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Introduction

The experimental realization in 1995 of Bose–Einstein condensation in dilute atomic gases marked the beginning of a very rapid development in the study of quantum gases. The initial experiments were performed on vapours of rubidium [1], sodium [2], and lithium [3].[1] So far, the atoms ^{1}H, ^{7}Li, ^{23}Na, ^{39}K, ^{41}K, ^{52}Cr, ^{85}Rb, ^{87}Rb, ^{133}Cs, ^{170}Yb, ^{174}Yb and ^{4}He* (the helium atom in an excited state) have been demonstrated to undergo Bose–Einstein condensation. In related developments, atomic Fermi gases have been cooled to well below the degeneracy temperature, and a superfluid state with correlated pairs of fermions has been observed. Also molecules consisting of pairs of fermionic atoms such as ^{6}Li or ^{40}K have been observed to undergo Bose–Einstein condensation. Atoms have been put into optical lattices, thereby allowing the study of many-body systems that are realizations of models used in condensed matter physics. Although the gases are very dilute, the atoms can be made to interact strongly, thus providing new challenges for the description of strongly correlated many-body systems. In a period of less than ten years the study of dilute quantum gases has changed from an esoteric topic to an integral part of contemporary physics, with strong ties to molecular, atomic, subatomic and condensed matter physics.

The dilute quantum gases differ from ordinary gases, liquids and solids in a number of ways, as we shall now illustrate by giving values of physical quantities. The particle density at the centre of a Bose–Einstein condensed atomic cloud is typically 10^{13}–10^{15} cm^{-3}. By contrast, the density of molecules in air at room temperature and atmospheric pressure is about 10^{19} cm^{-3}. In liquids and solids the density of atoms is of order 10^{22} cm^{-3}, while the density of nucleons in atomic nuclei is about 10^{38} cm^{-3}.

To observe quantum phenomena in such low-density systems, the tem-

[1] Numbers in square brackets are references, to be found at the end of each chapter.

perature must be of order 10^{-5} K or less. This may be contrasted with the temperatures at which quantum phenomena occur in solids and liquids. In solids, quantum effects become strong for electrons in metals below the Fermi temperature, which is typically 10^4–10^5 K, and for phonons below the Debye temperature, which is typically of order 10^2 K. For the helium liquids, the temperatures required for observing quantum phenomena are of order 1 K. Due to the much higher particle density in atomic nuclei, the corresponding degeneracy temperature is about 10^{11} K.

The path that led in 1995 to the first realization of Bose–Einstein condensation in dilute gases exploited the powerful methods developed since the mid 1970s for cooling alkali metal atoms by using lasers. Since laser cooling alone did not produce sufficiently high densities and low temperatures for condensation, it was followed by an evaporative cooling stage, in which the more energetic atoms were removed from the trap, thereby cooling the remaining atoms.

Cold gas clouds have many advantages for investigations of quantum phenomena. In a weakly interacting Bose–Einstein condensate, essentially all atoms occupy the same quantum state, and the condensate may be described in terms of a mean-field theory similar to the Hartree–Fock theory for atoms. This is in marked contrast to liquid ^{4}He, for which a mean-field approach is inapplicable due to the strong correlations induced by the interaction between the atoms. Although the gases are dilute, interactions play an important role as a consequence of the low temperatures, and they give rise to collective phenomena related to those observed in solids, quantum liquids, and nuclei. Experimentally the systems are attractive ones to work with, since they may be manipulated by the use of lasers and magnetic fields. In addition, interactions between atoms may be varied either by using different atomic species or, for species that have a Feshbach resonance, by changing the strength of an applied magnetic or electric field. A further advantage is that, because of the low density, 'microscopic' length scales are so large that the structure of the condensate wave function may be investigated directly by optical means. Finally, these systems are ideal for studies of interference phenomena and atom optics.

The theoretical prediction of Bose–Einstein condensation dates back more than 80 years. Following the work of Bose on the statistics of photons [4], Einstein considered a gas of non-interacting, massive bosons, and concluded that, below a certain temperature, a non-zero fraction of the total number of particles would occupy the lowest-energy single-particle state [5]. In 1938 Fritz London suggested the connection between the superfluidity of liquid ^{4}He and Bose–Einstein condensation [6]. Superfluid liquid ^{4}He is the prototype Bose–Einstein condensate, and it has played a unique role in the development of physical concepts. However, the interaction between helium atoms is strong, and this reduces the number of atoms in the zero-momentum state even at absolute zero. Consequently it is difficult to measure directly the occupancy of the zero-momentum state. It has been investigated experimentally by neutron scattering measurements of the structure factor at large momentum transfers [7], and the results are consistent with a relative occupation of the zero-momentum state of about 0.1 at saturated vapour pressure and about 0.05 near the melting pressure [8].

The fact that interactions in liquid helium reduce dramatically the occupancy of the lowest single-particle state led to the search for weakly interacting Bose gases with a higher condensate fraction. The difficulty with most substances is that at low temperatures they do not remain gaseous, but form solids or, in the case of the helium isotopes, liquids, and the effects of interaction thus become large. In other examples atoms first combine to form molecules, which subsequently solidify. As long ago as in 1959 Hecht [9] argued that spin-polarized hydrogen would be a good candidate for a weakly interacting Bose gas. The attractive interaction between two hydrogen atoms with their electronic spins aligned was then estimated to be so weak that there would be no bound state. Thus a gas of hydrogen atoms in a magnetic field would be stable against formation of molecules and, moreover, would not form a liquid, but remain a gas to arbitrarily low temperatures.

Hecht's paper was before its time and received little attention, but his conclusions were confirmed by Stwalley and Nosanow [10] in 1976, when improved information about interactions between spin-aligned hydrogen atoms was available. These authors also argued that because of interatomic interactions the system would be a superfluid as well as being Bose–Einstein condensed. This latter paper stimulated the quest to realize Bose–Einstein condensation in atomic hydrogen. Initial experimental attempts used a high magnetic field gradient to force hydrogen atoms against a cryogenically cooled surface. In the lowest-energy spin state of the hydrogen atom, the electron spin is aligned opposite the direction of the magnetic field (H↓), since then the magnetic moment is in the same direction as the field. Spin-polarized hydrogen was first stabilized by Silvera and Walraven [11]. Interactions of hydrogen with the surface limited the densities achieved in the early experiments, and this prompted the Massachusetts Institute of Technology (MIT) group led by Greytak and Kleppner to develop methods for trapping atoms purely magnetically. In a current-free region, it is impossible to create a local maximum in the magnitude of the magnetic field. To trap atoms by the Zeeman effect it is therefore necessary to work with a state of hydrogen in which the electronic spin is polarized parallel to the magnetic field (H↑). Among the techniques developed by this group is that of evaporative cooling of trapped gases, which has been used as the final stage in all experiments to date to produce a gaseous Bose–Einstein condensate. Since laser cooling is not feasible for hydrogen, the gas was precooled cryogenically. After more than two decades of heroic experimental work, Bose–Einstein condensation of atomic hydrogen was achieved in 1998 [12].

As a consequence of the dramatic advances made in laser cooling of alkali atoms, such atoms became attractive candidates for Bose–Einstein condensation, and they were used in the first successful experiments to produce a gaseous Bose–Einstein condensate. In later developments other atoms have been shown to undergo Bose–Einstein condensation: metastable ^{4}He atoms in the lowest-energy electronic spin-triplet state [13, 14], and ytterbium [15, 16] and chromium atoms [17] in their electronic ground states.

The properties of interacting Bose fluids are treated in many texts. The reader will find an illuminating discussion in the volume by Nozières and Pines [18]. A collection of articles on Bose–Einstein condensation in various systems, prior to its discovery in atomic vapours, is given in [19], while more recent theoretical developments have been reviewed in [20]. The 1998 Varenna lectures are a useful general reference for both experiment and theory on Bose–Einstein condensation in atomic gases, and contain in addition historical accounts of the development of the field [21]. For a tutorial review of some concepts basic to an understanding of Bose–Einstein condensation in dilute gases see Ref. [22]. The monograph [23] gives a comprehensive account of Bose–Einstein condensation in liquid helium and dilute atomic gases.

1.1 Bose–Einstein condensation in atomic clouds

Bosons are particles with integer spin. The wave function for a system of identical bosons is symmetric under interchange of the coordinates of any two particles. Unlike fermions, which have half-odd-integer spin and antisymmetric wave functions, bosons may occupy the same single-particle state. An estimate of the transition temperature to the Bose–Einstein condensed state may be made from dimensional arguments. For a uniform gas of free particles, the relevant quantities are the particle mass m, the number of particles per unit volume n, and the Planck constant $h = 2\pi\hbar$. The only quantity having dimensions of energy that can be formed from $\hbar$, n, and m is $\hbar^2 n^{2/3}/m$. By dividing this energy by the Boltzmann constant

k we obtain an estimate of the condensation temperature $T_{\rm c}$,

$$T_{\rm c} = C\frac{\hbar^2 n^{2/3}}{mk}. \tag{1.1}$$

Here C is a numerical factor which we shall show in the next chapter to be equal to approximately 3.3. When (1.1) is evaluated for the mass and density appropriate to liquid ^{4}He at saturated vapour pressure one obtains a transition temperature of approximately 3.13 K, which is close to the temperature below which superfluid phenomena are observed, the so-called lambda point[2] (T_λ= 2.17 K at saturated vapour pressure).

An equivalent way of relating the transition temperature to the particle density is to compare the thermal de Broglie wavelength λ_T with the mean interparticle spacing, which is of order $n^{-1/3}$. The thermal de Broglie wavelength is conventionally defined by

$$\lambda_T = \left(\frac{2\pi\hbar^2}{mkT}\right)^{1/2}. \tag{1.2}$$

At high temperatures, it is small and the gas behaves classically. Bose–Einstein condensation in an ideal gas sets in when the temperature is so low that λ_T is comparable to $n^{-1/3}$. For alkali atoms, the densities achieved range from 10^{13} cm^{-3} in early experiments to 10^{14}–10^{15} cm^{-3} in more recent ones, with transition temperatures in the range from 100 nK to a few μK. For hydrogen, the mass is lower and the transition temperatures are correspondingly higher.

In experiments, gases are non-uniform, since they are contained in a trap, which typically provides a harmonic-oscillator potential. If the number of particles is N, the density of gas in the cloud is of order N/R^3, where the size R of a thermal gas cloud is of order $(kT/m\omega_0^2)^{1/2}$, ω_0 being the angular frequency of single-particle motion in the harmonic-oscillator potential. Substituting the value of the density $n \sim N/R^3$ at $T = T_{\rm c}$ into Eq. (1.1), one sees that the transition temperature is given by

$$kT_{\rm c} = C_1\hbar\omega_0 N^{1/3}, \tag{1.3}$$

where C_1 is a numerical constant which we shall later show to be approximately 0.94. The frequencies for traps used in experiments are typically of order 10^2 Hz, corresponding to $\omega_0 \sim 10^3$ s^{-1}, and therefore, for particle numbers in the range from 10^4 to 10^8, the transition temperatures lie in the range quoted above. Estimates of the transition temperature based

[2] The name *lambda point* derives from the shape of the experimentally measured specific heat as a function of temperature, which near the transition resembles the Greek letter λ.

on results for a uniform Bose gas are therefore consistent with those for a trapped gas.

In the original experiment [1] the starting point was a room-temperature gas of rubidium atoms, which were trapped and cooled by lasers to about 20 μK. Subsequently the lasers were turned off and the atoms trapped magnetically by the Zeeman interaction of the electron spin with an inhomogeneous magnetic field. If we neglect complications caused by the nuclear spin, an atom with its electron spin parallel to the magnetic field is attracted to the minimum of the magnetic field, while one with its electron spin antiparallel to the magnetic field is repelled. The trapping potential was provided by a quadrupole magnetic field, upon which a small oscillating bias field was imposed to prevent loss of particles at the centre of the trap. Later experiments have employed a wealth of different magnetic field configurations, and also made extensive use of optical traps.

In the magnetic trap the cloud of atoms was cooled further by evaporation. The rate of evaporation was enhanced by applying a radio-frequency magnetic field which flipped the electronic spin of the most energetic atoms from up to down. Since the latter atoms are repelled by the trap, they escape, and the average energy of the remaining atoms falls. It is remarkable that no cryogenic apparatus was involved in achieving the record-low temperatures in the experiment [1]. Everything was held at room temperature except the atomic cloud, which was cooled to temperatures of the order of 100 nK.

So far, Bose–Einstein condensation has been realized experimentally in dilute gases of hydrogen, lithium, sodium, potassium, chromium, rubidium, cesium, ytterbium, and metastable helium atoms. Due to the difference in the properties of these atoms and their mutual interaction, the experimental study of the condensates has revealed a range of fascinating phenomena which will be discussed in later chapters. The presence of the nuclear and electronic spin degrees of freedom adds further richness to these systems when compared with liquid ^{4}He, and it gives the possibility of studying multi-component condensates.

From a theoretical point of view, much of the appeal of atomic gases stems from the fact that at low energies the effective interaction between particles may be characterized by a single quantity, the scattering length. The gases are often dilute in the sense that the scattering length is much less than the interparticle spacing. This makes it possible to calculate the properties of the system with high precision. For a *uniform* dilute gas the relevant theoretical framework was developed in the 1950s and 60s, but the presence of a confining potential gives rise to new features that are absent for uniform systems. The possibility of tuning the interatomic interaction by varying the magnitude of the external magnetic field makes it possible to study experimentally also the regime where the scattering length is comparable to or much larger than the interparticle spacing. Under these conditions the atomic clouds constitute strongly interacting many-body systems.

1.2 Superfluid ^{4}He

Many of the concepts used to describe properties of quantum gases were developed in the context of liquid ^{4}He. The helium liquids are exceptions to the rule that liquids solidify when cooled to sufficiently low temperatures, because the low mass of the helium atom makes the zero-point energy large enough to overcome the tendency to crystallization. At the lowest temperatures the helium liquids solidify only under a pressure in excess of 25 bar (2.5 MPa) for ^{4}He and 34 bar for the lighter isotope ^{3}He.

Below the lambda point, liquid ^{4}He becomes a superfluid with many remarkable properties. One of the most striking is the ability to flow through narrow channels without friction. Another is the existence of quantized vorticity, the quantum of circulation being given by h/m $(= 2\pi\hbar/m)$. The occurrence of frictionless flow led Landau and Tisza to introduce a two-fluid description of the hydrodynamics. The two fluids – the normal and the superfluid components – are interpenetrating, and their densities depend on temperature. At very low temperatures the density of the normal component vanishes, while the density of the superfluid component approaches the total density of the liquid. The superfluid density is therefore generally quite different from the density of particles in the condensate, which for liquid ^{4}He is only about 10% or less of the total, as mentioned above. Near the transition temperature to the normal state the situation is reversed: here the superfluid density tends towards zero as the temperature approaches the lambda point, while the normal density approaches the density of the liquid.

The properties of the normal component may be related to the elementary excitations of the superfluid. The concept of an elementary excitation plays a central role in the description of quantum systems. In a uniform ideal gas an elementary excitation corresponds to the addition of a single particle in a momentum eigenstate. Interactions modify this picture, but for low excitation energies there still exist excitations with well-defined energies. For small momenta the excitations in liquid ^{4}He are sound waves or *phonons.* Their dispersion relation is linear, the energy ϵ being proportional to the

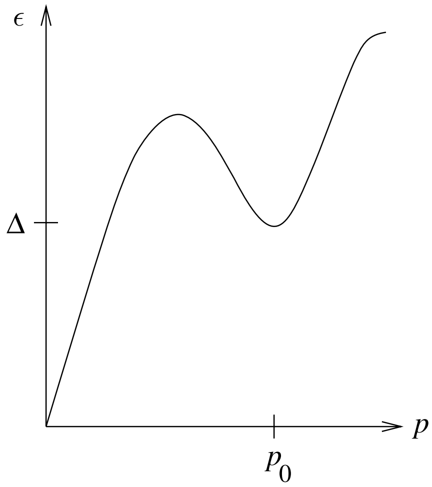

Fig. 1.1 The spectrum of elementary excitations in superfluid ^{4}He. The minimum roton energy is Δ.

magnitude of the momentum p,

$$\epsilon = sp, \tag{1.4}$$

where the constant s is the velocity of sound. For larger values of p, the dispersion relation shows a slight upward curvature for pressures less than 18 bar, and a downward one for higher pressures. At still larger momenta, $\epsilon(p)$ exhibits first a local maximum and subsequently a local minimum. Near this minimum the dispersion relation may be approximated by

$$\epsilon(p) = \Delta + \frac{(p - p_0)^2}{2m^*}, \tag{1.5}$$

where m^* is a constant with the dimension of mass and p_0 is the momentum at the minimum. Excitations with momenta close to p_0 are referred to as *rotons*. The name was coined to suggest the existence of vorticity associated with these excitations, but they should really be considered as short-wavelength phonon-like excitations. Experimentally, one finds at zero pressure that m^* is 0.16 times the mass of a ^{4}He atom, while the constant Δ, the minimum roton energy, is given by $\Delta/k = 8.7$ K. The roton minimum occurs at a wave number $p_0/\hbar$ equal to 1.9×10^8 cm^{-1} (see Fig. 1.1). For excitation energies greater than 2Δ the excitations become less well-defined since they can decay into two rotons.

The elementary excitations obey Bose statistics, and therefore in thermal equilibrium their distribution function f^0 is given by

$$f^0 = \frac{1}{e^{\epsilon(p)/kT} - 1}. \tag{1.6}$$

The absence of a chemical potential in this distribution function is due to the fact that the number of excitations is not a conserved quantity: the energy of an excitation equals the difference between the energy of an excited state and the energy of the ground state for a system containing the same number of particles. The number of excitations therefore depends on the temperature, just as the number of phonons in a solid does. This distribution function Eq. (1.6) may be used to evaluate thermodynamic properties.

1.3 Other condensates

The concept of Bose–Einstein condensation finds applications in many systems other than liquid ^{4}He and the atomic clouds discussed above. Historically, the first of these were superconducting metals, where the bosons are pairs of electrons with opposite spin. Many aspects of the behaviour of superconductors may be understood qualitatively on the basis of the idea that pairs of electrons form a Bose–Einstein condensate, but the properties of superconductors are quantitatively very different from those of a weakly interacting gas of pairs. The important physical point is that the binding energy of a pair is small compared with typical atomic energies, and at the temperature where the condensate disappears the pairs themselves break up. This situation is to be contrasted with that for the atomic systems, where the energy required to break up an atom is the ionization energy, which is of order electron volts. This corresponds to temperatures of tens of thousands of degrees, which are much higher than the temperatures for Bose–Einstein condensation.

Many properties of high-temperature superconductors may be understood in terms of Bose–Einstein condensation of pairs, in this case of holes rather than electrons, in states having predominantly d-like symmetry in contrast to the s-like symmetry of pairs in conventional metallic superconductors. The rich variety of magnetic and other behaviour of the superfluid phases of liquid ^{3}He is again due to condensation of pairs of fermions, in this case ^{3}He atoms in triplet spin states with p-wave symmetry. Considerable experimental effort has been directed towards creating Bose–Einstein condensates of excitons, which are bound states of an electron and a hole [24], and of biexcitons ('molecules' made up of two excitons) [25], but the strongest evidence for condensation of such excitations has been obtained for polaritons (hybrid excitations consisting of excitons and photons) [26].

Bose–Einstein condensation of pairs of fermions is also observed experimentally in atomic nuclei, where the effects of neutron–neutron and proton–proton pairing may be seen in the excitation spectrum as well as in reduced moments of inertia. A significant difference between nuclei and superconductors is that the size of a pair in bulk nuclear matter is large compared with the nuclear size, and consequently the manifestations of Bose–Einstein condensation in nuclei are less dramatic than they are in bulk systems. Theoretically, Bose–Einstein condensation of nucleon pairs is expected to play an important role in the interiors of neutron stars, and observations of glitches in the spin-down rate of pulsars have been interpreted in terms of neutron superfluidity. The possibility of mesons, either pions or kaons, forming a Bose–Einstein condensate in the cores of neutron stars has been widely discussed, since this would have far-reaching consequences for theories of supernovae and the evolution of neutron stars [27].

In the field of nuclear and particle physics the ideas of Bose–Einstein condensation also find application in the understanding of the vacuum as a condensate of quark–antiquark ($\mathrm{u\bar{u}}$, $\mathrm{d\bar{d}}$ and $\mathrm{s\bar{s}}$) pairs, the so-called chiral condensate. This condensate gives rise to particle masses in much the same way as the condensate of electron pairs in a superconductor gives rise to the gap in the electronic excitation spectrum. Condensation of pairs of quarks with different flavours and spins has been the subject of much theoretical work [28].

This brief account of the rich variety of contexts in which the physics of Bose–Einstein condensation plays a role shows that an understanding of the phenomenon is of importance in many branches of physics.

1.4 Overview

To assist the reader, we give here a brief survey of the material we cover. We begin, in Chapter 2, by discussing Bose–Einstein condensation for non-interacting gases in a confining potential. This is useful for developing understanding of the phenomenon of Bose–Einstein condensation and for application to experiment, since in dilute gases many quantities, such as the transition temperature and the condensate fraction, are close to those predicted for a non-interacting gas. We also discuss the density profile and the velocity distribution of particles in an atomic cloud at zero temperature. When the thermal energy kT exceeds the spacing between the energy levels of an atom in the confining potential, the gas may be described semi-classically in terms of a particle distribution function that depends on both