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978-0-521-84751-3 - Boolean Functions: Theory, Algorithms, and Applications

Yves Crama and Peter L. Hammer

Table of Contents

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## Contents

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Contributors	<i>page</i> xiii
Preface	xv
Acknowledgments	xix
Notations	xxi

### Part I Foundations

<b>1</b>	<b>Fundamental concepts and applications</b>	3
	1.1 Boolean functions: Definitions and examples	3
	1.2 Boolean expressions	8
	1.3 Duality	13
	1.4 Normal forms	14
	1.5 Transforming an arbitrary expression into a DNF	19
	1.6 Orthogonal DNFs and number of true points	22
	1.7 Implicants and prime implicants	24
	1.8 Restrictions of functions, essential variables	28
	1.9 Geometric interpretation	31
	1.10 Monotone Boolean functions	33
	1.11 Recognition of functional and DNF properties	40
	1.12 Other representations of Boolean functions	44
	1.13 Applications	49
	1.14 Exercises	65
<b>2</b>	<b>Boolean equations</b>	67
	2.1 Definitions and applications	67
	2.2 The complexity of Boolean equations: Cook's theorem	72
	2.3 On the role of DNF equations	74
	2.4 What does it mean to "solve a Boolean equation"?	78
	2.5 Branching procedures	80
	2.6 Variable elimination procedures	87

Cambridge University Press

978-0-521-84751-3 - Boolean Functions: Theory, Algorithms, and Applications

Yves Crama and Peter L. Hammer

Table of Contents

[More information](#)

viii	Contents	
2.7	The consensus procedure	92
2.8	Mathematical programming approaches	95
2.9	Recent trends and algorithmic performance	103
2.10	More on the complexity of Boolean equations	104
2.11	Generalizations of consistency testing	111
2.12	Exercises	121
<b>3</b>	<b>Prime implicants and minimal DNFs</b>	<b>123</b>
	<i>Peter L. Hammer and Alexander Kogan</i>	
3.1	Prime implicants	123
3.2	Generation of all prime implicants	128
3.3	Logic minimization	141
3.4	Extremal and typical parameter values	159
3.5	Exercises	165
<b>4</b>	<b>Duality theory</b>	<b>167</b>
	<i>Yves Crama and Kazuhisa Makino</i>	
4.1	Basic properties and applications	167
4.2	Duality properties of positive functions	176
4.3	Algorithmic aspects: The general case	183
4.4	Algorithmic aspects: Positive functions	189
4.5	Exercises	198
<b>Part II Special Classes</b>		
<b>5</b>	<b>Quadratic functions</b>	<b>203</b>
	<i>Bruno Simeone</i>	
5.1	Basic definitions and properties	203
5.2	Why are quadratic Boolean functions important?	205
5.3	Special classes of quadratic functions	207
5.4	Quadratic Boolean functions and graphs	209
5.5	Reducibility of combinatorial problems to quadratic equations	218
5.6	Efficient graph-theoretic algorithms for quadratic equations	230
5.7	Quadratic equations: Special topics	243
5.8	Prime implicants and irredundant forms	250
5.9	Dualization of quadratic functions (Contributed by Oya Ekin Karaşan)	263
5.10	Exercises	266
<b>6</b>	<b>Horn functions</b>	<b>269</b>
	<i>Endre Boros</i>	
6.1	Basic definitions and properties	269
6.2	Applications of Horn functions	273
6.3	False points of Horn functions	277

Cambridge University Press

978-0-521-84751-3 - Boolean Functions: Theory, Algorithms, and Applications

Yves Crama and Peter L. Hammer

Table of Contents

[More information](#)

Contents		ix
6.4	Horn equations	281
6.5	Prime implicants of Horn functions	286
6.6	Properties of the set of prime implicants	292
6.7	Minimization of Horn DNFs	297
6.8	Dualization of Horn functions	306
6.9	Special classes	309
6.10	Generalizations	314
6.11	Exercises	321
<b>7</b>	<b>Orthogonal forms and shellability</b>	<b>326</b>
7.1	Computation of orthogonal DNFs	326
7.2	Shellings and shellability	330
7.3	Dualization of shellable DNFs	336
7.4	The lexico-exchange property	338
7.5	Shellable quadratic DNFs and graphs	346
7.6	Applications	348
7.7	Exercises	349
<b>8</b>	<b>Regular functions</b>	<b>351</b>
8.1	Relative strength of variables and regularity	351
8.2	Basic properties	355
8.3	Regularity and left-shifts	362
8.4	Recognition of regular functions	365
8.5	Dualization of regular functions	369
8.6	Regular set covering problems	377
8.7	Regular minorants and majorants	380
8.8	Higher-order monotonicity	391
8.9	Generalizations of regularity	397
8.10	Exercises	401
<b>9</b>	<b>Threshold functions</b>	<b>404</b>
9.1	Definitions and applications	404
9.2	Basic properties of threshold functions	408
9.3	Characterizations of threshold functions	413
9.4	Recognition of threshold functions	417
9.5	Prime implicants of threshold functions	423
9.6	Chow parameters of threshold functions	428
9.7	Threshold graphs	438
9.8	Exercises	444
<b>10</b>	<b>Read-once functions</b>	<b>448</b>
	<i>Martin C. Golumbic and Vladimir Gurvich</i>	
10.1	Introduction	448
10.2	Dual implicants	450

Cambridge University Press

978-0-521-84751-3 - Boolean Functions: Theory, Algorithms, and Applications

Yves Crama and Peter L. Hammer

Table of Contents

[More information](#)

x	Contents	
	10.3 Characterizing read-once functions	456
	10.4 The properties of $P_4$ -free graphs and cographs	463
	10.5 Recognizing read-once functions	466
	10.6 Learning read-once functions	473
	10.7 Related topics and applications of read-once functions	476
	10.8 Historical notes	480
	10.9 Exercises	481
<b>11</b>	<b>Characterizations of special classes by functional equations</b>	<b>487</b>
	<i>Lisa Hellerstein</i>	
	11.1 Characterizations of positive functions	487
	11.2 Functional equations	488
	11.3 Characterizations of particular classes	491
	11.4 Conditions for characterization	495
	11.5 Finite characterizations by functional equations	500
	11.6 Exercises	506
	<b>Part III Generalizations</b>	
<b>12</b>	<b>Partially defined Boolean functions</b>	<b>511</b>
	<i>Toshihide Ibaraki</i>	
	12.1 Introduction	511
	12.2 Extensions of pdBfs and their representations	514
	12.3 Extensions within given function classes	531
	12.4 Best-fit extensions of pdBfs containing errors	547
	12.5 Extensions of pdBfs with missing bits	551
	12.6 Minimization with don't cares	558
	12.7 Conclusion	561
	12.8 Exercises	562
<b>13</b>	<b>Pseudo-Boolean functions</b>	<b>564</b>
	13.1 Definitions and examples	564
	13.2 Representations	570
	13.3 Extensions of pseudo-Boolean functions	578
	13.4 Pseudo-Boolean optimization	585
	13.5 Approximations	593
	13.6 Special classes of pseudo-Boolean functions	593
	13.7 Exercises	607
<b>A</b>	<b>Graphs and hypergraphs</b>	<b>609</b>
	A.1 Undirected graphs	609
	A.2 Directed graphs	612
	A.3 Hypergraphs	614

Cambridge University Press

978-0-521-84751-3 - Boolean Functions: Theory, Algorithms, and Applications

Yves Crama and Peter L. Hammer

Table of Contents

[More information](#)

	Contents	xi
<b>B</b>	<b>Algorithmic complexity</b>	615
	B.1 Decision problems	615
	B.2 Algorithms	617
	B.3 Running time, polynomial-time algorithms, and the class P	618
	B.4 The class NP	619
	B.5 Polynomial-time reductions and NP-completeness	620
	B.6 The class co-NP	621
	B.7 Cook's theorem	622
	B.8 Complexity of list-generation and counting algorithms	624
<b>C</b>	<b>JBool: A software tool</b>	627
	<i>Claude Benzaken and Nadia Brauner</i>	
	C.1 Introduction	627
	C.2 Work interface	628
	C.3 Creating a Boolean function	629
	C.4 Editing a function	632
	C.5 Operations on Boolean functions	633
	Bibliography	635
	Index	677