figure for Problem 7-11.	254
7.31	Figure for Problem 7-12.	255
7.32	Figure for Problem 7-13.	255
7.33	Figure for Problem 7-14.	256
7.34	Figure for Problem 7-15.	257
7.35	Figure for Problem 7-16.	257
7.36	Figure for Problem 7-17.	258
7.37	Figure for Problem 7-18.	258

Cambridge University Press

0521849667 - Fundamentals of Signals and Systems: A Building Block Approach

Philip D. Cha and John I. Molinder

Frontmatter

[More information](#)**xvii****Figures**

7.38	Figure for Problem 7-19.	259
7.39	Figure for Problem 7-20.	260
7.40	Figure for Problem 7-22.	261
7.41	Figure for Problem 7-23.	263
7.42	Figure for Problem 7-25.	265
7.43	Figure for Problem 7-26.	265
8.1	(a) Input spectrum, (b) frequency response function and (c) output spectrum.	269
8.2	Computing response using Fourier series and FRF.	270
8.3	Computing response using FFT.	270
8.4	Second-order electrical system.	273
8.5	Pulse train input.	274
8.6	Pulse train response for the underdamped second-order system of Figure 8.4.	275
8.7	Pulse train response found with Fourier technique.	276
8.8	Pulse train response obtained with Fourier technique (solid line) overlaid with the exact step response.	276
8.9	Pulse train response and exact step response for (a) $T_0 = 20$ s and (b) $T_0 = 3$ s.	279
8.10	Spectrum of music with annoying 1500 Hz tone.	280
8.11	Filtered audio signal.	281
8.12	Sampling a complex exponential.	282
8.13	(a) Gain and (b) phase of discrete-time frequency response function of Eq. (8.37).	284
8.14	System rejecting high frequencies.	286
8.15	Digital signal processing system.	287
8.16	(a) Gain and phase of the discrete-time frequency response function of Eq. (8.48) vs ω ; (b) gain vs. f in kHz.	289
8.17	Spectrum of the signal a) before and b) after filtering.	290
8.18	Frequency response of the modified filter.	291
8.19	Continuous- and discrete-time systems for comparison.	292
8.20	Continuous-time input spectrum.	292
8.21	(a) Continuous-time frequency response and (b) output spectrum.	293
8.22	Discrete-time input spectrum.	294
8.23	(a) Discrete-time frequency response and (b) output spectrum.	294
8.24	RC circuit: a first-order low-pass filter.	295
8.25	Bode plot for a low-pass RC filter.	297

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0521849667 - Fundamentals of Signals and Systems: A Building Block Approach

Philip D. Cha and John I. Molinder

Frontmatter

[More information](#)**xviii****Figures**

8.26	Frequency responses of continuous-time and discrete-time systems (linear scale).	298
8.27	Frequency responses of continuous-time and discrete-time systems (log scale).	298
8.28	Step responses of continuous-time and discrete-time systems.	299
8.29	Desired frequency response function.	301
8.30	Frequency response of discrete-time low-pass filter.	302
8.31	Frequency response of discrete-time high-pass filter.	303
8.32	Low-pass filter with 25 coefficients.	304
8.33	Frequency response of “world-famous” M filter.	304
8.34	Mixer: multiplication of signals.	305
8.35	(a) Input and (b) output spectrum of a mixer.	306
8.36	(a) Generic input spectrum for $x(t)$ and (b) the spectrum for its associated output $y(t)$.	307
8.37	Figure for Problem 8-4.	309
9.1	Mixing input with carrier wave.	311
9.2	Spectrum of $x(t)$.	311
9.3	Spectrum of $y(t)$.	311
9.4	(a) The demodulation process and (b) the resulting spectrum.	312
9.5	AM radio band.	312
9.6	Receiving AM station.	313
9.7	Spectrum of AM signal.	313
9.8	Time domain plots of (a) $x(t)$ and (b) $y(t)$.	314
9.9	Crystal radio receiver.	314
9.10	Amplitude modulation with envelope detection.	315
9.11	Systems representation of crystal radio receiver.	315
9.12	Time-domain view of AM radio receiver.	316
9.13	Frequency-domain view of AM radio receiver.	317
9.14	Time-domain view of AM radio receiver with overmodulation.	318
9.15	Vibration measuring device.	318
9.16	Free body diagram of seismic mass.	319
9.17	Gain of Eq. (9.11) as a function of ω/ω_n .	322
9.18	Model of a vibration absorber attached to an optical table.	324
9.19	Free body diagram for the system of Figure 9.18.	325
9.20	Free body diagram for the system of Figure 9.18 when $x = 0$.	325
9.21	Basis functions for the discrete cosine transform (DCT).	329
9.22	Approximating a slowly changing signal; $x[n]$ vs n .	330
9.23	Approximating an abruptly changing signal; $x[n]$ vs n .	332

Cambridge University Press

0521849667 - Fundamentals of Signals and Systems: A Building Block Approach

Philip D. Cha and John I. Molinder

Frontmatter

[More information](#)**xix****Figures**

9.24	Basis functions for two-dimensional DCT as a function of m, n .	333
9.25	Compressed photographs.	334
9.26	Figure for Problem 9-1.	335
9.27	Figure for Problem 9-2.	336
9.28	Figure for Problem 9-3.	336
9.29	Figure for Problem 9-4.	337
9.30	Figure for Problem 9-5.	338
9.31	Figure for Problem 9-6.	339
9.32	Figure for Problem 9-7.	340
10.1	Spectrum of sum of sinusoids.	344
10.2	Spectrum of pulse train.	345
10.3	Discrete-time spectrum showing folding and aliasing.	347
10.4	(a) An electrical and (b) a mechanical system.	348
10.5	Transient response characteristics.	349
10.6	Bode plot of a first-order system in the denominator using straight-line approximation.	350
10.7	Bode plot of an underdamped second-order system in the denominator using straight-line approximation.	353
10.8	Series and parallel combinations, and voltage dividers.	353
10.9	DSP with FFT.	354
10.10	DSP with FIR.	355
L1.1	Example MATLAB plot of 5 Hz sinusoidal waveform.	372
L1.2	Examples of MATLAB multiple data-sets and multiple separate plots in one window.	373
L1.3	Example MATLAB plot for the <code>sinc_5.m</code> script file.	374
L1.4	Example MATLAB plot for an echoed waveform.	376
L2.1	Layout of a piano keyboard. Key numbers are shaded. The notation C_4 means the C key in the fourth octave. (Source: McClellan, J. H., Schafer, R. W. and Yoder, M. A. <i>DSP First</i> , ©1998. Reprinted by permission of Pearson Education Inc., Upper Saddle River, NJ.)	383
L2.2	Musical notation is a time–frequency diagram where vertical position indicates the frequency of the note to be played. (Source: McClellan, J. H., Schafer, R. W. and Yoder, M. A. <i>DSP First</i> , ©1998. Reprinted by permission of Pearson Education Inc., Upper Saddle River, NJ.)	384
L2.3	The first few bars of Beethoven's <i>Für Elise</i> . (Source: McClellan, J. H., Schafer, R. W. and Yoder, M. A. <i>DSP First</i> , ©1998. Reprinted by permission of Pearson Education Inc., Upper Saddle River, NJ.)	385

L2.4	Example ADSR envelope to produce fading.	386
L3.1	One period of cosine waveform with four samples.	390
L3.2	Continuous-time spectrum for Figure L3.1.	390
L3.3	Discrete-time Fourier transform spectrum for Figure L3.1.	391
L3.4	DFT spectrum for Figure L3.1.	391
L3.5	Example spectrum for exercise.	391
L4.1	Example <i>RLC</i> circuit.	395
L4.2	Magnitude and phase plots for the circuit of Figure L4.1.	396
L4.3	Example sweep spectrum.	397
L4.4	Frequency response function of the circuit of Figure L4.1.	399
L5.1	A simple model of an automobile suspension system.	401
L6.1	Continuous-time frequency response of an ideal band-pass filter.	407
L7.1	The magnitude spectrum of y_1 in the Lab 7 example.	411
A1.1	Cartesian representation of a complex number.	413
A1.2	Polar representation of a complex number.	414
C1.1	Discrete-time upsampling with $M = 4$.	421
C1.2	Discrete-time sampling with $N = 24$ and $M = 2$.	422
C1.3	Discrete-time sampling with $N = 24$ and $M = 3$.	423
C1.4	Discrete-time sampling with $N = 24$ and $M = 4$.	423
C1.5	Discrete-time sampling with $N = 24$ and $M = 6$.	424

Tables

1.1	Touch-tone telephone tones.	7
1.2	Discrete-time signal values.	19
2.1	Coefficients of the pulse function of Figure 2.13 using sinusoids.	39
2.2	Coefficients of the sinusoid of Figure 2.15 using Walsh functions.	40
2.3	Coefficients of the sinusoid of Figure 2.17 using Haar Wavelet basis functions.	42
2.4	Periodic square wave coefficients.	47
2.5	Relation of $X[k]$ to c_k for $N = 4$.	61
2.6	General relation of $X[k]$ to c_k .	61
2.7	MATLAB relation of $X[k]$ to c_k .	62
2.8	Impact of the number of samples N on the quality of the approximation (c_k and $X[k]$ are given by Eqs. (2.59) and (2.66), respectively).	62
3.1	The effects of sampling frequency on the reconstructed signal.	95
3.2	Touch-tone telephone tones.	108
4.1	Constitutive equations for ideal translational mechanical elements.	131
5.1	Analogy between electrical and mechanical elements.	166
6.1	Homogeneous solutions of Eq. (6.40) as a function of ζ .	192
9.1	Coefficients for slowly changing signal.	331
9.2	Coefficients for abruptly changing signal.	331
9.3	File sizes of photographs shown in Figure 9.25.	335
10.1	Analogy between electrical and mechanical elements.	348
L	Laboratory exercises and relevant chapters.	364

Preface

This text results from a new approach to teaching the sophomore engineering course entitled *Introduction to Engineering Systems* at Harvey Mudd College. Since the course is required of all students regardless of major, the goal is to provide a clear understanding of concepts, tools, and techniques that will be beneficial in engineering, physics, chemistry, mathematics, biology, and computer science. Cha, Rosenberg, and Dym's *Fundamentals of Modeling and Analyzing Engineering Systems*,¹ written specifically for this course and providing an excellent introduction to the modeling and analysis of systems from a wide variety of disciplines, was used for several years. This text complements that text's focus on modeling with a strong emphasis on representation of continuous-time and discrete-time signals in both the time and frequency domains, modeling of mechanical and electrical systems, and the design of finite impulse response (FIR) discrete filters. *Fundamentals of Modeling and Analyzing Engineering Systems* and this text can be used together in a two-semester sequence to provide a thorough introduction to both signal processing and modeling, or selected topics from both can be used as the basis for a one-semester course. The sampling theorem, continuous-to-discrete and discrete-to-continuous converters, the discrete Fourier transform (DFT) and its computation with the fast Fourier transform (FFT) are explained in detail. Students are introduced to MATLAB and get hands-on experience with a series of laboratory assignments that illustrate and apply the theory. Single variable calculus is the only essential background although some knowledge of differential equations, linear algebra, and vector spaces is helpful. The materials covered in this text have

¹ *Fundamentals of Modeling and Analyzing Engineering Systems* by P. D. Cha, J. J. Rosenberg and C. L. Dym, Cambridge University Press, UK, 2000.

grown out of lectures given primarily to sophomores at Harvey Mudd College. These notes have been classroom-tested over a period of six semesters.

Engineers, scientists, and mathematicians are increasingly faced with acquiring, processing, interpreting, and extracting information from data, which are usually provided as a series of discrete samples, independent of whether the original underlying signals and systems were continuous or discrete in nature. Discrete-time techniques are used almost exclusively for simulating both continuous-time and discrete-time systems. Effective use of modern analysis, design, and simulation tools such as MATLAB require a clear understanding of the underlying theory as well as a good bit of practice with applications.

We begin by developing representations of continuous-time signals as functions and discrete-time signals as sequences (Chapter 1). We then explore various transformations such as shifts, reversal, compression, and expansion for continuous-time signals. We also cover upsampling and downsampling for discrete-time signals. Next, the construction of complicated signals from basic building blocks is introduced using the orthogonality principle (Chapter 2). This provides the foundation and context for the later focus on complex exponentials and the Fourier series. By starting with the generally applicable approach of minimizing the integrated squared error through the use of the orthogonality principle, the student is given a much deeper understanding and appreciation for a broad class of applications, including the use of Walsh functions in cellular phones to various series expansions using a variety of building blocks or basis functions. The difference between the orthogonality principle and orthogonal basis functions is carefully explained. An appendix provides a natural development of basis functions starting with the familiar three-dimensional vectors.

Complex exponentials as building blocks provide the foundation for the development of the spectrum of a continuous-time signal. Its importance in characterizing and extracting information from signals leads naturally to the need for numerical techniques to compute the spectrum of complicated signals produced by phenomena such as earthquakes, space photographs or communication systems. This, in turn, leads to the discrete Fourier transform and its efficient computation using the fast Fourier transform algorithm. While we do not derive the fast Fourier transform algorithm, the text describes its use in considerable detail and laboratory exercises provide the opportunity for students to explore practical applications.

Continuous-time and discrete-time signals are connected via the sampling theorem (Chapter 3). The spectrum of a discrete-time signal provides the basis for exploring the phenomena of aliasing and folding, which must be fully understood in order to correctly acquire, process, analyze and interpret data. The behavior of continuous-to-discrete (analog-to-digital) and discrete-to-continuous (digital-to-analog) converters is explored as the culmination of the first part of the text.

The next major division of the text is devoted to the lumped element modeling of mechanical and electrical systems (Chapters 4 and 5). We start with the basic elements (building blocks) for mechanical systems consisting of springs, dampers, and masses. First-order and second-order governing equations are developed and canonical forms are defined for both the translational and rotational systems. Parallel and series combinations of elements as well as the division of force and displacement are covered. A parallel development for electrical systems leads naturally to the force–current and velocity–voltage analogs. Solution of first-order and second-order differential governing equations is introduced, transient response specifications are defined that are used in system design, and a state space approach is formulated as an alternative means to analyze the free and forced responses of a system (Chapter 6).

Frequency response builds on the concept of the complex exponential building blocks that are covered in the first part of the text (Chapter 7). The complex exponentials also serve as the eigenfunctions of linear time-invariant systems, and the concept of frequency response provides the bridge between signals and systems. Bode plots of first-order and second-order factors are developed. The complex exponential building blocks and the concept of frequency response are then used to define impedance and its application in combining various elements of mechanical and electrical systems.

An introduction to the analysis and design of finite-impulse response filters forms the final major part of the text (Chapter 8). The ease with which arbitrary frequency response functions can be implemented is developed as another application of the Fourier series and demonstrated with a whimsical example. A series of applications from a variety of disciplines then follows, providing the student with an appreciation of the power and scope of the concepts, tools, and techniques developed throughout the text (Chapter 9). The text concludes with a short transition section designed to relate the fundamentals to concepts covered in more advanced texts (Chapter 10).

This text offers numerous special features that distinguish itself from other texts on signals and systems, and they are summarized in the following:

- A rigorous development of the construction of signals from building blocks (basis functions) via the orthogonality principle is given.
- The building block approach to develop a clear understanding of the spectra of both continuous-time and discrete-time signals as well as the frequency responses of continuous-time and discrete-time systems all without the use of the continuous-time impulse, Fourier transform, or Laplace transform is used.
- A solid understanding of the use of the FFT in extracting information from signals and determining the response of electrical and mechanical systems to realistic inputs is provided.
- Signal processing and systems modeling are treated on equal footing. Electrical engineering departments usually teach systems with primary emphasis on signal processing while mechanical engineering departments put the primary emphasis on dynamic modeling and control. In order to cover all of the desired topics, this text does not include the control of dynamical systems. This is a conscious decision the authors made in order to present a complete end-to-end analysis from characterizing the input signal, to modeling the physical system, to determining the response or output of the system to arbitrary inputs.
- A thorough treatment of the modeling of complicated mechanical and electrical systems is provided. The analogy between lumped mechanical and electrical systems is introduced in detail.
- Detailed examples of characterizing both simple and realistic (complicated) input signals, modeling physical systems, and determining their response to these inputs are provided. Most modeling texts focus extensively on how to describe the physical system and determine its response to classical inputs such as impulse, step, and sinusoid. In addition to these standard inputs, this text shows how complicated inputs can be represented using simple building blocks, thus allowing the determination of the response of systems to realistic inputs.

Finally, seven MATLAB laboratory exercises are included at the end of the text to allow students to gain a deeper understanding and mastery of the topics covered in the text. We hope that the use of computational software will enhance the learning experience and stimulate the students' interest in signals and systems.

Acknowledgments

The creation of this text was truly a team effort. As part of the teaching team Professor David Harris diligently took notes on his laptop during the authors' lectures and carefully edited them. Without these notes as a basis or building blocks, the writing of the text may never have gotten off the ground. In addition, David wrote the MATLAB-based laboratory exercises and created the review that is part of Chapter 10. Professors Lori Bassman, Mary Cardenas, David Harris, Elizabeth Orwin, Jennifer Rossmann, and Qimin Yang have all taught parts of the course over the past three years and contributed significantly to the final form and content of the text. Professor Anthony Bright, our department chair, gave us unwavering support and encouragement and allowed us to use his introduction to MATLAB that is the basis of Laboratory Exercise 1. Paul Nahin, a former colleague and long-time friend of the second author gave inspiration through his wide-ranging books from science fiction to a truly innovative blend of science technology, and history, as well as encouragement when this text was just an idea. During his senior year, student Warren Katzenstein provided support in developing the initial drafts of the text and figures. We also want to acknowledge the hundreds of students who have contributed through their questions, evaluations, and comments. Finally, we want to acknowledge the moral support of our families, who have all shared the joy and pain of the writing of this text. In particular, the first author would like to thank his Dad, Mom, brother Paul and sister Pauline, the second author his wife Janet, son Tim, and daughter Karen who always encouraged him but wondered if he would ever actually write a book.

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