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# 1

# Introduction to Multiphase Flow

## 1.1 Introduction

### 1.1.1 Scope

In the context of this book, the term *multiphase flow* is used to refer to any fluid flow consisting of more than one phase or component. For brevity and because they are covered in other texts, we exclude those circumstances in which the components are well mixed above the molecular level. Consequently, the flows considered here have some level of phase or component separation at a scale well above the molecular level. This still leaves an enormous spectrum of different multiphase flows. One could classify them according to the state of the different phases or components and therefore refer to gas/solids flows or liquid/solids flows or gas/particle flows or bubbly flows and so on; many texts exist that limit their attention in this way. Some treatises are defined in terms of a specific type of fluid flow and deal with low-Reynolds-number suspension flows, dusty gas dynamics, and so on. Others focus attention on a specific application such as slurry flows, cavitating flows, aerosols, debris flows, fluidized beds, and so on; again, there are many such texts. In this book we attempt to identify the basic fluid mechanical phenomena and to illustrate those phenomena with examples from a broad range of applications and types of flow.

Parenthetically, it is valuable to reflect on the diverse and ubiquitous challenges of multiphase flow. Virtually every processing technology must deal with multiphase flow, from cavitating pumps and turbines to electrophotographic processes to papermaking to the pellet form of almost all raw plastics. The amount of granular material, coal, grain, ore, and so on that is transported every year is enormous and, at many stages, that material is required to flow. Clearly the ability to predict the fluid flow behavior of these processes is central to the efficiency and effectiveness of those processes. For example, the effective flow of toner is a major factor in the quality and speed of electrophotographic printers. Multiphase flows are also a ubiquitous feature of our environment whether one considers rain, snow, fog, avalanches, mud slides, sediment transport, debris flows, and countless other natural phenomena, to say nothing of what happens beyond our planet. Very critical biological and medical flows are also multiphase, from blood flow to semen to the bends to lithotripsy to laser surgery cavitation and so on. No single list can adequately illustrate the diversity and ubiquity; consequently, any attempt at a comprehensive treatment of multiphase flows is flawed unless it focuses on common phenomenological themes and avoids the temptation to digress into lists of observations.

Two general topologies of multiphase flow can be usefully identified at the outset, namely *disperse flows* and *separated flows*. By *disperse flows* we mean those consisting of finite particles, drops, or bubbles (the disperse phase) distributed in a connected volume of the continuous phase. *Separated flows* consist of two or more continuous streams of different fluids separated by interfaces.

### **1.1.2** Multiphase Flow Models

A persistent theme throughout the study of multiphase flows is the need to model and predict the detailed behavior of those flows and the phenomena that they manifest. There are three ways in which such models are explored: (1) experimentally, through laboratory-sized models equipped with appropriate instrumentation; (2) theoretically, using mathematical equations and models for the flow; and (3) computationally, using the power and size of modern computers to address the complexity of the flow. Clearly there are some applications in which full-scale laboratory models are possible. But, in many instances, the laboratory model must have a very different scale from the prototype and then a reliable theoretical or computational model is essential for confident extrapolation to the scale of the prototype. There are also cases in which a laboratory model is impossible for a wide variety of reasons.

Consequently, the predictive capability and physical understanding must rely heavily on theoretical and/or computational models and here the complexity of most multiphase flows presents a major hurdle. It may be possible at some distant time in the future to code the Navier–Stokes equations for each of the phases or components and to compute every detail of a multiphase flow, the motion of all the fluid around and inside every particle or drop, the position of every interface. But the computer power and speed required to do this are far beyond present capability for most of the flows that are commonly experienced. When one or both of the phases become turbulent (as often happens), the magnitude of the challenge becomes truly astronomical. Therefore, simplifications are essential in realistic models of most multiphase flows.

In disperse flows two types of models are prevalent, *trajectory models* and *two-fluid models*. In trajectory models, the motion of the disperse phase is assessed by following the motion of either the actual particles or larger, representative *particles*. The details of the flow around each of the particles are subsumed into assumed drag, lift, and moment forces acting on and altering the trajectory of those particles. The thermal history of the particles can also be tracked if it is appropriate to do so. Trajectory models have been very useful in studies of the rheology of granular flows (see Chapter 13) primarily because the effects of the interstitial fluid are small. In the alternative approach, two-fluid models, the disperse phase is treated as a second continuous phase intermingled and interacting with the continuous phase. Effective conservation equations (of mass, momentum, and energy) are developed for the two fluid flows; these include interaction terms modeling the exchange of mass, momentum, and energy between the two flows. These equations are then solved either theoretically or computationally. Thus, two-fluid models neglect the discrete nature of the disperse phase and approximate its effects on the continuous phase. Inherent in this approach are averaging processes necessary to characterize the properties of the disperse phase; these involve significant difficulties. The boundary conditions appropriate in two-fluid models also pose difficult modeling issues.

In contrast, separated flows present many fewer issues. In theory one must solve the single-phase fluid-flow equations in the two streams, coupling them through appropriate kinematic and dynamic conditions at the interface. Free streamline theory (see, for example, Birkhoff and Zarantonello 1957, Tulin 1964, Woods 1961, Wu 1972) is an example of a successful implementation of such a strategy, though the interface conditions used in that context are particularly simple.

In the first part of this book, the basic tools for both trajectory and two-fluid models are developed and discussed. In the remainder of this first chapter, a basic notation for multiphase flow is developed and this leads naturally into a description of the mass, momentum, and energy equations applicable to multiphase flows and, particularly in two-fluid models. In Chapters 2, 3, and 4, we examine the dynamics of individual particles, drops, and bubbles. In Chapter 7 we address the different topologies of multiphase flows and, in the subsequent chapters, we examine phenomena in which *particle* interactions and the particle/fluid interactions modify the flow.

### 1.1.3 Multiphase Flow Notation

The notation that is used is close to the standard described by Wallis (1969). It has, however, been slightly modified to permit more ready adoption to the Cartesian tensor form. In particular the subscripts that can be attached to a property consist of a group of uppercase subscripts followed by lowercase subscripts. The lowercase subscripts ($i$, $ij$, etc.) are used in the conventional manner to denote vector or tensor components. A single uppercase subscript (N) refers to the property of a specific phase or component. In some contexts generic subscripts N = A, B are used for generality. However, other letters such as N = C (continuous phase), N = D (disperse phase), N = L (liquid), N = G (gas), N = V (vapor), or N = S (solid) are used for clarity in other contexts. Finally two uppercase subscripts imply the difference between the two properties for the two single uppercase subscripts.

Specific properties frequently used are as follows. *Volumetric fluxes* (volume flow per unit area) of individual components are denoted by $j_{\mathrm{A}i}$, $j_{\mathrm{B}i}$ ($i = 1, 2,$ or $3$ in three-dimensional flow). These are sometimes referred to as superficial component velocities. The *total volumetric flux*, $j_i$, is then given by the following:

$$j_i = j_{Ai} + j_{Bi} + \cdots = \sum_N j_{Ni}. \tag{1.1}$$

*Mass fluxes* are similarly denoted by $G_{Ai}$, $G_{Bi}$, or $G_i$. Thus if the densities of individual components are denoted by $\rho_A$, $\rho_B$ it follows that

$$G_{Ai} = \rho_A j_{Ai}; \quad G_{Bi} = \rho_B j_{Bi}; \quad G_i = \sum_N \rho_N j_{Ni}. \tag{1.2}$$

Velocities of the specific phases are denoted by $u_{Ai}$, $u_{Bi}$ or, in general, by $u_{Ni}$. The relative velocity between the two phases A and B is denoted by $u_{ABi}$ such that

$$u_{Ai} - u_{Bi} = u_{ABi}. \tag{1.3}$$

The volume fraction of a component or phase is denoted by $\alpha_N$ and, in the case of two components or phases, A and B, it follows that $\alpha_B = 1 - \alpha_A$. Though this is clearly a well-defined property for any finite volume in the flow, there are some substantial problems associated with assigning a value to an infinitesimal volume or point in the flow. Provided these can be resolved, it follows that the volumetric flux of a component, $N$, and its velocity are related by

$$j_{Ni} = \alpha_N u_{Ni} \tag{1.4}$$

and that

$$j_i = \alpha_A u_{Ai} + \alpha_B u_{Bi} + \cdots = \sum_N \alpha_N u_{Ni}. \tag{1.5}$$

Two other fractional properties are relevant only in the context of one-dimensional flows. The *volumetric quality*, $\beta_N$, is the ratio of the volumetric flux of the component, N, to the total volumetric flux, that is,

$$\beta_N = j_N / j, \tag{1.6}$$

where the index $i$ has been dropped from $j_N$ and $j$ because $\beta$ is only used in the context of one-dimensional flows and the $j_N$, $j$ refer to cross-sectionally averaged quantities.

The *mass fraction*, $x_A$, of a phase or component, A, is simply given by $\rho_A \alpha_A / \rho$ [see Eq. (1.8) for $\rho$]. Conversely, the *mass quality*, $\mathcal{X}_A$, is often referred to simply as *the quality* and is the ratio of the mass flux of component A to the total mass flux or

$$\mathcal{X}_A = \frac{G_A}{G} = \frac{\rho_A j_A}{\sum_N \rho_N j_N}. \tag{1.7}$$

Furthermore, when only two components or phases are present it is often redundant to use subscripts on the volume fraction and the qualities because $\alpha_A = 1 - \alpha_B$, $\beta_A = 1 - \beta_B$, and $\mathcal{X}_A = 1 - \mathcal{X}_B$. Thus unsubscripted quantities $\alpha$, $\beta$, and $\mathcal{X}$ are often used in these circumstances.

It is clear that a multiphase mixture has certain *mixture* properties, of which the most readily evaluated is the *mixture* density denoted by $\rho$ and given by the following:

$$\rho = \sum_{\mathrm{N}} \alpha_{\mathrm{N}} \rho_{\mathrm{N}}. \tag{1.8}$$

Conversely, the specific enthalpy, $h$, and specific entropy, $s$, being defined as per unit mass rather than per unit volume, are weighted according to the following:

$$\rho h = \sum_{\mathrm{N}} \rho_{\mathrm{N}} \alpha_{\mathrm{N}} h_{\mathrm{N}}; \quad \rho s = \sum_{\mathrm{N}} \rho_{\mathrm{N}} \alpha_{\mathrm{N}} s_{\mathrm{N}}. \tag{1.9}$$

Other properties such as the *mixture* viscosity or thermal conductivity cannot be reliably obtained from such simple weighted means.

Aside from the relative velocities between phases that were described earlier, there are two other measures of relative motion that are frequently used. The *drift velocity* of a component is defined as the velocity of that component in a frame of reference moving at a velocity equal to the total volumetric flux, $j_i$, and is therefore given by $u_{\mathrm{NJ}i}$, where

$$u_{\mathrm{NJ}i} = u_{\mathrm{N}i} - j_i. \tag{1.10}$$

Even more frequent use will be made of the *drift flux* of a component, which is defined as the volumetric flux of a component in the frame of reference moving at $j_i$. Denoted by $j_{\mathrm{NJ}i}$, this is given by the following:

$$j_{\mathrm{NJ}i} = j_{\mathrm{N}i} - \alpha_{\mathrm{N}} \mathrm{j}_i = \alpha_{\mathrm{N}}(u_{\mathrm{N}i} - j_i) = \alpha_{\mathrm{N}} u_{\mathrm{NJ}i}. \tag{1.11}$$

It is particularly important to notice that the sum of all the drift fluxes must be zero because from Eq. (1.11) we have the following:

$$\sum_{\mathrm{N}} j_{\mathrm{NJ}i} = \sum_{\mathrm{N}} j_{\mathrm{N}i} - j_i \sum_{\mathrm{N}} \alpha_{\mathrm{N}} = j_i - j_i = 0. \tag{1.12}$$

When only two phases or components, A and B, are present it follows that $j_{\mathrm{AJ}i} = -j_{\mathrm{BJ}i}$ and hence it is convenient to denote both of these drift fluxes by the vector $j_{\mathrm{AB}i}$, where we have the following:

$$j_{\mathrm{AB}i} = j_{\mathrm{AJ}i} = -j_{\mathrm{BJ}i}. \tag{1.13}$$

Moreover it follows from Eq. (1.11) that we have the following:

$$j_{\mathrm{AB}i} = \alpha_{\mathrm{A}} \alpha_{\mathrm{B}} u_{\mathrm{AB}i} = \alpha_{\mathrm{A}}(1 - \alpha_{\mathrm{A}}) u_{\mathrm{AB}i}, \tag{1.14}$$

and hence the drift flux, $j_{\mathrm{AB}i}$, and the relative velocity, $u_{\mathrm{AB}i}$, are simply related.

Finally, it is clear that certain basic relations follow from the preceding definitions and it is convenient to identify these here for later use. First the relations between the volume and mass qualities that follow from Eqs. (1.6) and (1.7) only involve ratios of

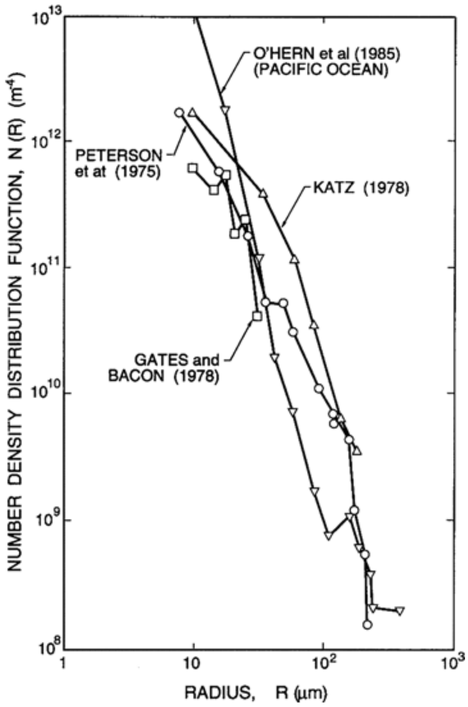

Figure 1.1. Measured size distribution functions for small bubbles in three different water tunnels (Peterson *et al*. 1975, Gates and Bacon 1978, Katz 1978) and in the ocean off Los Angeles, California (O'Hern *et al*. 1985).

the densities of the following components:

$$\mathcal{X}_A = \beta_A \Big/ \sum_N \left(\frac{\rho_N}{\rho_A}\right) \beta_N; \quad \beta_A = \mathcal{X}_A \Big/ \sum_N \left(\frac{\rho_A}{\rho_N}\right) \mathcal{X}_N. \tag{1.15}$$

Conversely, the relation between the volume fraction and the volume quality necessarily involves some measure of the relative motion between the phases (or components). The following useful results for two-phase (or two-component) one-dimensional flows can readily be obtained from Eqs. (1.11) and (1.6) as follows:

$$\beta_N = \alpha_N + \frac{j_{NJ}}{j}; \quad \beta_A = \alpha_A + \frac{j_{AB}}{j}; \quad \beta_B = \alpha_B - \frac{j_{AB}}{j}, \tag{1.16}$$

which demonstrate the importance of the drift flux as a measure of the relative motion.

### 1.1.4 Size Distribution Functions

In many multiphase flow contexts we shall make the simplifying assumption that all the disperse phase particles (bubbles, droplets, or solid particles) have the same size. However, in many natural and technological processes it is necessary to consider the distribution of particle size. One fundamental measure of this is the size distribution function, $N(v)$, defined such that the number of particles in a unit volume of the multiphase mixture with volume between $v$ and $v + dv$ is $N(v)dv$. For convenience,

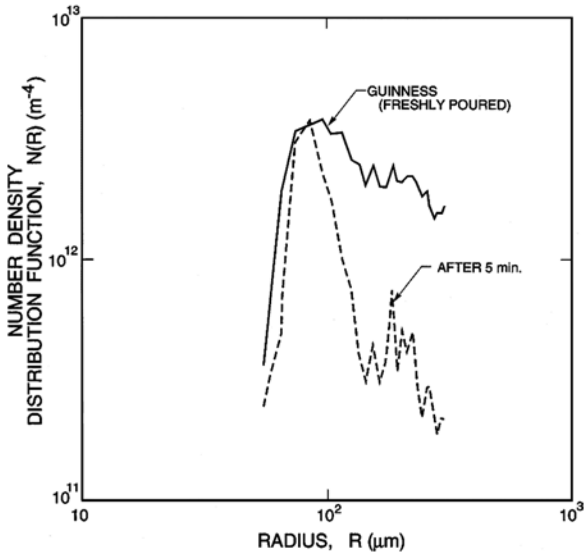

Figure 1.2. Size distribution functions for bubbles in freshly poured Guinness and after five minutes. Adapted from Kawaguchi and Maeda (2003).

it is often assumed that the particle size can be represented by a single linear dimension (for example, the diameter, $D$, or radius, $R$, in the case of spherical particles) so that alternative size distribution functions, $N'(D)$ or $N''(R)$, may be used. Examples of size distribution functions based on radius are shown in Figures 1.1 and 1.2.

Often such information is presented in the form of cumulative number distributions. For example, the cumulative distribution, $N^*(v^*)$, defined as

$$N^*(v^*) = \int_0^{v^*} N(v)dv, \tag{1.17}$$

is the total number of particles of volume less than $v^*$. Examples of cumulative distributions (in this case for coal slurries) are shown in Figure 1.3.

In these disperse flows, the evaluation of global quantities or characteristics of the disperse phase will clearly require integration over the full range of particle sizes using the size distribution function. For example, the volume fraction of the disperse phase,

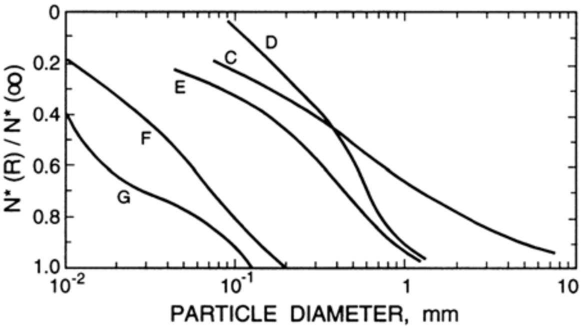

Figure 1.3. Cumulative size distributions for various coal slurries. Adapted from Shook and Roco (1991).

$\alpha_D$, is given by the following:

$$\alpha_D = \int_0^\infty v N(v) dv = \frac{\pi}{6}\int_0^\infty D^3 N'(D) dD, \tag{1.18}$$

where the last expression clearly applies to spherical particles. Other properties of the disperse phase or of the interactions between the disperse and continuous phases can involve other moments of the size distribution function (see, for example, Friedlander 1977). This leads to a series of mean diameters (or sizes in the case of nonspherical particles) of the form, $D_{jk}$, where

$$D_{jk} = \left[\frac{\int_0^\infty D^j N'(D) dD}{\int_0^\infty D^k N'(D) dD}\right]^{\frac{1}{j-k}}. \tag{1.19}$$

A commonly used example is the *mass mean* diameter, $D_{30}$. Conversely, processes that are controlled by particle surface area would be characterized by the *surface area mean* diameter, $D_{20}$. The surface area mean diameter would be important, for example, in determining the exchange of heat between the phases or the rates of chemical interaction at the disperse phase surface. Another measure of the average size that proves useful in characterizing many disperse particulates is the Sauter mean diameter, $D_{32}$. This is a measure of the ratio of the particle volume to the particle surface area and, as such, is often used in characterizing particulates (see, for example, Chapter 14).

## 1.2 Equations of Motion

### 1.2.1 Averaging

In Section 1.1.3 it was implicitly assumed that there existed an *infinitesimal* volume of dimension, $\epsilon$, such that $\epsilon$ was not only very much smaller than the typical distance over which the flow properties varied significantly but also very much larger than the size of the individual phase elements (the disperse phase particles, drops, or bubbles). The first condition is necessary to define derivatives of the flow properties within the flow field. The second is necessary so that each *averaging* volume (of volume $\epsilon^3$) contains representative samples of each of the components or phases. In the sections that follow (Sections 1.2.2 to 1.2.9), we proceed to develop the effective differential equations of motion for multiphase flow assuming that these conditions hold.

However, one of the more difficult hurdles in treating multiphase flows is that the preceding two conditions are rarely both satisfied. As a consequence the averaging volumes contain a finite number of finite-sized particles and therefore flow properties such as the continuous phase velocity vary significantly from point to point within these averaging volumes. These variations pose the challenge of how to define appropriate average quantities in the averaging volume. Moreover, the gradients of those averaged flow properties appear in the equations of motion that follow and the mean of the gradient is not necessarily equal to the gradient of the mean. These difficulties are addressed in Section 1.4 after we have explored the basic structure of the Equations in the absence of such complications.

### 1.2.2 Continuum Equations for Conservation of Mass

Consider now the construction of the effective differential equations of motion for a disperse multiphase flow (such as might be used in a two-fluid model) assuming that an appropriate elemental volume can be identified. For convenience this elemental volume is chosen to be a *unit* cube with edges parallel to the $x_1, x_2, x_3$ directions. The mass flow of component N through one of the faces perpendicular to the $i$ direction is given by $\rho_N j_{Ni}$ and therefore the net outflow of mass of component N from the cube is given by the divergence of $\rho_N j_{Ni}$ or

$$\frac{\partial(\rho_N j_{Ni})}{\partial x_i}. \tag{1.20}$$

The rate of increase of the mass of component N stored in the elemental volume is $\partial(\rho_N \alpha_N)/\partial t$ and hence conservation of mass of component N requires that

$$\frac{\partial}{\partial t}(\rho_N \alpha_N) + \frac{\partial(\rho_N j_{Ni})}{\partial x_i} = \mathcal{I}_N, \tag{1.21}$$

where $\mathcal{I}_N$ is the rate of transfer of mass to the phase N from the other phases per unit total volume. Such mass exchange would result from a phase change or chemical reaction. This is the first of several phase interaction terms that are identified and, for ease of reference, the quantities $\mathcal{I}_N$ are termed the *mass interaction terms*.

Clearly there is a continuity equation such as Eq. (1.21) for each phase or component present in the flow. They are referred to as the individual phase continuity equations (IPCE). However, because mass as a whole must be conserved whatever phase changes or chemical reactions are happening, it follows that

$$\sum_N \mathcal{I}_N = 0 \tag{1.22}$$

and hence the sum of all the IPCEs results in a combined phase continuity equation (CPCE) that does not involve $\mathcal{I}_N$ as follows:

$$\frac{\partial}{\partial t}\left(\sum_N \rho_N \alpha_N\right) + \frac{\partial}{\partial x_i}\left(\sum_N \rho_N j_{Ni}\right) = 0 \tag{1.23}$$

or, using Eqs. (1.4) and (1.8),

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}\left(\sum_N \rho_N \alpha_N u_{Ni}\right) = 0. \tag{1.24}$$

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Notice that only under the conditions of *zero* relative velocity in which $u_{Ni} = u_i$ does this reduce to the mixture continuity equation (MCE), which is identical to that for an equivalent single-phase flow of density $\rho$ as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0. \tag{1.25}$$

We also record that for one-dimensional duct flow the individual phase continuity equation [Eq. (1.21)] becomes the following:

$$\frac{\partial}{\partial t}(\rho_N \alpha_N) + \frac{1}{A}\frac{\partial}{\partial x}(\mathrm{A}\rho_N \alpha_N u_N) = \mathcal{I}_N, \tag{1.26}$$

where $x$ is measured along the duct; $A(x)$ is the cross-sectional area; $u_N$, $\alpha_N$ are cross-sectionally averaged quantities, and $A\mathcal{I}_N$ is the rate of transfer of mass to the phase N per unit length of the duct. The sum over the constituents yields the following combined phase continuity equation

$$\frac{\partial p}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(A \sum_N \rho_N \alpha_N u_n\right) = 0. \tag{1.27}$$

When all the phases travel at the same speed, $u_N = u$, this reduces to the following:

$$\frac{\partial \rho}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}(\rho A u) = 0. \tag{1.28}$$

Finally, we should make note of the form of the equations when the two components or species are intermingled rather than separated because we analyze several situations with gases diffusing through one another. Then both components occupy the entire volume and the void fractions are effectively unity so that the continuity equation [Eq. (1.21)] becomes the following:

$$\frac{\partial \rho_N}{\partial t} + \frac{\partial(\rho_N u_{Ni})}{\partial x_i} = \mathcal{I}_N. \tag{1.29}$$

### 1.2.3 Disperse Phase Number Continuity

Complementary to the equations of conservation of mass are the equations governing the conservation of the number of bubbles, drops, particles, and so on that constitute a disperse phase. If no such particles are created or destroyed within the elemental volume and if the number of particles of the disperse component, D, per unit total volume is denoted by $n_D$, it follows that

$$\frac{\partial n_D}{\partial t} + \frac{\partial}{\partial x_i}(n_D u_{Di}) = 0. \tag{1.30}$$

This is referred to as the disperse phase number equation (DPNE).

If the volume of the particles of component $D$ is denoted by $v_D$ it follows that

$$\alpha_D = n_D v_D \tag{1.31}$$