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AN INTRODUCTION TO PARTIAL DIFFERENTIAL EQUATIONS

A complete introduction to partial differential equations, this textbook provides a rigorous yet accessible guide to students in mathematics, physics and engineering. The presentation is lively and up to date, with particular emphasis on developing an appreciation of underlying mathematical theory.

Beginning with basic definitions, properties and derivations of some fundamental equations of mathematical physics from basic principles, the book studies first-order equations, the classification of second-order equations, and the one-dimensional wave equation. Two chapters are devoted to the separation of variables, whilst others concentrate on a wide range of topics including elliptic theory, Green's functions, variational and numerical methods.

A rich collection of worked examples and exercises accompany the text, along with a large number of illustrations and graphs to provide insight into the numerical examples.

Solutions and hints to selected exercises are included for students whilst extended solution sets are available to lecturers from solutions@cambridge.org.

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To our parents

The equation of heaven and earth
remains unsolved.
(Yehuda Amichai)

הַמִּשְׁוָאָה שֶׁל שָׁמַיִם וָאָרֶץ
נִשְׁאֶרֶת בְּלִי פִּתְרוֹן.
(יהודה עמיחי)

Contents

Preface

This book presents an introduction to the theory and applications of partial differential equations (PDEs). The book is suitable for all types of basic courses on PDEs, including courses for undergraduate engineering, sciences and mathematics students, and for first-year graduate courses as well.

Having taught courses on PDEs for many years to varied groups of students from engineering, science and mathematics departments, we felt the need for a textbook that is concise, clear, motivated by real examples and mathematically rigorous. We therefore wrote a book that covers the foundations of the theory of PDEs. This theory has been developed over the last 250 years to solve the most fundamental problems in engineering, physics and other sciences. Therefore we think that one should not treat PDEs as an abstract mathematical discipline; rather it is a field that is closely related to real-world problems. For this reason we strongly emphasize throughout the book the relevance of every bit of theory and every practical tool to some specific application. At the same time, we think that the modern engineer or scientist should understand the basics of PDE theory when attempting to solve specific problems that arise in applications. Therefore we took great care to create a balanced exposition of the theoretical and applied facets of PDEs.

The book is flexible enough to serve as a textbook or a self-study book for a large class of readers. The first seven chapters include the core of a typical one-semester course. In fact, they also include advanced material that can be used in a graduate course. Chapters 9 and 11 include additional material that together with the first seven chapters fits into a typical curriculum of a two-semester course. In addition, Chapters 8 and 10 contain advanced material on Green's functions and the calculus of variations. The book covers all the classical subjects, such as the separation of variables technique and Fourier's method (Chapters 5, 6, 7, and 9), the method of characteristics (Chapters 2 and 9), and Green's function methods (Chapter 8). At the same time we introduce the basic theorems that guarantee that the problem at hand is well defined (Chapters 2–10), and we took care to include modern ideas such as variational methods (Chapter 10) and numerical methods (Chapter 11).

The first eight chapters mainly discuss PDEs in two independent variables. Chapter 9 shows how the methods of the first eight chapters are extended and enhanced to handle PDEs in higher dimensions. Generalized and weak solutions are presented in many parts of the book.

Throughout the book we illustrate the mathematical ideas and techniques by applying them to a large variety of practical problems, including heat conduction, wave propagation, acoustics, optics, solid and fluid mechanics, quantum mechanics, communication, image processing, musical instruments, and traffic flow.

We believe that the best way to grasp a new theory is by considering examples and solving problems. Therefore the book contains hundreds of examples and problems, most of them at least partially solved. Extended solutions to the problems are available for course instructors using the book from solutions@cambridge.org. We also include dozens of drawing and graphs to explain the text better and to demonstrate visually some of the special features of certain solutions.

It is assumed that the reader is familiar with the calculus of functions in several variables, with linear algebra and with the basics of ordinary differential equations. The book is almost entirely self-contained, and in the very few places where we cannot go into details, a reference is provided.

The book is the culmination of a slow evolutionary process. We wrote it during several years, and kept changing and adding material in light of our experience in the classroom. The current text is an expanded version of a book in Hebrew that the authors published in 2001, which has been used successfully at Israeli universities and colleges since then.

Our cumulative expertise of over 30 years of teaching PDEs at several universities, including Stanford University, UCLA, Indiana University and the Technion – Israel Institute of Technology guided to us to create a text that enhances not just technical competence but also deep understanding of PDEs. We are grateful to our many students at these universities with whom we had the pleasure of studying this fascinating subject. We hope that the readers will also learn to enjoy it.

We gratefully acknowledge the help we received from a number of individuals. Kristian Jenssen from North Carolina State University, Lydia Peres and Tiferet Saadon from the Technion – Israel Institute of Technology, and Peter Sternberg from Indiana University read portions of the draft and made numerous comments and suggestions for improvement. Raya Rubinstein prepared the drawings, while Yishai Pinchover and Aviad Rubinstein assisted with the graphs. Despite our best efforts, we surely did not discover all the mistakes in the draft. Therefore we encourage observant readers to send us their comments at pincho@techunix.technion.ac.il. We will maintain a webpage with a list of errata at http://www.math.technion.ac.il/~pincho/PDE.pdf.