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A. A BRIEF HISTORICAL PERSPECTIVE OF TRANSPORT PHENOMENA IN CHEMICAL ENGINEERING

“Transport phenomena” is the name used by chemical engineers to describe the subjects of fluid mechanics and heat and mass transfer. The earliest step toward the inclusion of specialized courses in fluid mechanics and heat or mass transfer processes within the chemical engineering curriculum probably occurred with the publication in 1923 of the pioneering text *Principles of Chemical Engineering* by Walker, Lewis, and McAdams.¹ This was the first major departure from curricula that regarded the techniques involved in the production of specific products as largely unique, to a formal recognition of the fact that certain physical or chemical processes, and corresponding fundamental principles, are common to many widely differing industrial technologies.

A natural outgrowth of this radical new view was the gradual appearance of fluid mechanics and transport in both teaching and research as the underlying basis for many of the unit operations. Of course, many of the most important unit operations take place in equipment of complicated geometry, with strongly coupled combinations of heat and mass transfer, fluid mechanics, and chemical reaction, so that the exact equations could not be solved in a context of any direct relevance to the process of interest. Hence, insofar as the large-scale industrial processes of chemical technology were concerned, even at the unit operations level, the impact of fundamental studies of fluid mechanics or transport phenomena was certainly less important than a well-developed empirical approach (and this remains true today in many cases). Indeed, the great advances and discoveries of fluid mechanics during the first half of the twentieth century took place almost entirely without the participation (or even knowledge) of chemical engineers.

Gradually, however, chemical engineers began to accept the premise that the generally “blind” empiricism of the “lumped-parameter” approach to transport processes at the unit operations scale should at least be supplemented by an attempt to understand the basic physical principles. This finally led, in 1960, to the appearance of the landmark textbook of Bird, Stewart, and Lightfoot,² which not only introduced the idea of detailed analysis of transport processes at the continuum level, but also emphasized the mathematical similarity of the governing field equations, along with the simplest constitutive approximations for fluid mechanics and heat and mass transfer. The presentation of Bird *et al.* was primarily focused on results and solutions rather than on the methods of solution or analysis. However, the combination of the more fundamental approach that it pioneered within the chemical engineering community and the appearance of chemical engineers with very strong mathematics backgrounds produced the most recent transitions in our ways of thinking about and understanding transport processes.

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Initially, this was focused largely on the use of asymptotic and numerical methods for detailed analysis and understanding of the important correlations between the dependent and independent dimensionless groups in flow and transport processes relevant to large-scale engineering applications. Although asymptotic approximation methods were initially the product of applied mathematicians, chemical engineers now played an extremely important role in their application to many transport processes and viscous flow phenomena. A critical component in this approach is nondimensionalization by means of characteristic scales to identify dominant physical balances and the use of this approach in approximating (simplifying) the governing equations and boundary conditions.

Another simultaneous development within chemical engineering was a major focus on flows at very low Reynolds number, at least partly motivated by the classic book *Low Reynolds Number Hydrodynamics* by Happel and Brenner,³ originally published in 1965. Initially, the primary application of creeping-flow theory was to the analysis of suspensions, emulsions, and other particulate materials, in combination with the effects of Brownian motion that typically play an important role for particulates with length scales of 10 μm or less. More recently, the scale of many processes of interest has decreased, to the point that there is sometimes a need to incorporate “nonhydrodynamic” forces such as van der Waals forces that act over very short length scales. Furthermore, the recent development of microelectromechanical system technology, mainly focused on very small-scale flow systems, and many of the applications of fluid mechanics and biotransport, have provided a further emphasis on the importance of a fundamental understanding of viscous flow and transport phenomena. Finally, the relevance of interfacial phenomena and of non-Newtonian rheology associated with complex fluids such as polymeric liquids has additionally broadened the scope of what chemical engineers are likely to encounter as “transport phenomena.”

Finally, we cannot overlook the development of computational tools for the solution of problems in fluid mechanics and transport processes. Methods of increasing sophistication have been developed that now enable quantitative solutions of some of the most complicated and vexing problems at least over limited parameter regimes, including direct numerical simulation of turbulent flows; so-called free-boundary problems that typically involve large interface or boundary deformations induced by flow; and methods to solve flow problems for complex fluids, which are typically characterized by viscoelastic constitutive equations and complicated flow behavior.

B. THE NATURE OF THE SUBJECT

The study of fluid mechanics and convective transport processes for heat or molecular species is an old subject. Provided that we limit ourselves to Newtonian fluids and to flow domains involving only solid boundaries, there is no question of understanding the underlying physical principles that govern a problem, at least from a continuum mechanical point of view. On a point-by-point basis, these are represented by the Navier–Stokes and thermal energy (or species transport) equations, with boundary conditions that are generally well established.⁴ These equations and boundary conditions (at least for solid boundaries) have been known for more than a century. Yet these subjects are still extremely active topics of basic research, and, for the most part, this research is aimed at the discovery of new phenomena and principles of fluid motion, rather than the engineering application of previously discovered phenomena to new systems. Although the underlying physical principals are completely understood for this class of fluids, the macroscopic phenomena that are inherent in these physical principles can be extremely complex. From a mathematical point of view, this is largely because the governing equations (and the boundary conditions too at a fluid interface) are nonlinear. However, one need think only of the amazing variety

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of fluid flow phenomena that are encountered in everyday experience to recognize the complexity allowed by the well-known fundamental principles.

Examples include the dynamics of waves and of breaking waves at a beach; the complex “mixing” flows created by a spoon moving through a cup of coffee; the dripping of water from a tap; the bathtub vortex as water drains from a tub; or the complicated coiling motion of honey dripping from a knife onto a slice of toast. We may also think of the changes in flow structure that can be caused by variations in the flow rate, even when the geometry of the boundaries and all the fluid properties are fixed. Likely familiar to most readers is the transition in pressure-driven flow through a circular tube, from a one-dimensional time-independent “laminar” flow at low flow rates to a fully three-dimensional time-dependent “turbulent” flow when the flow rate is increased beyond some critical value. Perhaps less familiar is the flow produced by the translational motion of a cylindrical body perpendicular to its axis through an otherwise stationary Newtonian fluid. The motion of the fluid in all circumstances is governed by the Navier–Stokes equations, yet the range of observable flow phenomena that occur as the velocity of the cylinder is increased is quite amazing, beginning at low speeds with steady motion that follows the body contour, and then followed by a transition to an asymmetric motion that includes a standing pair of recirculating vortices at the back side of the cylinder, which are at first steady and attached and then become unsteady and alternately detached as the velocity increases. Finally, at even higher speeds the whole motion in the vicinity of the cylinder and downstream becomes three dimensional and chaotic in the well-known regime of turbulent flow. All of these phenomena are inherent in the physical principles encompassed in the Navier–Stokes equations. It is “only” that the solutions of these equations (namely, the physical phenomena) become increasingly complex with increase in the velocity of the cylinder through the fluid. Generally, then, for Newtonian fluids, the basic objective is to understand the physics of the flow rather than the underlying physical principles.

In a sense, we can visualize the physics of a flow by carrying out laboratory experiments, or by observing natural flows directly. When this is not possible or is not feasible, the advent of increasingly powerful computers and numerical techniques sometimes allow a “computational experiment” to be carried out, based on the known governing equations and boundary conditions. It is unfortunate that a qualitative description, or even flow-visualization pictures, of complex phenomena do not translate immediately into “understanding.” Obviously, if this were the case, it would be possible to provide students with a much more realistic picture of real phenomena than they can hope to achieve in the normal classroom (or textbook) environment. The difficulty with a qualitative description is that it can never go much beyond a case-by-case approach, and it would clearly be impossible to encompass all of the many flow or transport systems that will be encountered in technological applications. The present book does not provide anything like a catalog of physically interesting phenomena. Hopefully, the reader will have already encountered some of these in the context of undergraduate laboratory studies or personal experience. There is also at least one textbook⁵ that attempts (with some success) to fill the gap between “analytic technique” and “physical phenomena” in fluid mechanics, and this can provide an important complement to the material presented here. In fluid mechanics, there is also a very useful video series available from Cambridge University Press for a very nominal cost, “Multi-Media Fluid Mechanics,” which is an excellent source of visual exposure to real phenomena, coupled with useful physical explanations as well.⁶ Finally, every student and teacher of fluid mechanics should examine the wonderful collection of photos in the book *An Album of Fluid Motions* by Van Dyke,⁷ the series of articles “A Gallery of Fluid Motion,” published annually in *Physics of Fluids*, and the recent compilation of highlights from these articles published as a book by Cambridge University Press, and also titled *A Gallery of Fluid Motion*.⁸ The events depicted in these latter photographs provide a graphic reminder of the vast wealth of complex, important,

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L. Gary Leal

Excerpt

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and interesting phenomena that are encompassed within fluid mechanics. Clearly the fluid mechanics and heat and mass transfer presented in the classroom or by any textbook only scratch the surface of this fascinating subject.

C. A BRIEF DESCRIPTION OF THE CONTENTS OF THIS BOOK

The material in this book is the basis of an introductory (two-term) graduate course on transport phenomena. It starts (in Chap. 2 of the book that is subsequently described in more detail) with a derivation of all of the necessary governing equations and boundary conditions in a context that is intended to focus on the underlying fundamental principles and the connections between this topic and other topics in continuum physics and thermodynamics. Some emphasis is also given to the limitations of both equations and boundary conditions (for example, “non-Newtonian” behavior, the “no-slip” condition, surfactant and thermocapillary effects at interfaces, etc.). It should be noted, however, that, though this course starts at the very beginning by deriving the basic equations from first principles and thus can be taken successfully even without an undergraduate transport background, there are important topics from the undergraduate curriculum that are *not* included, especially macroscopic balances, friction factors, correlations for turbulent flow conditions, etc.

The remainder of the book is more or less concerned with how to solve transport and fluids problems analytically, but with a lot of emphasis on basic physics, scaling and nondimensionalization, and approximations that can be used to obtain solutions that are due either to geometric simplifications or large or small values of dimensionless parameters. I am more specific in the following subsections, but it is important to note that *there is a strong emphasis on setting the problem up and extracting as much information as possible short of obtaining detailed solutions of differential equations*. The book reflects my bias that it is more important for students to see moderate numbers of problems with enough detail so that they can follow the analysis and the thinking behind the analysis from the beginning to the end. Although the problems chosen are obviously not going to be identical in most cases to a research problem that students may encounter later, they are chosen to expose students to many qualitative phenomena, and one may hope that with this background behind them they may be able to actually use the material in some future “application.” At the minimum, they should be able to read and understand the research literature on any transport-related problem that arises in their later work, including an understanding of the approximations or limitations, etc. In what follows, I outline the content of the various chapters, including the most important ideas or concepts that I hope a student or reader will extract.

Chapter 2: The Basic Principles

This book begins with a detailed derivation of the governing equations and boundary conditions for fluid mechanics and convective transport processes. Some emphasis is placed on understanding the limitations of these equations and boundary conditions, including the origins of non-Newtonian behavior. We also consider, in some detail, the boundary conditions at a fluid interface and the role of surfactants when these are present. At several points in this chapter, we begin to think qualitatively about flows, and particularly what we may anticipate about flows that are driven by body forces (gravity) in the presence of density gradients and by capillary forces that are due either to gradients in the interface curvature or to surface-tension gradients.

Chapter 3: Unidirectional and One-Dimensional Flow and Heat Transfer Problems

This chapter is *primarily* concerned with the most general class of problems for which an “exact” analytic solution is possible. Thus it is used to review classical methods of solution

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for linear partial differential equations, but that is not really the main point. The main point is to introduce the concepts of characteristic scales, nondimensionalization, dynamic similarity, diffusive time scales and their role in the transient evolution of flows and transport processes, and self-similarity for problems that do not exhibit characteristic scales. There is also a discussion of Taylor diffusion that does not exhibit an exact solution of the transport equations, but is an important and interesting problem of transport in a unidirectional flow with many applications. By the time we finish this chapter, there should be no doubt about how to nondimensionalize problems, how to solve problems that can be reduced to a linear form, and the reader will also have seen the first examples of using characteristic scales to think about transport problems.

Chapter 4: An Introduction to Asymptotic Approximations⁹

In this chapter, we discuss general concepts about asymptotic methods and illustrate a number of different types of asymptotic methods by considering relatively simple transport or flow problems. We do this by first considering pulsatile flow in a circular tube, for which we have already obtained a formal exact solution in Chap. 3, and show that we can obtain useful information about the high- and low-frequency limits more easily and with more physical insight by using asymptotic methods. Included in this is the concept of a boundary layer in the high-frequency limit. We then go on to consider problems for which no exact solution is available. The problems are chosen to illustrate important physical ideas and also to allow different types of asymptotic methods to be introduced:

- (a) We consider viscous dissipation effects in shear flow and indicate what it may have to do with the use of a shear rheometer to measure viscosities.
- (b) We consider flow in a tube that is slightly curved. This illustrates that the flows in straight and curved tubes are fundamentally different with potentially important implications for transport processes.
- (c) We consider flow in a wavy-wall channel primarily to show how “domain perturbation methods” can be used to turn this problem into a simpler problem that we can solve as flow in a straight-wall channel with “slip” at the boundaries.
- (d) We consider a simple model problem of transport inside a catalyst pellet with fast reaction to illustrate another example of a boundary-layer-type problem.
- (e) Finally there is a longish section on the dynamics of a gas bubble in a time-dependent pressure field that introduces ideas about linear stability analysis and its connection to perturbation methods, resonance when the forcing and natural frequencies of oscillation match, and multiple-time-scale asymptotic methods to analyze resonant behavior.

Chapter 5: The Thin-Gap Approximation – Lubrication Problems

One important class of problems for which we can obtain significant results at the first level of approximation is the motion of fluids in thin films. In this and the subsequent chapter, we consider how to analyze such problems by using the ideas of scaling and asymptotic approximation. In this chapter, we consider thin films between two solid surfaces, in which the primary physics is the large pressures that are set up by relative motions of the boundaries, and the resulting ideas about “lubrication” in a general sense.

- (a) The basic ideas are introduced by use of the classic problem of the eccentric Couette problem, called the “journal-bearing problem” in the lubrication literature. This problem is advantageous because the thin-gap approximation is uniformly valid throughout the domain in the so-called narrow-gap limit.
- (b) Following this, we derive the thin-film/lubrication equations from a more general point of view; one result of this general analysis is the famous Reynolds equation of lubrication

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theory, but we consider how to analyze such problems from a fundamental point of view that can be adapted to many applications even when it may not be immediately obvious how to apply the Reynolds equation.

(c) In the last sections, we consider several examples:

- (1) the so-called “slider-block problem”;
- (2) the motion of a sphere near a solid bounding wall, which leads to the conclusion that the sphere will not come into contact with the wall in finite time if it is moving under the action of a finite force and the surfaces are smooth;
- (3) an analysis of the dynamics of the disk on an *air hockey table*. This problem is amenable to “standard” lubrication theory when the blowing velocity is small enough (though still large enough to maintain a finite gap between the disk and the tabletop), but requires a boundary-layer-like analysis when the blowing velocity is large (even though the thin-film approximation is still valid).

Chapter 6: The Thin-Gap Approximation – Films with a Free Surface

The second basic class of thin-film problems involves the dynamics of films in which the upper surface is an interface (usually with air). In this case, the same basic scaling ideas are valid, but the objective is usually to determine the shape of the upper boundary (i.e., the geometry of the thin film), which is usually evolving in time.

A typical example is a spreading film on a solid substrate, and we begin with this class of problems. Analysis of this class of thin-film problems requires use of the interface boundary conditions derived in Chap. 2 and also revisits a number of examples of capillary and Marangoni flow problems that were discussed qualitatively in Chap 2. The governing equation for the thin-film shape function often takes the form of a “nonlinear diffusion equation,” and this allows the scaling behavior of the thin-film dynamics to be deduced by means of a similarity transformation (“advanced” dimensional analysis), without necessarily solving the resulting nonlinear equation. For example, for the spreading of an axisymmetric film (or drop) on a solid substrate caused by capillary effects, we can deduce the famous Tanner’s law that the radius of the contact circle should increase as $R(t) \sim t^{1/10}$ without solving equations. These are great examples for illustrating what we can get from seeking the form of self-similar solutions.

We then go on consider the role of van der Waals forces on the dynamics of a thin film. First we consider the *stability* of a horizontal fluid layer (which is bounded either above or below by a solid substrate) due to the coupled interactions of gravity, capillary forces, and van der Waals forces across the film. This allows us to introduce the ideas of a *linear stability analysis* and leads to interesting and important results. We then consider the actual rupture process of a thin film with van der Waals forces present. In particular, we show that the final stages of the rupture process, including the geometry of the film and the scaling of the process with time, can be analyzed again by means of a similarity transformation (without solving equations).

Finally, we consider a number of problems involving nonisothermal flows in a shallow cavity. The motion in this cavity may be due to buoyancy caused by differential heating of the end walls or to thermocapillary flow that is due to Marangoni stresses at the upper interface, again with differential heating at the end walls. These problems are idealized models for a number of important applications; for example, the latter case is a model for the “liquid bridge” in containerless processing of single crystals. The objective of analysis is the flow and temperature fields, but also the shape of the free surface. It is shown that the interface shape problems can be analyzed both by means of the classic thin-gap approach of preceding sections of this chapter and also by the method of “domain perturbations,” first

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introduced in Chap. 4. These latter problems focus again on the important issue of interfacial boundary conditions and the role of capillary and thermocapillary effects in flow.

Chapter 7: Creeping Flows (Two-Dimensional and Axisymmetric Problems)¹⁰

We begin, in this chapter and the next, with the class of flow problems for general geometries in which the dynamics is dominated by the balance between pressure gradients and the viscous terms in the Navier–Stokes equation. This class of problem is known collectively as “creeping” flows. In the first of these two chapters, we initially consider nondimensionalization, the role of the Reynolds number for this general class of problems, the concepts of quasi-steady flow, and some extremely important consequences of the fact that the governing equations in the creeping-flow limit are linear. The latter material is important beyond the several examples considered, because it forces the student to think about what can be said about the solution of linear problems without actually solving any equations. We then go on in this chapter to consider two-dimensional and axisymmetric problems that can be solved by introducing the concept of a streamfunction. This leads to a single fourth-order partial differential equation and the natural use of general eigenfunction expansions. The following specific problems are solved:

- (a) 2D corner flows (scraping, mixing, etc.),
- (b) uniform flow past a solid sphere (the classic Stokes problem),
- (c) axisymmetric extensional flow in the vicinity of a solid sphere and the use of this result to derive the famous Einstein expression for the viscosity of a dilute suspension of spheres,
- (d) the buoyancy-driven translation of a drop through a quiescent fluid including the fact that the shape is a sphere independent of the interfacial tension,
- (e) motions of drops driven by Marangoni stress in a nonuniform temperature field,
- (f) the effects of surfactants on the buoyancy-driven motion of a drop.

These problems are chosen because they illustrate important ideas and concepts in addition to simply solving problems. However, the analysis in this chapter is completely based on classical eigenfunction expansions.

Chapter 8: Creeping Flows (Three-Dimensional Problems)¹¹

We begin this chapter in Sections A–C by discussing the construction of solutions to the creeping-flow equations by representing the solutions in terms of “vector harmonic functions.” It is shown that one can literally write the solutions of a *whole class of problems* almost by inspection, thus eliminating the need for the laborious eigenfunction expansions of Chap. 7 even for the two-dimensional and axisymmetric problems for which they can be used, but also simultaneously obtaining the solutions for fully three-dimensional problems (e.g., a sphere in a linear shear flow). The main requirement is that the boundaries of the flow domain must correspond approximately to surfaces in a known analytic coordinate system. In this chapter, we consider problems that we can solve by using vector harmonic functions in a spherical coordinate system. The method is illustrated for a number of examples including both problems with axisymmetric symmetry that we could solve by using the methods of Chap. 7, and problems such as particle motion in a linear shear flow that we could not solve by using these methods. We conclude in Section C by considering the motion of drops in general linear (shearlike) flows, including an illustration of how to estimate the deformed shape of the drop in the flow.

In subsequent sections of this chapter we discuss the use of *fundamental solutions* of the creeping-flow equations to construct solutions for which the flow domain has a more

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general geometry. This includes “slender-body” theory for slender rodlike objects, and an introduction to a powerful method known as the “boundary-integral” technique that can be implemented numerically to solve virtually any creeping-flow problem, including those with complex or unknown (possibly evolving) geometries. Other sections illustrate general results that can be obtained because of the linearity of creeping-flow problems, with an emphasis on illustrating general physical phenomena for this class of problem.

Chapter 9: Convection Effects and Heat Transfer for Viscous Flows

Now that we have learned how to solve for the detailed velocity fields for at least one class of flow problems (creeping/viscous flows), we turn to a general introduction to convection effects for heat transfer (primarily) for this class of flows.

We begin by considering the nondimensional form of the thermal energy equation, leading to the recognition of the Peclet number (the product of the Reynolds number and the Prandtl number) as the critical independent parameter for “forced” convection heat transfer problems. At the end of this section, we briefly discuss the analogy with mass transfer in a two-component system, with the Schmidt number replacing the Prandtl number and the Sherwood number replacing the Nusselt number.

The limit $Pe \rightarrow 0$ yields the pure conduction heat transfer case. However, for a fluid in motion, we find that the pure conduction limit is not a uniformly valid first approximation to the heat transfer process for $Pe \ll 1$, but breaks down “far” from a heated or cooled body in a flow. We discuss this in the context of the “Whitehead” paradox for heat transfer from a sphere in a uniform flow and then show how the problem of forced convection heat transfer from a body in a flow can be understood in the context of a singular-perturbation analysis. This leads to an estimate for the first correction to the Nusselt number for small but finite Pe – this is the first “small” effect of convection on the correlation between Nu and Pe for a heated (or cooled) sphere in a uniform flow.

We then return briefly to consider the creeping-flow approximation of the previous two chapters. We do this at this point because we recognize that the creeping-flow solution is exactly analogous to the pure conduction heat transfer solution of the preceding section and thus should also not be a uniformly valid first approximation to flow at low Reynolds number. We thus explain the sense in which the creeping-flow solution can be accepted as a first approximation (i.e., why does it play the important role in the analysis of viscous flows that it does?), and in principle how it might be “corrected” to account for convection of momentum (or vorticity) for the realistic case of flows in which Re is small but nonzero.

We then go on to consider the generalization of the analysis of heat transfer problems for small Peclet numbers. *These generalizations clearly illustrate the power of the asymptotic method to provide insight into the form of correlations between dimensionless parameters, with a minimum of detailed analysis.* First, we show that the detailed analysis that we developed for a sphere actually can be applied with no extra work to obtain the first correction to Nu for bodies of *arbitrary shape* in a uniform flow (or where the body is sedimenting through an otherwise motionless fluid). Next, we consider heat transfer from a sphere in a shear flow. The purpose of this is to show that the same theoretical framework can be applied again, but that the form of the correlation between Nu and Pe changes if the nature of the flow is changed. Again, the analysis for a sphere in linear shear flow can be generalized with little additional work to obtain the correlation for any linear flow and for bodies of arbitrary shape.

The second half of this chapter considers the opposite limit in which $Pe \gg 1$. In this case, the superficial conclusion is that heat transfer must be dominated everywhere by convection. However, this cannot be true, as the only mechanism for heat transfer from the surface of a body to a surrounding fluid is by conduction. This leads to the concept of the

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thermal boundary layer and a fundamentally different form for the correlation between Nu and Pe .

Chapter 10: Boundary-Layer Theory for Laminar Flows

The concept of a boundary layer is one of the most important ideas in understanding transport processes. It is based on the idea that transport systems often generate internal length scales so that dissipative effects (or diffusive effects in the case of heat transfer or mass transfer) continue to play an essential role even in the limit as the viscosity (or the diffusivity) becomes smaller and smaller. In this chapter, we continue the development of these ideas, first introduced at the end of the previous chapter, by considering their application to the approximate solution of fluid mechanics problems in the asymptotic limit of large Reynolds number. The chapter begins with a section on potential-flow theory, namely the solutions of the equations of motion when viscous effects are completely neglected. We find that the predictions that leave out viscous effects are fatally flawed for some problems such as flow past a circular cylinder, leading to the famous d’Alembert’s paradox, which says that the drag on bodies at high Reynolds number is zero. This occurs mainly because potential-flow theory cannot predict the asymmetry that is responsible for boundary-layer separation and the dominance of “form” drag for nonstreamlined bodies. The next section of the chapter develops the key ideas of the asymptotic boundary-layer theory. This is first applied to the classic Blasius problem of flow past a horizontal flat plate and then considers the class of problems in which self-similar solutions of the boundary-layer equations are possible. This is followed by the Blasius series solution for flow past nonstreamlined bodies and the application of this theory to the problem of flow past a circular cylinder. This exposes a key result, which is the ability of boundary-layer theory to predict the onset of “separation” and thus to determine whether a two-dimensional body (such as an airfoil) is sufficiently streamlined to avoid “form” drag. We then consider the generalization of boundary-layer theory to axisymmetric geometries. Finally, we address the question of boundary layers on a free surface, such as an interface, by considering the application of boundary-layer concepts to the motion of a spherical bubble at high Reynolds number. This section is perhaps the most important one in the chapter from a pedagogical point of view, because it challenges most of the simplistic ideas that students may have from undergraduate transport courses, and forces them to see that boundary layers are applicable to a very broad class of problems. For example, the question of a boundary layer on a bubble forces students to reconsider the simplistic (and often incorrect) idea that a boundary layer exists because of the no-slip condition.

Chapter 11: Heat and Mass Transfer at Large Reynolds Number

In this chapter, we return to forced convection heat and mass transfer problems when the Reynolds number is large enough that the velocity field takes the boundary-layer form. For this class of problems, we find that there must be a correlation between the dimensionless transport rate (i.e., the Nusselt number for heat transfer) and the independent dimensionless parameters, Reynolds number Re and either Prandtl number Pr or Schmidt number Sc of the form

$$Nu = cRe^a Pr^b \quad \text{or} \quad Nu = cRe^a Sc^b.$$

The coefficient $a = 0.5$ for laminar flow conditions and $Re \gg 1$. On the other hand, the coefficient b depends on the magnitude of the Prandtl (or Schmidt) number and also changes depending on whether the boundary is a no-slip surface or a fluid interface. For example, for a no-slip surface, $b = 1/2$ in the limit Pr (or Sc) $\rightarrow 0$ but $b = 1/3$ for Pr (or Sc) $\rightarrow \infty$. By now, students can easily analyze and understand qualitatively the reasons for these changes, as well as the effect of changes in the fluid mechanics or thermal boundary conditions. The coefficient c is an order 1 number that depends on the geometry, but we show that

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very general solutions for “arbitrary” body shapes can be obtained by means of similarity transformations. Finally, we readdress the issue of the analogy between heat transfer and single-component mass transfer by considering the effects of finite interfacial velocities that must exist at a boundary that acts as a source or sink of material in the mass transfer problem but not in the thermal problem.

Chapter 12: Hydrodynamic Stability

All of the preceding chapters seek solutions for various transport and fluid flow problems, without addressing the stability of the solutions that are obtained. The ideas of linear stability theory are very important both within the transport area and also in a variety of other problem areas that students are likely to encounter. Too often, it is not addressed in transport courses, even at the graduate level. The purpose of this chapter is to introduce students to the ideas of linear stability theory and to the methods of analysis. The problems chosen are selected because it is possible to make analytic progress and because they are of particular relevance to chemical engineering applications. The one topic that is only lightly covered is the stability of parallel shear flows. This is primarily because it is such a subtle and complicated subject that one cannot do justice to it in this type of presentation (it is the subject of complete books all by itself).

We begin with *capillary instability* of a liquid thread. This is a problem that was discussed qualitatively already in Chap. 2. It is a problem with a physically clear mechanism for instability and thus provides a good framework for introducing the basic ideas of linear stability theory. This problem is one of several examples in which the viscosity of the fluid plays no role in determining stability, but only influences the rate of growth or decay of the infinitesimal disturbances that are analyzed in a linear theory.

Next, we turn to the classic problem of *Rayleigh–Taylor instability* for the gravitationally driven “overturning” of a pair of immiscible superposed fluids in which the upper fluid has a higher density than the lower fluid. This is another example of a problem in which the viscosity of the fluid is not an essential factor in its instability.

The third problem is known as the *Saffman–Taylor instability* of a fluid interface for motion of a pair of fluids with different viscosities in a porous medium. It is this instability that leads to the well-known and important phenomenon of viscous fingering. In this case, we first discuss Darcy’s law for motion of a single-phase fluid in a porous medium, and then we discuss the instability that occurs because of the displacement of one fluid by another when there is a discontinuity in the viscosity and permeability across an interface. The analysis presented ignores surface-tension effects and is thus valid strictly for “miscible displacement.”

Next we turn to the *stability of Couette flow* for parallel rotating cylinders. This is an important flow for various applications, and, though it is a shear flow, the stability is dominated by the centrifugal forces that arise because of centripetal acceleration. This problem is also an important contrast with the first two examples because it is a case in which the flow can actually be stabilized by viscous effects. We first consider the classic case of an inviscid fluid, which leads to the well-known criteria of Rayleigh for the stability of an inviscid fluid. We then analyze the role of viscosity for the case of a narrow gap in which analytic results can be obtained. We show that the flow is stabilized by viscous diffusion effects up to a critical value of the Reynolds number for the problem (here known as the Taylor number).

We then go on to consider three examples of instabilities that arise because of buoyancy and Marangoni effects in a nonisothermal system. This is preceded by a brief discussion of the Bousinesq approximation of the Navier–Stokes and thermal energy equations.

The first problem considered is the classic problem of *Rayleigh–Benard convection* – namely the instability that is due to buoyancy forces in a quiescent fluid layer that is heated