Path Integrals and Anomalies in Curved Space

Path integrals provide a powerful method for describing quantum phenomena, first introduced in physics by Dirac and Feynman. This book introduces the quantum mechanics of particles that move in curved space by employing the path integral method, and uses this formalism to compute anomalies in quantum field theories.

The authors start by deriving path integrals for particles moving in curved space (one-dimensional nonlinear sigma models), and their supersymmetric generalizations. Coherent states are used for fermionic particles. They then discuss the regularization and renormalization schemes essential to constructing and computing these path integrals.

In the second part of the book, the authors apply these methods to discuss and calculate anomalies in quantum field theories, with external gravitational and/or (non) abelian gauge fields. Anomalies constitute one of the most important aspects of quantum field theory; requiring that there are no anomalies is an enormous constraint in the search for physical theories of elementary particles, quantum gravity and string theories. In particular, the authors include explicit calculations of the gravitational anomalies, reviewing the seminal work of Alvarez-Gaumé and Witten in an original way, and their own work on trace anomalies.

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who championed quantum mechanics in curved space

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Fiorenzo Bastianelli and Peter Van Nieuwenhuizen
Frontmatter
More information

Contents

Preface

In 1983, L. Alvarez-Gaumé and E. Witten (AGW) wrote a fundamental article in which they calculated the one-loop gravitational anomalies (anomalies in the local Lorentz symmetry of $(4k + 2)$-dimensional Minkowskian quantum field theories coupled to external gravity) of complex chiral spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ fields and real self-dual antisymmetric tensor fields[1] [1]. They used two methods: a straightforward Feynman graph calculation in $4k + 2$ dimensions with Pauli–Villars regularization, and a quantum mechanical (QM) path integral method in which corresponding nonlinear sigma models appeared. The former has been discussed in detail in an earlier book [3]. The latter method is the subject of this book.

AGW applied their formulas to $N = 2B$ supergravity in 10 dimensions, which contains precisely one field of each kind, and found that the sum of the gravitational anomalies cancels. Soon afterwards, M. B. Green and J.H. Schwarz [4] calculated the gravitational anomalies in one-loop string amplitudes, and concluded that these anomalies cancel in string theory, and therefore should also cancel in $N = 1$ supergravity in 10 dimensions with suitable gauge groups for the $N = 1$ matter couplings. Using the formulas of AGW, one can indeed show that the sum of anomalies in $N = 1$ supergravity coupled to super Yang–Mills theory with gauge group $SO(32)$ or $E_8 \times E_8$, though nonvanishing, is in the technical sense exact:

[1] Just as one can always shift the axial anomaly from the vector current to the axial current by adding a suitable counterterm to the action or by using a different regularization scheme, one can also shift the gravitational anomaly from the general coordinate symmetry to the local Lorentz symmetry [2]. Conventionally one chooses to preserve general coordinate invariance. AGW chose the symmetric vielbein gauge, so that the symmetry for which they computed the anomalies was a linear combination of a general coordinate transformation and a compensating local Lorentz transformation. However, they used a regulator that manifestly preserved general coordinate invariance, so that their calculation yielded the anomaly in the local Lorentz symmetry.

it can be removed by adding a local counterterm to the action. These two papers led to an explosion of interest in string theory.

We discussed these two papers in a series of internal seminars for advanced graduate students and faculty at Stony Brook (the "Friday seminars"). Whereas the basic philosophy and methods of the paper by AGW were clear, we stumbled on numerous technical problems and details. Some of these became clearer upon closer reading, some became more baffling. In a desire to clarify these issues we decided to embark on a research project: the AGW program for trace anomalies. Since gravitational and chiral anomalies only contribute at the one-worldline-loop level in the QM method, one does not need to be careful with definitions of the measure for the path integral, choice of regulators, regularization of divergent graphs, etc. This is explicitly discussed in [1]. However, we soon noticed that for the trace anomalies the opposite is true: if the field theory is defined in $n = 2k$ dimensions, one needs $(k + 1)$-loop graphs on the worldline in the QM method. Consequently, every detail in the calculation matters. Our program of calculating trace anomalies turned into a program of studying path integrals for nonlinear sigma models in phase space and configuration space, a notoriously difficult and controversial subject. As already pointed out by AGW, the QM nonlinear sigma models needed for space-time fermions (or self-dual antisymmetric tensor fields in spacetime) have $N = 1$ (or $N = 2$) worldline supersymmetry (susy), even though the original field theories were not spacetime supersymmetric. Thus, we also had to wrestle with the role of susy in the careful definitions and calculations of these QM path integrals.

Although it only gradually dawned upon us, we have come to recognize the problems with these susy and nonsusy QM path integrals as problems one should expect to encounter in any quantum field theory (QFT), the only difference being that these particular field theories have a one-dimensional (finite) spacetime, as a result of which infinities in the sum of Feynman graphs for a given process cancel. (They must cancel since this sum can also be written as a matrix element in quantum mechanics which is manifestly finite, see section 2.5). However, individual Feyn-man graphs are power-counting divergent (because these models contain double-derivative interactions just like quantum gravity). This cancellation of infinities in the sum of graphs is perhaps the psychological reason why there is no systematic discussion of regularization issues in the early literature on the subject (in the 1950s and 1960s). With the advent of the renormalization of gauge theories in the 1970s, issues of regularization of nonlinear sigma models were also studied. It was found that the regularization schemes used at that time (the time slicing method and the mode regularization method) broke general coordinate invariance at intermediate stages, but it was also noted that by adding particular noncovariant counterterms [5–9], the final physical results were still general coordinate invariant (we shall use the shorter term Einstein invariance for this symmetry in this book). The question thus arose how to determine those counterterms, and to understand the relation between the counterterms in one regularization scheme and those in other schemes. Once again, the answer to this question could be found in the general literature on QFT: the imposition of suitable renormalization conditions.

As we tackled more and more difficult problems (four-loop graphs for trace anomalies in six dimensions) it became clear to us that a scheme which needed only covariant counterterms would be very welcome. Dimensional regularization (DR) is such a scheme [10]. It had been used by Kleinert and Chervyakov [11] for the QM of a one-dimensional target space on an infinite worldline time interval (with a mass term added to regulate infrared divergences). For our purposes we have developed instead a version of dimensional regularization on a compact space; because the space is compact we do not need to add by hand a mass term to regulate the infrared divergences due to massless fields. The counterterms needed in such an approach are indeed covariant (both Einstein and locally Lorentz invariant).

The quantum mechanical path integral formalism can be used to compute anomalies in quantum field theories. This application forms the second part of this book. Chiral spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ fields and selfdual antisymmetric tensor (SAT) fields can produce anomalies in loop graphs with external gravitons and/or external gauge (Yang–Mills) fields. The treatment of the spin $\frac{3}{2}$ and SAT fields formed a major obstacle. For example, in the article by AGW the SAT fields are described by a bispinor $\psi_{\alpha\beta}$. However, the vector index of the spin-$\frac{3}{2}$ field and the β index of $\psi_{\alpha\beta}$ are treated differently from the spinor index of the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ fields and the α index of $\psi_{\alpha\beta}$. In [1] one finds the following transformation rule for the spin-$\frac{3}{2}$ field (in their notation):

$$-\delta_\eta \psi_A = \eta^i D_i \psi_A + D_a \eta_b (T^{ab})_{AB} \psi_B \tag{1}$$

where $\eta^i(x)$ parametrizes an infinitesimal coordinate transformation $x^i \to x^i + \eta^i(x)$, and $A = 1, 2, \ldots, n$ is the flat vector index of the spin-$\frac{3}{2}$ (gravitino) field, while $(T^{ab})_{AB} = -i(\delta^a_A \delta^b_B - \delta^b_A \delta^a_B)$ are the matrix elements of the Euclidean Lorentz group $SO(n)$ in the vector representation. One would expect that this transformation rule is a linear combination of an Einstein transformation $\delta_E \psi_{A\alpha} = \eta^i \partial_i \psi_{A\alpha}$ (the vector index A of $\psi_{A\alpha}$ is flat and α is the spin index) and a local Lorentz rotation $\delta_{lL} \psi_{A\alpha} = \frac{1}{4} \eta^i \omega_{iBC} (\gamma^B \gamma^C)_\alpha{}^\beta \psi_{A\beta} + \eta^i \omega_{iA}{}^B \psi_{B\alpha}$. However, on top of this Lorentz rotation with parameter $\eta^i \omega_{iAB}$, one finds the second term in (1) which describes a local Lorentz rotation with parameter $(D_a \eta_b - D_b \eta_a)$ and this local Lorentz transformation only acts on the vector index of the gravitino. If one assumes (1), one finds a beautiful simple relation between the gravitational contribution to the axial (γ_5) anomaly in $4k+4$ dimensions and the gravitational (local Lorentz) anomaly in $4k+2$ dimensions. We shall derive (1) from first principles, and show that it is correct, but only if one uses a particular regulator $\mathcal{R}$.

The regulators for the spin-$\frac{1}{2}$ field λ, for the gravitino ψ_A, and for the bispinor $\psi_{\alpha\beta}$ are in all cases the square of the field operators for the nonchiral spinors $\tilde{\lambda}$, $\tilde{\psi}_A$ and $\tilde{\psi}_{\alpha\beta}$, where the "twiddled fields" $\tilde{\lambda}$, $\tilde{\psi}_A$ and $\tilde{\psi}_{\alpha\beta}$ are obtained from λ, ψ_A and $\psi_{\alpha\beta}$ by multiplication by $g^{1/4} = (\det e_\mu{}^m)^{1/2}$. These regulators are covariant regulators, not consistent regulators, and the anomalies we will obtain are covariant anomalies, not consistent anomalies [2]. However, when we come to the cancellation of anomalies, we shall use the descent equations to convert these covariant anomalies to consistent anomalies, and then construct counterterms whose variations cancel these consistent anomalies.

The twiddled fields were used by Fujikawa, who pioneered the path integral approach to anomalies [12]. An ordinary Einstein transformation of $\tilde{\lambda}$ is given by $\delta\tilde{\lambda} = \frac{1}{2}(\xi^\mu\partial_\mu + \partial_\mu\xi^\mu)\tilde{\lambda}$, where the second derivative ∂_μ can also act on $\tilde{\lambda}$, and if one evaluates the corresponding regulated anomaly $An_E = \mathrm{Tr}\frac{1}{2}(\xi^\mu\partial_\mu + \partial_\mu\xi^\mu)\mathrm{e}^{-\beta\mathcal{R}}$ by inserting a complete set of eigenfunctions $\tilde{\varphi}_k$ of $\mathcal{R}$ with non-negative eigenvalues λ_k, one finds

$$An_E = \lim_{\beta\to 0}\sum_k \int d^n x\, \tilde{\varphi}_k^*(x)\frac{1}{2}(\xi^\mu\partial_\mu + \partial_\mu\xi^\mu)\mathrm{e}^{-\beta\lambda_k}\tilde{\varphi}_k(x)\ . \qquad (2)$$

Thus, the Einstein anomaly vanishes (partially integrate the second ∂_μ) as long as the regulator is self-adjoint with respect to the inner product $\langle\tilde{\lambda}_1|\tilde{\lambda}_2\rangle = \int dx\, \tilde{\lambda}_1^*(x)\tilde{\lambda}_2(x)$ (so that $\tilde{\varphi}_k$ form a complete set), and as long as both $\tilde{\varphi}_k(x)$ and $\tilde{\varphi}_k^*(x)$ belong to the same complete set of eigenstates, as in the case of plane waves e^{ikx}. One can always make a unitary transformation from $\tilde{\varphi}_k$ to the set e^{ikx}, and using these plane waves, the calculation of anomalies in the framework of quantum field theory is reduced to a set of n-dimensional Gaussian integrals over k. We shall use the regulator $\mathcal{R}$ discussed above, and twiddled fields, but then cast the calculation of anomalies in terms of quantum mechanics and path integrals. Calculating anomalies using quantum mechanics is much simpler than evaluating the Gaussian integrals of quantum field theory. Using path integrals simplifies the calculations even further.

When we first started studying the problems discussed in this book, we used the shortcuts and plausible arguments which are used by researchers and sometimes mentioned in the literature. However, the more we tried to clarify and complete these shortcuts and arguments, the more we were driven to basic questions and theoretical principles. We have been studying these issues now for over 15 years, and have accumulated a wealth of facts and insights. We decided to write a book in which all ideas and calculations were developed from scratch, with all intermediate steps worked out. The result looks detailed, and at places technical. We have made every effort to keep the text readable by providing verbal descriptions next to formulas, and providing introductory sections and historical reviews. In the end, however, we felt there is no substitute for a complete and fundamental treatment.

We end this preface by summarizing the content of this book. In the first part of this book we give a complete derivation of the path integrals for supersymmetric and nonsupersymmetric nonlinear sigma models describing bosonic and fermionic point particles (commuting coordinates $x^i(t)$ and anticommuting variables $\psi^a(t) = e_i^a(x(t))\psi^i(t)$) in a curved target space with metric $g_{ij}(x) = e_i^a(x)e_j^b(x)\delta_{ab}$. All of our calculations are performed in Euclidean target space. We consider a finite time interval because this is what is needed for the applications to anomalies. As these models contain double-derivative interactions, they are divergent according to power-counting, just as in quantum gravity, but ghost loops arising from the path integral measure cancel the divergences. Only the one-and two-loop graphs are power-counting divergent, hence in general the action may contain extra finite local one- and two-loop counterterms, the coefficients of which should be fixed. They are fixed by imposing suitable renormalization conditions. To regularize individual diagrams we use three different regularization schemes:

(i) time slicing (TS), known from the work of Dirac and Feynman;
(ii) mode regularization (MR), known from instanton and soliton physics;[2] and
(iii) dimensional regularization on a finite time interval (DR), discussed in this book.

The renormalization conditions relate a given quantum Hamiltonian $\hat{H}$ to a corresponding quantum action S, by which we mean the action that appears in the exponent of the path integral. The particular finite one-and two-loop counterterms in S thus obtained are different for each regularization scheme. In practice we fix the counterterms for the three regularization schemes in different ways. In the case of time-slicing we begin with an operator $\hat{H}$, and by careful transition to the discretized path integral, we deduce the counterterms by using Weyl ordering. In the case of mode and dimensional regularization, we begin with a path integral which contains arbitrary one- and two-loop counterterms, and then proceed to fix this freedom by requiring that the path integral satisfies a Schrödinger equation with the same $\hat{H}$.

In principle, any $\hat{H}$ with a definite ordering of the operators can be taken as the starting point, and gives a corresponding path integral

[2] Actually, the mode expansion had already been used by Feynman and Hibbs to compute the path integral for the harmonic oscillator.

(with different counterterms for different regularization schemes), but for our physical applications we shall consider quantum Hamiltonians that maintain reparametrization and local Lorentz invariance in target space (i.e. commute with the quantum generators of these symmetries. The chiral anomaly is then due to the chirality matrix in the Jacobians). Then there are no one-loop counterterms in the three schemes, but only two-loop counterterms. Having defined the regulated path integrals, the continuum limit can be taken and reveals the correct "Feynman rules" (the rules of how to evaluate the integrals over products of distributions and equal-time contractions) for each regularization scheme. All three regularization schemes give the same final answer for the transition amplitude, although the Feynman rules are different.

In the second part of this book we apply our methods to the evaluation of anomalies in n-dimensional relativistic quantum field theories with bosons and fermions in the loops (spin $0, \frac{1}{2}, 1, \frac{3}{2}$ and self-dual antisymmetric tensor fields) coupled to external gauge fields and/or gravity. We regulate the field-theoretical Jacobian for the symmetries whose anomalies we want to compute with a factor of $\exp(-\beta\mathcal{R})$, where $\mathcal{R}$ is the covariant regulator which follows from the corresponding quantum field theory, as discussed before, and β tends to zero only at the end of the calculation. Next, we introduce a quantum mechanical representation of the operators which enter in the field-theoretical calculation. The regulator $\mathcal{R}$ yields a corresponding quantum mechanical Hamiltonian $\hat{H}$. We rewrite the quantum mechanical operator expression for the anomalies as a path integral on the finite time interval $-\beta \leq t \leq 0$ for a linear or nonlinear sigma model with action S. For given spacetime dimension n, in the limit $\beta \to 0$ only graphs with a finite number of loops on the worldline contribute. In this way the calculation of the anomalies is transformed from a field-theoretical problem to a problem in quantum mechanics. We give details of the derivation of the chiral and gravitational anomalies as first given by Alvarez-Gaumé and Witten, and discuss our own work on trace anomalies. For the former one only needs to evaluate one-loop graphs on the worldline, but for the trace anomalies in two dimensions we need two-loop graphs, and for the trace anomalies in four dimensions we compute three-loop graphs. Here a technical but important problem was settled: using time-slicing or mode regularization, counterterms proportional to the product of two Christoffel symbols were found, but it is incorrect to invoke normal coordinates and to ignore these counterterms. Their expansion produces products of two Riemann curvatures which do contribute at 3 loops to trace anomalies. We obtain complete agreement with the results for these anomalies obtained from other methods. We conclude with a detailed analysis of the gravitational anomalies in 10-dimensional supergravities, both for classical and for exceptional gauge groups.

Twenty years have passed since AGW wrote their renowned article. We believe we have solved all major and minor problems we initially ran into.[3] The quantum mechanical approach to quantum field theory can be applied to more problems than only anomalies. If future work on such problems will profit from the detailed account given in this book, our scientific and geographical Odyssey has come to a good ending.

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Bologna and Stony Brook, January 2005

[3] Except one problem: a rigorous derivation, based only on quantum mechanical path integrals, of the overall normalization of the gravitational anomaly of self-dual antisymmetric tensor fields, see Chapter 8. We fix this normalization by requiring agreement with bosonization formulas of two-dimensional quantum field theories.