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Excerpt
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1

Kepler, Newton, and the mass function

What we learn in this chapter

Binary star systems serve as laboratories for the measurement of star masses through the gravitational effects of the two stars on each other. Three observational types of binaries – namely, **visual**, **eclipsing**, and **spectroscopic** – yield different combinations of parameters describing the **binary orbit** and the **masses** of the two stars. We consider an example of each type – respectively, **α Centauri**, **β Persei (Algol)**, and **φ Cygni**.

Kepler described the orbits of solar planets with his **three laws**. They are grounded in **Newton's laws**. The **equation of motion** from Newton's second and gravitational force laws may be solved to obtain the **elliptical motions** described by Kepler for the case of a very large central mass, $M \gg m$. The results can then be extended to the case of **two arbitrary masses** orbiting their common barycenter (center of mass). The result is a **generalized Kepler's third law**, a relation between the masses, period, and relative semimajor axis. We also obtain expressions for the **system angular momentum** and **energy**. Kepler's laws are useful in determining the **orbital elements** of a binary system.

The generalized third law can be restated so that the measurable quantities for a star in a **spectroscopic binary** yield the **mass function**, a combination of the two masses and inclination. This provides a **lower limit to the partner mass**. Independent measures of the partner star's mass function and also of **orbital tidal light variations** or an **eclipse duration**, if available, can provide the information needed to obtain the masses of both stars and the inclination of their orbits. The track of one of the two stars in a **visual binary** relative to the other yields the **sum of the two masses** if the distance to the system is known. The tracks of both stars in inertial space (relative to the background galaxies) together with the distance yield the **two individual masses**.

Measurement of the optical mass function of the partner of the x-ray source Cyg X-1 revealed the first credible evidence for the **existence of a black hole**. Timing the arrival of pulses from a **radio or x-ray pulsar** provides information equivalent to that from a spectroscopic binary. Such studies have made possible the determination of the **masses of several dozens of neutron stars**. They have also provided the first evidence of **exoplanets**, which are planets outside the solar system. Almost **300 exoplanets** have now been discovered – most of them through optical radial velocity measures that detect the **minuscule wobble of the parent star**.

At the **center of the Galaxy**,* orbits of stars near the central dark mass have yielded the **mass of the central object**, $\sim 3 \quad 10^6 M_\odot$, and give strong evidence that it is indeed a **massive black hole**. Spectral and imaging data from orbiting bodies, used together, have yielded the **distance to the center of the Galaxy** and to the nearby star cluster, **the Pleiades**.

* In this text, we use "Galaxy" for our own galaxy (the Milky Way) and "galaxy" for other galaxies.

1.1 Introduction

Between one-third and two-thirds of all stars are in *binary stellar systems*. In such a system, two stars are gravitationally bound to one another; they each orbit the common center of mass (*barycenter*) with periods ranging from days to years for normal stars and down to hours or less for systems containing a compact star. In this chapter, we examine the motions of the individual stars and describe how these movements can be deduced from observations. We then learn how to deduce the masses of the component stars. Finally, we examine some contemporary applications of Kepler's and Newton's laws.

The motions of stars in a binary system can be understood in terms of the second law ($\boldsymbol{F} = m\boldsymbol{a}$) of Isaac Newton (1643–1727). This is worked out initially for a massive star M orbited by a much smaller mass m (i.e., $M \gg m$). The motion of a body in a gravitational r^{-2} force field is found to follow an elliptical path. The derived motions satisfy *Kepler's laws*, which were empirically discovered by Johannes Kepler (1571–1630). Thereafter, the "two-body" problem is worked out for two bodies of arbitrary masses. The results for the $M \gg m$ case provide a useful shortcut to the solution of the more general case.

In many binary systems, the stars are so far apart that they evolve quite independently of each other. In this case, their binary membership is only of incidental interest. In many systems, however, the two stars are so close to each other that their mutual interactions greatly affect their structure and evolution through tidal distortion and interchange of matter. The creation of white dwarfs, neutron stars, and black holes can follow directly from the modified evolutionary paths; see Section 4.5. In this chapter our analyses pertain only to the effects of gravitational interaction of point masses.

1.2 Binary star systems

The binary systems observable in optical (visible) light are of three general types: *visual*, *eclipsing*, and *spectroscopic*. The classes are not mutually exclusive; for example, a system can be eclipsing *and* spectroscopic. The distinctions between the classes arise from the sizes of the two stars, their closeness to each other, and their distance from the observer.

An additional observational class is that of (*visual*) *astrometric binaries*, wherein only one star is detectable but is observed to wobble on the sky owing to its orbital motion about a stellar companion. An example is AB Doradus, which has been tracked to milliarcsecond precision with very long baseline (radio) interferometry (VLBI). It is now known to be a quadratic system of late-type stars.

Celestial laboratories

Binary stellar systems may be considered laboratories in space. One star interacts with the other, and its response to the environment of the other can be measured by the signals (photons) reaching us. One measurable quantity is the orbital period, and another is the line-of-sight velocity. If the orbit is viewed edge-on, the mass of a massive star can be determined by measurements of a much lighter companion. Similar information can be obtained from systems in which the masses of the stars have arbitrary values. Such studies have long been important in optical astronomy; since 1971, they have been important in x-ray astronomy.

There can, of course, be forces on the entire two-star binary system due to other (external) gravitational systems. These external forces will accelerate the system's center of mass. In this chapter, we assume that there are no significant external forces on the two-star system. In this case, the center of mass will be stationary or will drift through space with a constant velocity. Our attention will be focused on the motion of the two stars relative to each other and to their center of mass (barycenter).

The large proportion of stars in binary systems (about one-half) is one indication that stars are formed from the interstellar medium in groups. Triple systems are also common. Another indication is the existence of groups of stars in the Galaxy (open clusters) such as the Pleiades. The stars in such clusters were all formed at about the same time and probably condensed out of a single interstellar cloud. If two or more stars are formed sufficiently close to each other to be gravitationally bound, they will orbit each other and will thus be a binary or triple system.

The nature of the component stars in binary systems is as varied as the types of stars known to us. Almost any type of star can be in a binary system. Two main-sequence stars (Section 4.3) are common, but there are also highly evolved systems such as (*i*) *cataclysmic variables*, in which one component is a white dwarf; (*ii*) *neutron-star binaries*, in which one component is a neutron star; and (*iii*) *RS CVn binaries*, in which one component is a flaring K giant. (The latter class is named after the star RS in the constellation Canes Venatici = Hunting Dogs.) A binary system that includes a main-sequence star may contain a giant star because one of the stars will have moved off the main sequence to become a red giant; see Section 4.3.

In many of these cases, the close proximity of the two stars to each other leads to direct and complex interactions between them. For instance, as a star expands to become a giant, its gas envelope can overflow its potential well and flow onto a close partner. This process is called *accretion*. If the partner is a neutron star, the accretion leads to the emission of x rays. If it is a white dwarf, highly variable optical emission is seen as well as some x rays. The energy from the emission comes from the release of gravitational potential energy by the infalling material.

The close proximity of two stars can also disturb the atmosphere of a star, giving rise to turbulence and flaring (RS CVn binaries) caused by tidal effects. Their proximity can also distort the shape of a star in the same way that the moon changes the levels of the earth's oceans.

Accretion of gas from one star to another in a binary system can dramatically modify the evolution of the two stars. For example, accreted gas will increase the mass of a normal,

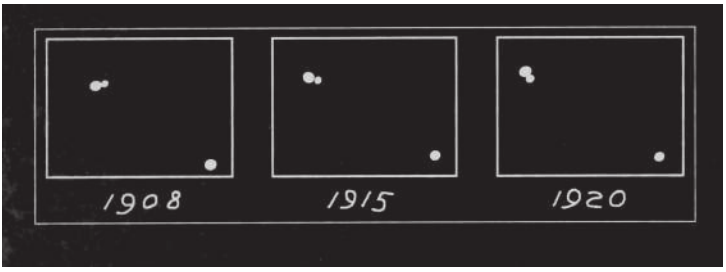

Figure 1.1. Kruger 60, a visual binary. The relative positions of the two stars (upper left) are seen to vary as they orbit each other with a period of 44.6 yr. The two stars are M stars with visual magnitudes 9.8 and 11.4. They are distant from the earth 12.9 LY with relative angular semimajor axis 2.4″ ($a_s = 9.5$ AU). [Yerkes Observatory]

gaseous star, leading to faster nuclear burning and a shorter life. The interactive evolution can lead to millisecond pulsars, which are neutron stars spinning with periods of a few milliseconds. The study of these objects can therefore provide great insight into the underlying physics of stars. Binary stellar systems are a diverse and fascinating breed of objects worthy of study in their own right; see Section 4.5.

Visual binaries

Visual binaries are systems that can be seen as two adjacent stars on an image of the sky, such as a photographic plate (e.g., Kruger 60 shown in Fig. 1 and Sirius). Over a period of some years, the two stars can be seen to orbit about each other. In such systems, the motion of one or both stars on the sky can be mapped to yield important parameters of the system.

An example of such mapping is α Centauri (Fig. 2). In this case, the asymmetry of the path is a consequence of an elliptical (eccentric) orbit. The orientation of the orbit to the line of sight gives it a strange appearance. The degree of eccentricity, the 80-yr period, and the angular size of the orbit (projected angular semimajor axis) provide information about the masses of the stars.

The inclination of the orbit relative to the line of sight is conventionally defined with the inclination angle i (Fig. 3a). If the observer is viewing the orbit face-on (i.e., normal to the orbital plane), the inclination is zero, $i = 0°$. If the observer is in the orbital plane, the inclination is $i = 90°$.

Eclipsing binaries

Eclipsing binaries are systems in which one star goes behind the other. This will happen if (*i*) the stars are very close to each other, (*ii*) one of the stars is sufficiently large, and (*iii*) the orbital plane is viewed more or less edge-on (inclination $\sim 90°$). Conditions (*i*) and (*ii*) can be summarized by requiring that the ratio of star size R to the distance s between the stars be of order unity:

$$R/s \approx 1. \qquad \text{(Approximate condition for eclipsing binary)} \qquad (1.1)$$

If this condition is met, the inclination need not be particularly close to 90°.

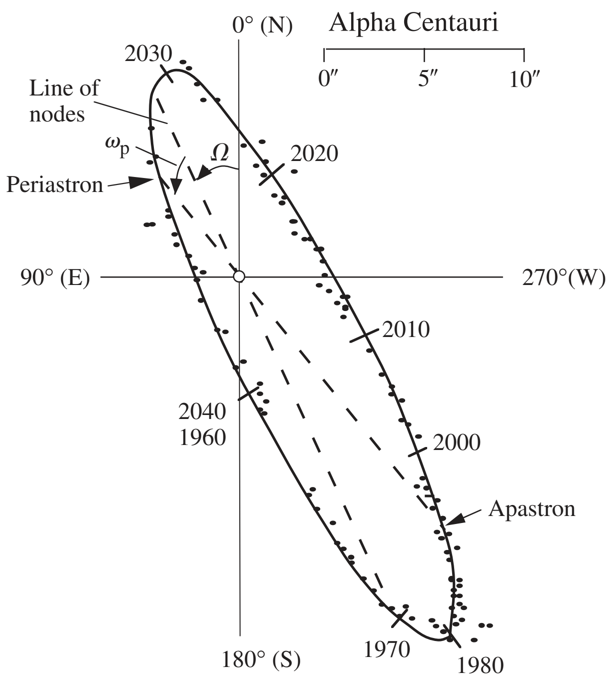

Figure 1.2. The α Centauri visual binary system. This is a plot of the positions projected onto the plane of the sky of one of the two stars relative to the other as a function of time (years). The origin is one of the components and the radial distances are in arcseconds (see scale). The track of the star in the plot is the projection of an ellipse, which is also an ellipse, but with shifted focus. The stars are at their smallest physical separation at the position marked *periastron* and at their largest at *apastron*. The *line of nodes* is the intersection at the focus (origin) of the plane of the orbit and the plane of the sky (see Fig. 11). The position angle Ω of the line of nodes and the (projected) longitude of periastron ω_p are indicated; they are defined in Fig. 11. The stellar components are bright main-sequence stars (G2 V and K IV) of visual magnitudes $m_V = 0.0$ and 1.36, respectively, with a period of 79.9 yr. This system is very close to the sun, 4.4 LY. (A faint, outlying additional companion, Proxima Cen, is the closest star to the sun.) [After D. Menzel, F. Whipple, and G. deVaucouleurs, *Survey of the Universe*, Prentice Hall, 1970, p. 467]

The stars in eclipsing systems are sufficiently close to each other that they can not be resolved on a traditional optical photograph; they appear to be a single star. The binary character is detected by the reduction of light emanating from the system during eclipse. Modern high-resolution imaging such as interferometry or adaptive optics, however, can sometimes resolve the two stellar components in these systems.

When the smaller of the two stars of a hypothetical binary (Fig. 4a) moves behind its companion, it is *occulted*, and only the light from the larger star reaches the observer. The *light curve* (flux density versus time) thus shows a reduction of light. The light also dims when the small star covers part of the big star. The edges of the dips in the light curve are not vertical; they show a gradual diminution of the light. This is due to the finite size of the small star. The two eclipses per orbit are of different depths; it is instructive to understand this (see

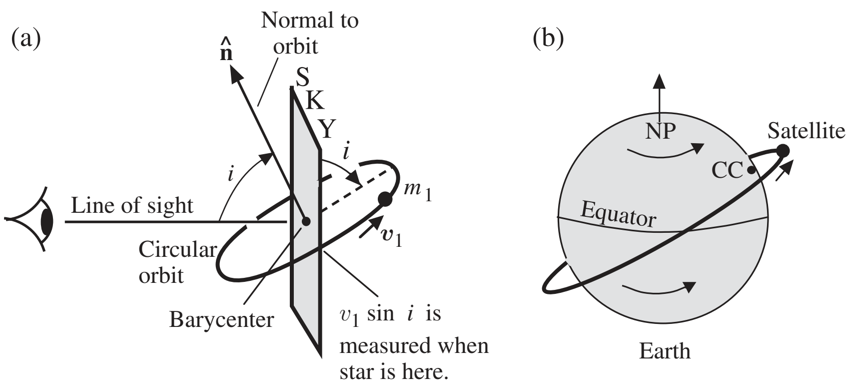

Figure 1.3. (a) Definition of the inclination angle of an orbital plane for a circular orbit. The position of the star when the line-of-sight velocity equals $v_1 \sin i$ is indicated. An orbit lying exactly in the plane of the sky has inclination $i = 0°$ and will exhibit no Doppler shifts. (b) Inclination of low-altitude satellite orbit. If the satellite is launched eastward from Cape Canaveral (CC), its greatest latitude will be that of the Cape. The earth rotates under the orbit, and the orbit precesses with a period of $\sim$50 d.

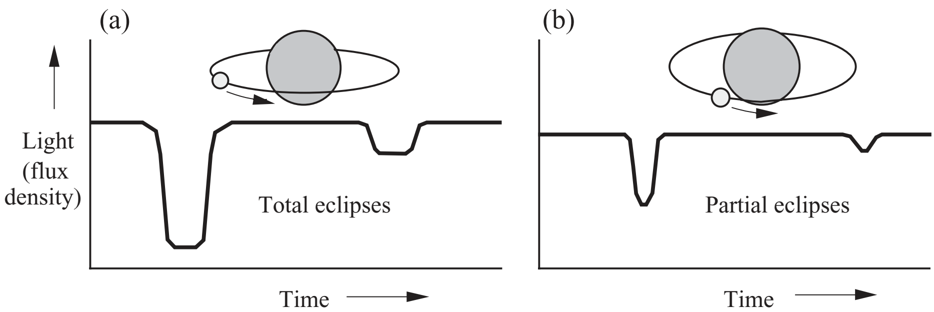

Figure 1.4. Schematic and hypothetical light curves of (a) a totally eclipsing binary and (b) a partially eclipsing binary. A light curve is a plot of flux density versus time. One eclipse is deeper than the other because the stars are assumed to have different surface brightnesses. Larger main-sequence (hydrogen-core-burning) stars are brighter per fixed solid angle than smaller main-sequence stars. The deeper eclipse occurs when the larger star is partially covered.

discussion immediately following). Sometimes the star merely grazes its companion, giving rise to *partial eclipses*. This case is shown schematically in Fig. 4b.

An actual (partial) eclipsing system, Algol, is shown schematically in Fig. 5. It contains one main-sequence star (B8 V) and one subgiant (K2 IV). They orbit each other with a period of 2.9 d. There is a third companion (not shown) at a greater distance that orbits the close pair in 1.9 yr. The system is $\sim$100 LY distant, and the close pair are separated by 14 $R_\odot$. Their separation is thus only $\sim$2 milliarcsec, and so they appear as a single star through most telescopes. Those now equipped with optical interferometry do resolve the components of this triple system.

The spectral type (Table 4.2) is a measure of the stellar color or temperature T, and this in turn determines the energy outflow per unit area from the stellar surface, which is approximately $\mathscr{F} = \sigma T^4$ (W/m^2), the flux density from a blackbody (6.18). Thus, in the

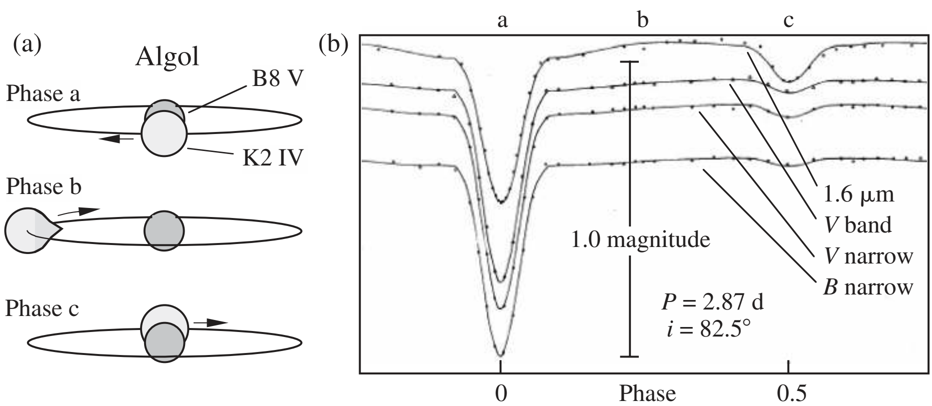

Figure 1.5. (a) Schematic drawing of the partially eclipsing system Algol (β Persei) approximately to scale. Darker shadings represent brighter surfaces. (b) Its light curve in four frequency bands. The designations a, b, c are used to associate portions of the light curve with particular phases of the orbit. The B8 V and K2 IV stars orbit one another with a period of 2.87 d and inclination 82.5° and are separated by 14 $R_\odot$. The K star fills and overflows its pseudopotential well (Roche lobe) and hence accretes matter onto the B star. The changing flux between the two eclipses is due to the changing aspect of the distorted K star and to backscattering of B-star light from the surface of the cooler K star. [(b) R. Wilson *et al., ApJ* **177**, 191 (1972)]

visual band, the effect of partially covering the hotter B8 star is much more pronounced than is the effect of covering the same area of the cooler K2 star, as seen in Fig. 5.

The details of such light curves can tell astronomers a great deal about the stars in the binary system. The existence of the eclipse constrains the orbital plane to lie roughly in the line of sight; the duration and shape of the eclipse are related to the inclination, the separation of the stars, and their physical sizes. The changes of intensity are related to the surface brightnesses and hence the classes of the stars. Can you speculate about the cause of the gradual changes of light during the phases between the two eclipses of Fig. 5 (see caption)?

Spectroscopic binaries

Some close binaries do not eclipse each other because the orbit has low inclination, the stars are sufficiently separated, or both. In these cases, the binary nature of the stars can be identified only through the detection of periodic Doppler shifts in the spectral lines of one or both stars. These binaries are called *spectroscopic binaries*. The Doppler shifts are due to the motions about the system barycenter.

The radial velocity of the star must be great enough to be detected as a spectral Doppler shift, and it must be bright enough to yield sufficient photons for high-resolution spectroscopy. The motions are greatest for binaries of close separation (see (3) below); most known spectroscopic binaries have separations less than 1 AU. Not surprisingly, therefore, systems showing orbital spectroscopic variations often also exhibit eclipses; these are called *eclipsing spectroscopic binaries*.

The orbits and Doppler velocities of a hypothetical binary system are shown in Fig. 6. For simplicity, the orbits are circular and oriented such that the observer (astronomer) is in the plane of the orbit, $i = 90°$. Thus, once each orbit, each star approaches directly toward

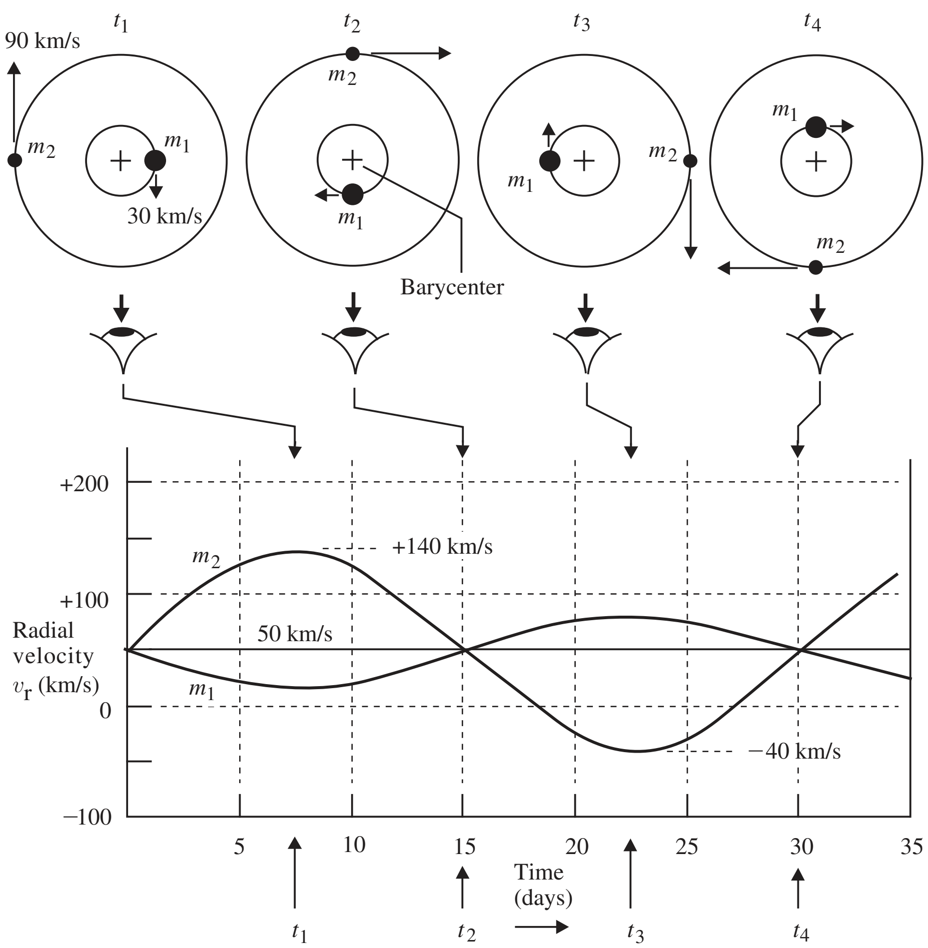

Figure 1.6. Hypothetical spectroscopic binary with circular orbits and a 30-d period shown at four phases of the orbit. The stationary observer is in the plane of the orbit. Star 1 is three times more massive than star 2. At any given time the two stars are on opposite sides of the barycenter, which is moving steadily away from the observer with a radial velocity component of +50 km/s. The star speeds relative to the barycenter are shown in the upper left. Star 2, with its smaller mass, is three times farther out from the barycenter than star 1; hence, it must travel three times faster to get around the orbit in the same time as star 1. The direction to the astronomer and the observed Doppler velocities are shown. [Adapted from G. Abell, *Exploration of the Universe*, 3rd Ed., Holt Rinehart Winston, 1975, p. 439, with permission of Brooks/Cole]

the observer, and a half-period later it recedes directly away. If the orbit were oriented such that it would lie in the plane of the sky, $i = 0°$, the stars would have no component of velocity along the line of sight. In this case, there would be no detectable Doppler shift.

The line-of-sight (radial) velocities are shown in Fig. 6 as a function of time for each star. They are not centered about zero radial velocity because the barycenter of the system is (in our example) receding from the observer with a radial velocity of $v_r = +50$ km/s. Star 1 is more massive than star 2 ($m_1 = 3m_2$). It is therefore closer to the barycenter (upper sketches) and is moving at a lesser velocity relative to the barycenter; see plots.

The radial-velocity curves are obtained spectroscopically from observations of the Doppler shifts of the frequency of stellar absorption (or emission) lines. The shift is toward higher frequency (blue shift) if the object approaches the observer and to lower frequency (red shift) if the object recedes. The usual sign convention for radial velocity in astronomy is "+" for a receding object and "−" for an approaching object.

The Doppler relation between the radial velocity υ_r and the frequency shift $\Delta\nu$ for nonrelativistic speeds ($\upsilon \ll c$) is

$$\frac{\nu - \nu_0}{\nu_0} = -\frac{\upsilon_r}{c}, \qquad (\upsilon \ll c) \qquad (1.2)$$

where ν_0 is the *rest frequency* of the absorption line (as would be seen by an observer moving with the star), ν is the observed frequency at the earth, and υ_r is the radial component of the star's velocity relative to that of the earth.

Additional features of Fig. 6 are as follows:

(*i*) zero Doppler velocity (relative to the barycenter) at t_2 and t_4 when the stars are moving at right angles to the line of sight;
(*ii*) sinusoidal radial-velocity curves as expected for projected circular motion at any inclination;
(*iii*) amplitudes that reflect the 3-to-1 mass ratio; and
(*iv*) phases that differ by exactly 180° owing to momentum conservation.

Data from an actual spectroscopic binary, ϕ Cygni, are shown in Fig. 7. Spectral lines from each of the two stars yield, from (2), the plotted radial velocity points. They show asymmetries introduced by the orientation of the elliptical orbits relative to the observer's line of sight and by the varying speeds of the stars as they move in their elliptical orbits. Note that the curves cross at a nonzero velocity. Again, this is due to the motion of the barycenter relative to the observer.

In many actual spectroscopic binary systems, astronomers obtain only one curve because one star is too faint – either in an absolute sense or because its light is swamped by its much brighter companion. These are called *single-line spectroscopic binaries*. If the brighter star is much more massive than its companion, as is likely (see Section 4.3), its motion may be too small to be measured. In this case only an upper limit to υ_r is obtained. On the other hand, if the Doppler shifts of both stars are measurable, and if eclipses occur, a wealth of information is obtained. Such a system would be a *double-line eclipsing binary*.

1.3 Kepler and Newton

The laws of Kepler are described here together with an analysis of the ellipse and a presentation of Newton's equations of motion in polar coordinates. The latter lead to Kepler's laws, as we demonstrate in Sections 4 and 5. Here we also discuss how Kepler's laws govern the motions of earth-orbiting satellites.

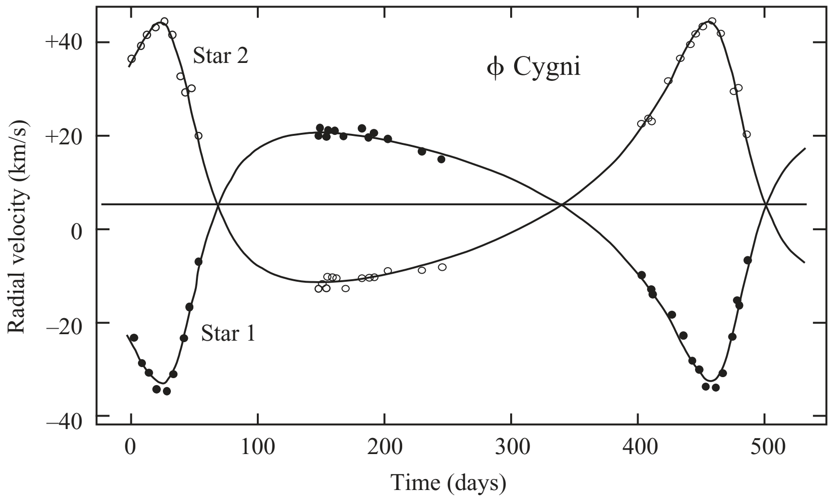

Figure 1.7. The radial velocities as a function of time for ϕ Cygni derived from the Doppler frequency shifts of the spectral lines. This is a double-line spectroscopic binary system consisting of two giants of about equal masses. The smooth curves are theoretical fits to the data points. The orientation of the elliptical orbit with respect to the line of sight and the nonconstant speeds in the orbit yield the strange shape. The barycenter recedes from the observer. [Adapted from R. Rach and G. Herbig, *ApJ* **133**, 143 (1961)]

Kepler's laws $(M \gg m)$

Kepler carried out a detailed analysis of the celestial tracks of the sun's planets as recorded with good precision by Tycho Brahe (1546–1601). The sun is so much more massive than the planets that it can be considered to be stationary (i.e., the condition $M \gg m$ holds). He discovered three simple laws that well describe the tracks of the planets, the speed variations of a planet in its orbit, and the relative periods of the orbits of the several planets. They are known as Kepler's laws and are as follows:

Kepler I. The orbital track of a given planet is an ellipse with the sun at one of the foci. (A circular orbit is a special case of an ellipse.)

Kepler II. The radius vector (sun to planet) sweeps out equal areas in equal time.

Kepler III. The square of the orbital period P^2 is proportional to the cube of the semimajor axis a^3 of the orbit. That is, $P^2 = c_1 a^3$, where c_1 is a constant *independent of the mass of the planet*. The physical constants that make up c_1 are now known (see (45) below), and so the law becomes

$$GMP^2 = 4\pi^2 a^3, \qquad \text{(Kepler III; } M \gg m\text{)} \tag{1.3}$$

where M is the mass of the central object, if $M \gg m$.

The first law tells us that the orbits are elliptical (Fig. 8a) and that the sun is at one of the foci. It is remarkable, as we later demonstrate, that, according to a Newtonian analysis, an ellipse is precisely the expected track for an inverse-squared gravitational force law. Kepler was not just close; he was exactly right.

The second law (Fig. 8b) tells us that, as a planet traverses its orbit, it speeds up as it approaches the sun. It is fastest at the closest approach (*perihelion*) and slowest at its