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978-0-521-84783-4 - Random Fields and Spin Glasses: A Field Theory Approach

Cirano De Dominicis and Irene Giardina

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RANDOM FIELDS AND SPIN GLASSES

A Field Theory Approach

Disordered magnetic systems enjoy nontrivial properties which are different from and richer than those observed in their pure, non-disordered counterparts. These properties dramatically affect the thermodynamic behaviour and require specific theoretical treatment.

In this book the authors deal with the theory of magnetic systems in the presence of frozen disorder, and in particular paradigmatic and well known spin models such as the Random Field Ising Model and the Ising Spin Glass. They describe some of the most successful approaches to the physics of disordered systems, such as the replica method and Langevin dynamics, together with lesser known results in finite dimension. This is a unified presentation using a field theory language which covers mean field theory, dynamics and perturbation expansion within the same theoretical framework. Particular emphasis is given to the connections between different approaches such as statics vs. dynamics, microscopic vs. phenomenological models. The book introduces some useful and little known techniques in statistical mechanics and field theory including multiple Legendre transforms, supersymmetry, Fourier transforms on a tree, infinitesimal permutations and Ward Takahashi Identities.

This book will be of great interest to graduate students and researchers in statistical physics and basic field theory.

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*To Florence, Ariane,
Marion and Bruno*

To Elsa and Andrea

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Preface

I vividly remember the academic year 1977–1978. I was a Loeb lecturer at Harvard that year. The Wilsonian revolution had been blossoming everywhere and I was teaching ‘field theory approach to critical phenomena’ in the wake of the works of my colleagues and friends Edouard Brezin and Jean Zinn Justin. I had not yet been exposed to the novel intricacies that were being uncovered in the critical behaviour of quenched random systems. But, during that year, several seminars were to deal with them and I began learning and interacting with Mike Stephen, Jo Rudnick and the late Sheng Ma. This is how it all started for me. A good quarter of a century later, the two central problematic systems of the field, the Random Field Ising Model and the Ising Spin Glass, despite several thousand papers and a huge amount of efforts dedicated to them, remain objects of controversy for what concerns how to describe their glassy phase. So why add a book on top of that? Perhaps I will tell how it all occurred.

At the origin the book was a mere set of lecture notes for a course given in this laboratory, a course that was largely repeated two years ago in the theory group of the physics department at UFRS in Porto Alegre. The Lecture Notes Series of Cambridge University Press having been discontinued, it was gracefully suggested that the notes be transformed into a book. I was lucky enough to have had Irene Giardina visiting as a postdoc here. She, as a learned student of Giorgio Parisi, was able to cast a critical ear and eye on the lectures, and then accepted to join in transforming the set of notes into a book. It must be said frankly that, without Irene, the book would not have been born, while with her expertise, several chapters were thoroughly remade and bear her imprint.

In its final version, one third of the book is dedicated to the statics and dynamics of the Random Field Ising Model, one half of it to the Ising Spin Glass, the rest being occupied by the statics and aging dynamics of spherical spins, and by Chapter 5, the lecture delivered by Marc Mézard on the Random Energy Model and the Simplest

Spin Glass. One motivation was to clarify and unify via field theory language what were often found as cryptic and very pointed papers, and to present in some details a few chosen results via useful but little known techniques (e.g. dynamics vs. replicas, multiple Legendre Transforms, supersymmetry vs. Fluctuation–Dissipation Theorem, Fourier Transforms on a tree, infinitesimal permutations vs. Ward–Takahashi Identities). On the controversy that concerns spin glasses, we have chosen to develop the viewpoint according to which the Parisi approach that gives the exact solution for the Sherrington–Kirkpatrick model (i.e. the spin glass in infinite dimension) remains a reasonable starting point to work out what is happening in finite dimension. In contrast with the alternative approach, the so-called droplet theory, which is briefly reviewed in the last chapter, this spin glass field theory approach is a microscopic one in the same sense as the ϕ^4 field theory is a microscopic approach for the pure Ising ferromagnet. A microscopic formulation for a spin glass with droplet-like characteristic properties, that would put both approaches on the same footing, is yet to emerge.

The expertise I may have gathered on the complex subjects presented in this book is, to a considerable extent, the product of enduring discussions I have pursued in the course of years with colleagues and friends. First and foremost I want to mention Edouard Brezin. I remain deeply indebted to him for all the benefits I drew from our discussions, and for the pleasure derived from work accomplished together, some of which is largely used in several chapters of the book. Irene and I are also warmly thankful to Marc Mézard for allowing us to present his views in Chapter 5 and for illuminating exchanges. I am also very grateful to colleagues and friends from the centre, with whom I had inspiring discussions for many years, Alain Billoire, Giulio Biroli, Jean-Philippe Bouchaud, Philippe di Francesco, Thomas Garel, Henri Orland, Jean Zinn-Justin and many others. My thanks also go to my faraway coworkers from past and present with whom I learned so much, Jairo de Almeida from Universidade Federal de Pernambuco in Recife, Andrea Crisanti from Rome University, Imre Kondor from The Collegium, Budapest, Iveta Pimentel from Lisbon University, Tamas Temesvari from Budapest University and Peter Young from California University in Santa Cruz. Loic Bervas typed the lecture notes version on which the book was built and I would like to warmly thank him. Finally I am very grateful to the Service de Physique Théorique and its Director for having extended to us all facilities to conclude this project.

Cirano De Dominicis

My collaboration and friendship with Cirano go back to the period I spent as a postdoc in Saclay, in 1999–2001. The idea and nucleus of this book came out at that time when Cirano gave a set of lectures and we started collaborating on spin glasses. My perspective on the subject was somewhat different from his, being mostly based on the analysis of solvable models and tied to a direct intuitive interpretation of the results. On the other hand, spin glass field theory may sometimes appear obscure, due to the technical complications arising from the nature of the order parameter. I was lucky enough to find someone like Cirano, undisputed master in the field, who introduced me to the field theory approach and who was always ready to discuss any single issue. This book, I hope, bears also the mark of our numerous discussions, of his effort to explain the many subtleties of the theory, and of my attempts to link them with my background and previous works. The writing of the book, was for us the occasion to present in a unified perspective some of the most striking features of disordered systems models. In this light, we added a few more subjects to the original structure of the lecture notes, but decided not to include an explicit treatment of the dynamics of spin glasses. This is surely one of the most promising approaches to these systems, but would have required too much space, rendering the book disproportionate. We have addressed in detail only the simpler case of a disguised ferromagnet, while for real spin glasses we limited ourselves to introducing some of the main concepts characterizing off-equilibrium behaviour and to quoting some important results.

I had the chance to work and discuss on disordered systems with many people, and benefitted from fruitful and stimulating exchanges. There are, however, a few people who had a prominent role in my experience as a physicist and who deserve explicit acknowledgement. I would like to thank first Andrea Cavagna for sharing with me passion, curiosity and enthusiasm in our numerous collaborations. He was kind enough to read extensive parts of this book and his comments have always been precious to me. I am greatly indebted to Giorgio Parisi for having taught me so many things in the past, and for all the inspiring discussions we keep having here in Rome. Cirano and I also thank him warmly for carefully reading this manuscript and for his comments and criticisms. In Oxford, as a postdoc, I worked with David Sherrington and benefitted from his great experience on spin glasses, to him goes my deepest gratitude. I warmly thank Marc Mézard and Olivier Martin for the many discussions we had on spin glasses while I was in Paris, and particularly Jean-Philippe Bouchaud with whom I enjoyed working together on several different subjects. I am also thankful to Alan Bray and Mike Moore for their critical and

stimulating comments on my work, during my frequent visits to Manchester in that period. Here in Rome there are many friends and colleagues with whom I constantly discuss on complex systems, and I am grateful to them all. In particular, Enzo Marinari, with whom I also had the pleasure of collaborating on some of the issues discussed in this book, Federico Ricci-Tersenghi and Tomas Grigera, who is now in La Plata. Finally, I would like to thank Alba and Emilio Giardina for their constant support, and for their practical help last summer. I would like to acknowledge the support to this project of the Department of Physics of the University of Rome La Sapienza and of the Institute of Complex Systems of the National Research Council ISC-CNR.

Irene Giardina

Abbreviations

AT	Almeida Thouless
BRST	Becchi–Rouet–Stora–Tyutin
EA	Edwards–Anderson
FDT	Fluctuation–Dissipation Theorem
FT	Fourier Transform
IR	infrared
L-A	longitudinal-anomalous
MSR	Martin–Siggia–Rose
REM	Random Energy Model
RFIM	Random Field Ising Model
RFT	Replica Fourier Transform
RG	Renormalization Group
RS	Replica Symmetric
RSB	Replica Symmetry Broken
SK	Sherrington–Kirkpatrick
TAP	Thouless–Anderson–Palmer
TTI	time translational invariance
UV	ultraviolet