

ASTROPHYSICS PROCESSES

Bridging the gap between physics and astronomy textbooks, this book provides step-by-step physical and mathematical development of fundamental astrophysical processes underlying a wide range of phenomena in stellar, galactic, and extragalactic astronomy. The book has been written for upper-level undergraduates and beginning graduate students, and its strong pedagogy ensures solid mastery of each process and application. It contains over 150 tutorial figures, numerous examples of astronomical measurements, and 202 exercises. Topics covered include the Kepler–Newton problem, stellar structure, binary evolution, radiation processes, special relativity in astronomy, radio propagation in the interstellar medium, and gravitational lensing. Applications presented include Jeans length, Eddington luminosity, the cooling of the cosmic microwave background (CMB), the Sunyaev–Zeldovich effect, Doppler boosting in jets, and determinations of the Hubble constant. This text is a stepping stone to more specialized books and primary literature. Password-protected solutions to the exercises are available to instructors at www.cambridge.org/9780521846561.

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Cover information

Views of the entire sky at six wavelengths in galactic coordinates; the equator of the Milky Way system is the central horizontal axis and the galactic center direction is at the center. Five of the views may be seen in greater detail on NASA's Web site "Astronomy Picture of the Day" (APOD). Except for the far-infrared and x-ray views, the colors represent intensity levels, with the greatest intensities lying along the equator. In all cases, the radiation shows an association with the galactic equator, the general direction of the galactic center, or both. The views are in frequency sequence as listed below and are located, respectively, as follows: background and upper and lower insets on the front cover; top, middle, and bottom insets on the back cover.

Radio sky at 408 MHz exhibiting a diffuse glow of synchrotron radiation from the entire sky. High-energy electrons spiraling in the magnetic fields of the Galaxy emit this radiation. Note the *North Polar Spur* projecting above the equator to the left of center. From three observatories: Jodrell Bank, MPIfR, and Parkes. [Glyn Haslam *et al.*; MPIfR, SkyView; APOD, 14 December 1997]

Radio emission at 1420 MHz, the spin-flip (hyperfine) transition in the ground state of hydrogen, which shows the location of clouds of neutral hydrogen gas. The gas is heavily concentrated in the galactic plane and manifests pronounced filamentary structure off the plane. [Courtesy of National Radio Astronomy Observatory/AUI/NSF; APOD, 13 January 2001]

Far-infrared (60–240 μm) sky from the COBE satellite showing primarily emission from small grains of graphite and silicates ("dust") in the interstellar medium of the Galaxy. The faint, whitish, large S-shaped curve (on its side) is emission from dust and rocks in the solar system; reflection of solar light from this material causes the zodiacal light at optical wavelengths. Color coding: 60 μm (blue), 100 μm (green), 240 μm (red). [E. L. Wright (UCLA); COBE; DIRBE; NASA; APOD, 17 May 2000]

Optical sky from a mosaic of 51 wide-angle photographs showing mostly stars in our (Milky Way) Galaxy, with significant extinction by dust along the galactic plane. Galaxies are visible at higher galactic latitudes, the most prominent being the two nearby Magellanic Clouds (lower right). [©Axel Mellinger; APOD, 2 February 2001]

X-ray sky at 1–20 keV from the A1 experiment on the HEAO-1 satellite showing 842 discrete sources. The circle size represents intensity of the source and the color denotes the type of object. The most intense sources shown (green, larger circles) signify compact binary systems containing white dwarfs, neutron stars, and black holes. Other objects are supernova remnants (blue), clusters of galaxies (pink), active galactic nuclei (orange), and stellar coronae (white). [Kent Wood (NRL); see *ApJ Suppl.* **56**, 507 (1984)]

Gamma-ray sky above 100 MeV from the EGRET experiment on the Compton Gamma-Ray Observatory. The diffuse glow from the galactic equator is due to the collisions of cosmic-ray protons with the atoms of gas clouds; the nuclear interactions produce neutral pions that decay to the detected gamma rays. Discrete sources include pulsars and jets associated with active galaxies ("blazars"). [The EGRET team; NASA; CGRO; APOD, 21 March 1998]

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To my wife Dottie, my daughters Elizabeth
and Dorothy, and my sisters Val, Abby,
and Dale Anne

They are my fans and I, theirs.

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Preface

This volume is based on notes that evolved during my teaching of astrophysics classes for junior and senior physics students at MIT beginning in 1973 and thereafter on and off until 1997. The course focused on a physical, analytical approach to underlying processes in astronomy and astrophysics. In each class, I would escort the students through a mathematical and physical derivation of some process relevant to astrophysics in the hope of giving them a firm comprehension of the underlying principles.

The approach in the text is meant to be accessible to undergraduates who have completed the fundamental calculus-based physics courses in mechanics and electromagnetic theory. Additional physics courses such as quantum mechanics, thermodynamics, and statistics would be helpful but are not necessary for large parts of this text. Derivations are developed step by step – frequently with brief reviews or reminders of the basic physics being used because students often feel they do not remember the material from an earlier course. The derivations are sufficiently complete to demonstrate the key features but do not attempt to include all the special cases and finer details that might be needed for professional research.

This text presents twelve “processes” with derivations and focused, limited examples. It does not try to acquaint the student with all the associated astronomical lore. It is quite impossible in a reasonable-sized text to give both the physical derivations of fundamental processes and to include all the known applications and lore relating to them across the field of astronomy. The assumption here is that many students will have had an elementary astronomy course emphasizing the lore. Nevertheless, selected germane examples of the processes are presented together with background information about them. These examples cover a wide and rich range of astrophysical phenomena.

The twelve processes, with the principal applications presented, are the Kepler–Newton problem (*mass functions, exoplanets, galactic center orbits*); stellar equilibrium (*nuclear burning, Eddington luminosity*); stellar equations of state (*ideal and degenerate gases, the distribution function*); stellar structure and evolution (*normal and compact stars, binary evolution*); thermal bremsstrahlung (*clusters of galaxies*); blackbody radiation (*cosmological cooling*); synchrotron (*Crab nebula*) and curvature radiation (*pulsars*); 21-cm radiation (*galaxy rotation, dark matter, Zeeman absorption*); Compton scattering (*Sunyaev–Zeldovich effect*); relativity in astronomy (*jets, photon absorption in the cosmic microwave background*); dispersion (*interstellar medium*) and Faraday rotation (*galactic magnetic field*); and gravitational lensing (*quasars, Fermat approach, Hubble constant, weak lensing*). Cosmology as

such is not systematically covered to limit the size of the text. Several related topics, however, are addressed: (i) the dark matter in galaxies and in clusters of galaxies, (ii) the cooling of the background blackbody radiation of the CMB, and (iii) determinations of the Hubble constant through both the S-Z effect and gravitational lensing.

Knowledge of the material in my previous textbook, *Astronomy Methods – A Physical Approach to Astronomical Observations* (AM), is not required for this text. The topics are largely complementary to those herein. I do, though, occasionally refer to it as an optional background reference. (The chapter numbers are those of the original edition.) The AM text, in Section 11.5, develops the formation of spectral lines in stellar atmospheres, one of the most basic processes in astronomy; hence, regrettably, this topic is not included in this book. Students and instructors are advised to download the PDF of this section from the resources link at www.cambridge.org/9780521846561. Other supplementary material and errata will also be posted at this site.

Again, SI units are used throughout to be consistent with most standard undergraduate science texts. Professional astronomers use cgs units. Unfortunately, this precludes progress in bringing the various science communities together to one system of units. It is also a significant hindrance to the student exploring astronomy or astrophysics. In this work I vote for ease of student access. Rather than use the customary and highly specialized astronomical unit of distance, the “parsec,” I instead employ the better understood, but non-SI, unit, the “light year” (LY), which is the distance light travels in one year. This is a well-defined quantity if one specifies the Julian year of exactly 365.25 days each of exactly 86 400 SI seconds for a total for 31 557 600 s.

Other features of the book to note are as follows:

- (i) Problems are provided for each chapter, and approximate answers indicated by the $\sim$ symbol are given when appropriate. Password protected solutions for instructors are available at the above mentioned resources link.
- (ii) The problems are generally constructed to help carry the student through them and hence are mostly multipart.
- (iii) Units are often given gratuitously (in parentheses) for algebraic variables to remind the reader of the meaning of the symbol.
- (iv) For improved readability, equation, table, figure, and section numbers in the text do not carry the chapter prefix if they refer to the current chapter.
- (v) For ease of referencing during class discussions, all equations are numbered, labels are provided for many of them, and important equations are each marked with a boldface arrow in the left margin.
- (vi) Tables of useful units, symbols, and constants are given in the appendix.
- (vii) A glossary of acronyms and abbreviations is provided just before the appendix.
- (viii) Logarithms are base 10 if “log” and base e if “ln”.
- (ix) Quantitative information is meant to be up to date and correct but should not be relied upon for professional research. The goal here is to teach underlying principles.

In teaching this course from my notes, I adopted a seminar, or Socratic, style of teaching that turned out to be extremely successful and personally rewarding. I recommend this approach to teachers using this text. I sat with the students (up to about 20) around a table, or we would rearrange classroom desks and chairs in a circular or rectangular pattern so that we were all more or less facing each other. I would then have the students explain the material to their fellow students (“Don’t look at me,” I often said). One student would do a bit, and I would

move on to another. I made my prompts easy and straightforward, avoided disparaging incorrect or confusing answers, and encouraged discussion among the students. I would synthesize arguments and describe the broader implications of the material interspersed by stories of real-life astronomy, personalities, discoveries, and so on.

The class would often become quite animated. During the discussions, the text was available to all and freely referenced. The students had to work hard to prepare for class, and thus they gained much from the class discussion. The course was also great fun for the teacher. In good weather, we would move outdoors and have our class on the lawn of MIT's Killian Court.

The author asks his readers' forbearance with the inevitable errors in the current text and requests to be notified of them. He also welcomes other comments and suggestions.

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Note for 2010 printing: The only substantial corrections in this printing pertain to the Sunyaev-Zeldovich effect. These include (i) a revised Figure 9.7c with thanks to E. L. Wright of UCLA, (ii) a revised Eq. 9.26 and discussion pertaining thereto including clarification of the distinction between the "scattered spectrum" and the "modified" (observed) spectrum, with thanks to Alan Levine of MIT, and (iii) an addition to Problem 9.52. John Fulton of Austin Community College helpfully pointed out several other errors in both the text and the problem solutions, which are not cited here because they were not fundamentally misleading. I look forward to further comments from interested readers.

Hale Bradt
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