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166 The Lévy Laplacian

The Lévy Laplacian is an infinite-dimensional generalization of the well-known classical Laplacian. Its theory has been increasingly well-developed in recent years and this book is the first systematic treatment of it.

The book describes the infinite-dimensional analogues of finite-dimensional results, and more especially those features that appear only in the generalized context. It develops a theory of operators generated by the Lévy Laplacian and the symmetrized Lévy Laplacian, as well as a theory of linear and nonlinear equations involving it. There are many problems leading to equations with Lévy Laplacians and to Lévy–Laplace operators, for example superconductivity theory, the theory of control systems, the Gauss random field theory, and the Yang–Mills equation.

The book is complemented by exhaustive bibliographic notes and references. The result is a work that will be valued by those working in functional analysis, partial differential equations and probability theory.

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