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When a term is encountered on numerous occasions, the page number of only three of its appearances are given; the number in italics indicates the page where the term was defined.

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