Index

*abc*-conjecture
  Baker’s explicit version, 404
  implies Catalan’s conjecture, 403
  implies Fermat’s last theorem, 403
  implies Roth’s theorem, 406
  $K$-rational, 497
  strong form, 402
  over number field, 494
  weak form, 403
*abc*-ratio, 423, 435
*abc*-theorem
  for meromorphic functions, 456, 477
  for polynomials, 416, 504
Abel’s theorem, 271
Abelian extension, 113, 344
Abelian subvariety, 233
Abelian variety, 232
  as a complex torus, 239
  is commutative, 234
  is projective, 254
  simple, 267
Absolute value, 1–4
  archimedean, 2
  discrete, 4
  equivalent, 2
  non-archimedean, 2
  normalization of, 6, 9, 11
  $p$-adic, 2
  trivial, 2
Absolute values
  of a function field, 12
  of a number field, 10
Additive reduction
  of an elliptic curve, 427
Adeles, 604–607
Admissible pair, 372
Affine chart, 521
Affine open subset, 522
Affine space, 514
Affine variety, 515
  associated complex analytic variety, 519
  associated scheme, 516
  basis of topology, 517
  complex topology, 519
  geometrically irreducible, 519
  geometrically reduced, 519
  open subsets are quasicompact, 517
Ahlfors–Shimizu characteristic, 458
  of a holomorphic curve, 468
Albanese variety, 394
Algebraic equivalence
  of divisors, 562
  of line bundles, 561–563
  on an abelian variety, 255, 265
Algebraic subgroup, 82
Algebraic variety, 584
Amoroso–Dvornicich theorem, 113
Ample
  Cartier divisor, 548
  line bundle, 534
  finite pull-back, 578
  on abelian variety, 252–255
  on curve, 583
Approximation
  class, 164
  nontrivial, 164
Approximation class
  primitive, 200
Approximation domain, 201
$v$-adic, 201
Approximation theorem, 4
  strong form, 11
Arakelov height
  multiplicative, 66
  of a matrix, 67, 69, 74
  of a subspace, 67, 395
  on projective space, 66
Archimedean, see also Absolute value
644  Index

Arithmetic genus, 583
Artin–Schreier equation, 154
Automorphism of group varieties, 232

Bad reduction of elliptic curve, 426
Banach’s fixed point theorem, 368
Base change
and (very) ample, 534
and closed embeddings, 534
of a variety, 522
of a vector bundle, 529
of affine variety, 517
of coherent sheaf, 570
of cycle, 555

Base-point
of a line bundle, 530
of complete linear system, 550

Base-point-free
Cartier divisor, 548
line bundle, 530
Belyi’s lemma, 404
Belyi’s theorem, 413
Bertram–Ein–Lazarsfeld theorem, 53
Bi-elliptic curve, 391
Bilu’s theorem, 102, 399
Binet’s formula, 68

Birational map, 574
Birational to a hypersurface, 575–576
Birational varieties, 575
Birthday paradox, 423
Bogomolov conjecture, 399
Bogomolov property (B), 120
Bogomolov’s conjecture, 400
Bombieri–Lang conjecture, 486
Bounded subset, 37, 54–57
Bézout’s theorem, 561

Canonical divisor, 583, 599
Canonical form, 294
Canonical line bundle, 473, 542, 582, 583
Canonical metric, 303, 308
has harmonic Chern form, 311
Cartan’s formula, 448
Cartier divisor, 548
Čech cocycle, 566
associated Weil divisor, 549
effective, 550
of rational function, 549
Catalan’s conjecture, 403
Cauchy inequalities, 366
Čech boundary, 565

Čech cocycle, 565
Čech cohomology group, 565
Čech complex, 564
Character, 607
Characteristic function, 448
of a holomorphic curve, 466, 476
of Ahlfors–Shimizu, 458
Chevalley–Weil theorem, 341
for abelian varieties, 341
for discrete valuations, 339
for number fields, 339
global version, 338
local version, 336
local version for abelian varieties, 340
Chordal distance, 457
Chow group, 553
of projective space, 560
Chow’s lemma, 562
Class group, 142
of \( S \)-integers, 348
Closed embedding, 523
Closed map, 535
Closed subgroup, 232
Closed subvariety, 523
ideal sheaf, 523, 528, 569
Cocycle rule, 526
Codifferent, 590
Codimension, 523
Coherent sheaf, 567–574
Cohomology group
of coherent sheaf, 569–574
Compact, xv
Complete linear system, 550
Complete variety, 535–536
Completion, 2
Complex analytic variety, 584
Complex embedding, 7
Complex manifold, 583–585
Complex space, 584
Complex topology, 519, 584
Complex torus, 239
Component
of a cycle, 551
of Weil divisor, 544
Composition series, 552
Conductor
of a meromorphic function, 456
of a point, 489
over function field, 499
Conjugates
of a point in an affine variety, 516
of a variety, 524
Conormal bundle, 543
Constancy lemma, 233
Coordinate ring, 515
is noetherian, 517
Correspondence, 309
Cotangent bundle, 542
Cotangent space, 542
Counting function, 446
in diophantine approximation, 481
in diophantine geometry, 483
of a holomorphic curve, 466, 476
over function field, 499
Covering group, 411
Cube-slicing inequality, 618
Curve, 581–583
rational, 237
Cycle, 551
of a scheme, 555
Darmon–Granville theorem, 441
Davenport–Estermann theorem, 611
Decomposition group, 597
Dedekind domain, 589
Dedekind’s different theorem, 594
Dedekind’s discriminant theorem, 595
Defect inequality, 452
Defect of meromorphic function, 452
Deficient value, 453
Degree
essential, 88
is positive, 561
of a cycle, 559, 561
of a line bundle, 582
of a variety, 559
of an algebraic number, xvi
of morphism, 578
of zero-dimensional cycle, 557
Derivation, 345
Derivative, 536
Descente inﬁnie, 350
Different, 590–591
Dedekind’s theorem, 594
Differential
of a function, 543
of a morphism, 537, 538
Differential forms, 542
Dimension
of cycle, 551
of topological space, 518
Dimension theorem
for group varieties, 236
of varieties, 577
Direct image of coherent sheaf, 568
Direct sum of vector bundles, 527
Dirichlet’s theorem, 181
Dirichlet’s unit theorem, 18
for orders, 189
Discrete valuation ring, 544
Discriminant
absolute logarithmic, 487
for function fields, 499
Hermite’s theorem, 595
minimal global form, 429
minimal local form, 426
Minkowski’s theorem, 596
of a polynomial, 586–589, 592
of elliptic curve, 425
of free algebra, 586
over Dedekind domain, 589–591
Discriminant ideal, 586
Distorsion factor, 69
Divisor, 544–551
Néron, 61
on regular variety, 549
special, 270
with normal crossings, 473, 484
Dobrowolski’s theorem, 29, 107
Domain, 574
Dominant rational map, 574
Drasin’s results on deﬁciencies, 454
Dual abelian variety, 255
biduality, 256
Dual vector bundle, 528
Effective Cartier divisor, 550
Effective cycle, 561
Effective divisor, 549
Effective methods, 146
for unit equation, 146
Effective Weil divisor, 544
Elliptic curve, 240–246, 425–431
addition law, 244
additive reduction, 427
good reduction, 426
multiplicative reduction, 427
split multiplicative reduction, 314
Elliptic function, 453
Endomorphism of group varieties, 232
Enflo’s theorem, 29
Enumerating function, 446
Essential degree, 88
Étale morphism, 580

differential criterion, 581
for schemes, 598
local behaviour, 581, 598
Euler characteristic
of coherent sheaf, 571
Even elements, 258
Even line bundle, 258
Exact sequence
of sheaves, 564
Exponent of field extension, 117, 344
Exterior product of vector bundles, 529
Faltings’s big theorem, 391, 486
Faltings’s theorem, 352, 406, 485, 497
Vojta’s proof, 352
Faltings–Wüthholz theorem, 229
Fano surface for cubic threefold, 394
Fano variety, 393
Fenchel’s conjecture, 437
Fermat curve, 495
Fermat descent, 349, 350
Fermat equation, 403
generalized, 435, 441
Fermat’s conjecture, 403, 429
Fermat’s last theorem, 403, 429
Fibre of vector bundle, 526
Fibre product of varieties, 535
Filtration
Harder–Narasimhan, 229
jointly semistable, 228
Finite length, 552
Finite morphism, 577–579
First Chern class, 554
First main theorem
Ahlfors–Shimizu version, 458
for a holomorphic curve, 469
of Nevanlinna, 448
Fischer’s inequality, 68
Flat module, 579
Flat morphism, 579–580
for schemes, 598
is open, 580
Fourier transform, 607
Fox’s theorem, 437
Free $O_X$-module, 566
Frey curve, 429
Frobenius map, 299
Fubini–Study form, 457
Fubini–Study metric, 58, 467
Full module, 189
Function field, 524
and birationality, 575
extension of, 12
of projective variety, 531
places of, 12
Fundamental group, 411
Fundamental inequality, 20
GAGA-principle, 584
Galois cohomology, 345
Gap principle, 139
strong, 171
Gauss norm, 22
Gauss’s lemma, 22
Gelfond’s lemma, 27
general abc-theorem
for function field, 511
for several polynomials, 418, 511
General Lang conjecture, 486
General position
hyperplanes, 470
linear forms, 180
Genus, 582
Geometrically connected, 235
Geometrically irreducible
affine variety, 519
smooth variety, 539
variety, 522, 524
Geometrically reduced
affine variety, 519
variety, 522, 524
smooth points are dense, 540
Global height, 40
Global section, 549
of a vector bundle, 527
Good reduction
of a curve, 438
of an abelian variety, 312, 340
of an elliptic curve, 426
Group variety, 232
Haar measure, 602–604
normalized on adeles, 605–607
Hadamard’s inequality, 26, 68
Hall’s conjecture, 425
strong form, 425
Hall–Lang–Waldschmidt–Szpiro conjecture, 434
Height, 15–21
function, 41, 63
<table>
<thead>
<tr>
<th>Metric</th>
<th>p.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, 62</td>
<td></td>
</tr>
<tr>
<td>canonical, 303, 308</td>
<td></td>
</tr>
<tr>
<td>Fubini–Study, 58</td>
<td></td>
</tr>
<tr>
<td>locally bounded, 58</td>
<td></td>
</tr>
<tr>
<td>on a line bundle, 58</td>
<td></td>
</tr>
<tr>
<td>standard, 58</td>
<td></td>
</tr>
<tr>
<td>trivial, 58</td>
<td></td>
</tr>
<tr>
<td>Metrized line bundle, 58</td>
<td></td>
</tr>
<tr>
<td>isometric, 58</td>
<td></td>
</tr>
<tr>
<td>pull-back of, 58</td>
<td></td>
</tr>
<tr>
<td>Minimal polynomial, xvi</td>
<td></td>
</tr>
<tr>
<td>Minimal subset for unit equation, 510</td>
<td></td>
</tr>
<tr>
<td>Minkowski’s discriminant theorem, 596</td>
<td></td>
</tr>
<tr>
<td>Minkowski’s first theorem, 615</td>
<td></td>
</tr>
<tr>
<td>Minkowski’s second theorem, 611, 614</td>
<td></td>
</tr>
<tr>
<td>Monoidal transformation, 83</td>
<td></td>
</tr>
<tr>
<td>Mordell conjecture, 352, 406</td>
<td></td>
</tr>
<tr>
<td>Mordell–Weil group, 355</td>
<td></td>
</tr>
<tr>
<td>Mordell–Weil theorem, 349</td>
<td></td>
</tr>
<tr>
<td>for finitely generated fields, 350</td>
<td></td>
</tr>
<tr>
<td>Morphism</td>
<td></td>
</tr>
<tr>
<td>of affine varieties, 515</td>
<td></td>
</tr>
<tr>
<td>of topological coverings, 411</td>
<td></td>
</tr>
<tr>
<td>of varieties, 521</td>
<td></td>
</tr>
<tr>
<td>Moving lemma, 305</td>
<td></td>
</tr>
<tr>
<td>Multiplicative reduction of elliptic curve, 427</td>
<td></td>
</tr>
<tr>
<td>Multiplicity</td>
<td></td>
</tr>
<tr>
<td>of a cycle, 551</td>
<td></td>
</tr>
<tr>
<td>of a divisor, 544</td>
<td></td>
</tr>
<tr>
<td>Multiprojective space, 534</td>
<td></td>
</tr>
<tr>
<td>cohomology, 574</td>
<td></td>
</tr>
<tr>
<td>Mumford’s formula, 294</td>
<td></td>
</tr>
<tr>
<td>generalization, 359</td>
<td></td>
</tr>
<tr>
<td>Mumford’s gap principle, 298</td>
<td></td>
</tr>
<tr>
<td>Natural density, 319</td>
<td></td>
</tr>
<tr>
<td>Néron divisor, 61</td>
<td></td>
</tr>
<tr>
<td>Néron model, 312, 327, 340</td>
<td></td>
</tr>
<tr>
<td>Néron symbol</td>
<td></td>
</tr>
<tr>
<td>on abelian varieties, 304</td>
<td></td>
</tr>
<tr>
<td>on complete varieties, 308</td>
<td></td>
</tr>
<tr>
<td>on curves, 310</td>
<td></td>
</tr>
<tr>
<td>Néron–Tate height, 284–289</td>
<td></td>
</tr>
<tr>
<td>associated bilinear form, 289–293</td>
<td></td>
</tr>
<tr>
<td>Nevanlinna’s inverse problem, 454</td>
<td></td>
</tr>
<tr>
<td>Noetherian ring, 517</td>
<td></td>
</tr>
<tr>
<td>Non-archimedean, see also Absolute value</td>
<td></td>
</tr>
<tr>
<td>Norm, 586</td>
<td></td>
</tr>
<tr>
<td>Norm-form equation, 189</td>
<td></td>
</tr>
<tr>
<td>family of solutions, 190</td>
<td></td>
</tr>
<tr>
<td>Term</td>
<td>Page</td>
</tr>
<tr>
<td>-----------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>Normal bundle, and self-intersection</td>
<td>543, 556</td>
</tr>
<tr>
<td>Normal variety</td>
<td>546</td>
</tr>
<tr>
<td>Normalization</td>
<td></td>
</tr>
<tr>
<td>of a curve, of a variety</td>
<td>581, 578</td>
</tr>
<tr>
<td>in a field</td>
<td>12</td>
</tr>
<tr>
<td>Normalized variance</td>
<td>122</td>
</tr>
<tr>
<td>Northcott property</td>
<td>117, 298</td>
</tr>
<tr>
<td>Northcott’s theorem</td>
<td></td>
</tr>
<tr>
<td>for algebraic numbers, for varieties</td>
<td>25, 44</td>
</tr>
<tr>
<td>Number field</td>
<td>4</td>
</tr>
<tr>
<td>Numerical equivalence</td>
<td>563</td>
</tr>
<tr>
<td>Odd elements</td>
<td>258</td>
</tr>
<tr>
<td>Odd line bundle</td>
<td>258, 265</td>
</tr>
<tr>
<td>Open subvariety</td>
<td>522</td>
</tr>
<tr>
<td>Order</td>
<td></td>
</tr>
<tr>
<td>of a meromorphic function</td>
<td>450</td>
</tr>
<tr>
<td>of a number field</td>
<td>189</td>
</tr>
<tr>
<td>of a rational function</td>
<td>545, 547, 552</td>
</tr>
<tr>
<td>of a section</td>
<td>547</td>
</tr>
<tr>
<td>Ostrowski’s theorem</td>
<td>3</td>
</tr>
<tr>
<td>$p$-adic, totally</td>
<td>120</td>
</tr>
<tr>
<td>$p$-adic integers</td>
<td>3</td>
</tr>
<tr>
<td>$p$-adic numbers</td>
<td>3</td>
</tr>
<tr>
<td>Padé approximant</td>
<td>129</td>
</tr>
<tr>
<td>Pell equation</td>
<td>191</td>
</tr>
<tr>
<td>Picard group</td>
<td>529, 548, 549</td>
</tr>
<tr>
<td>Čech cohomology group</td>
<td>566</td>
</tr>
<tr>
<td>isomorphic to first Chow group</td>
<td>555</td>
</tr>
<tr>
<td>of affine space</td>
<td>559</td>
</tr>
<tr>
<td>of multiprojective space</td>
<td>559</td>
</tr>
<tr>
<td>of projective space</td>
<td>559</td>
</tr>
<tr>
<td>Picard scheme</td>
<td>249</td>
</tr>
<tr>
<td>Picard variety</td>
<td>246–252</td>
</tr>
<tr>
<td>Picard’s little theorem, Pisot–Vijayaraghavan number</td>
<td>455, 116</td>
</tr>
<tr>
<td>Place</td>
<td>2</td>
</tr>
<tr>
<td>complex</td>
<td>7</td>
</tr>
<tr>
<td>division of, extension of</td>
<td>2</td>
</tr>
<tr>
<td>finite</td>
<td>605</td>
</tr>
<tr>
<td>infinite</td>
<td>605</td>
</tr>
<tr>
<td>lying over</td>
<td>2</td>
</tr>
<tr>
<td>ramified</td>
<td>592</td>
</tr>
<tr>
<td>real</td>
<td>7</td>
</tr>
<tr>
<td>tamely ramified</td>
<td>594</td>
</tr>
<tr>
<td>totally ramified</td>
<td>596</td>
</tr>
<tr>
<td>unramified</td>
<td>592, 598</td>
</tr>
<tr>
<td>wildly ramified</td>
<td>594</td>
</tr>
<tr>
<td>Plane curve</td>
<td>582</td>
</tr>
<tr>
<td>Poincaré class</td>
<td>249</td>
</tr>
<tr>
<td>is even</td>
<td>266</td>
</tr>
<tr>
<td>Poincaré’s complete reducibility theorem</td>
<td>267</td>
</tr>
<tr>
<td>complex analytically</td>
<td>268</td>
</tr>
<tr>
<td>Poisson’s formula</td>
<td>445</td>
</tr>
<tr>
<td>Poisson–Jensen formula</td>
<td>445</td>
</tr>
<tr>
<td>Pole-divisor</td>
<td>544</td>
</tr>
<tr>
<td>Preperiodic point</td>
<td>288</td>
</tr>
<tr>
<td>Presentation</td>
<td></td>
</tr>
<tr>
<td>degree of</td>
<td>47</td>
</tr>
<tr>
<td>height of</td>
<td>47</td>
</tr>
<tr>
<td>of a Cartier divisor</td>
<td>35</td>
</tr>
<tr>
<td>of a morphism</td>
<td>47</td>
</tr>
<tr>
<td>pull-back of</td>
<td>36</td>
</tr>
<tr>
<td>sum of</td>
<td>36</td>
</tr>
<tr>
<td>Presheaf</td>
<td>520</td>
</tr>
<tr>
<td>associated sheaf</td>
<td>563</td>
</tr>
<tr>
<td>Prevariety</td>
<td>521</td>
</tr>
<tr>
<td>Prime cycle</td>
<td>551</td>
</tr>
<tr>
<td>Prime divisor</td>
<td>544</td>
</tr>
<tr>
<td>Primitive solution</td>
<td>425</td>
</tr>
<tr>
<td>Primitive subgroup</td>
<td>83</td>
</tr>
<tr>
<td>Principal Cartier divisor</td>
<td>549</td>
</tr>
<tr>
<td>Principal Weil divisor</td>
<td>547, 553</td>
</tr>
<tr>
<td>Product formula</td>
<td>9</td>
</tr>
<tr>
<td>for function fields</td>
<td>12</td>
</tr>
<tr>
<td>for number fields</td>
<td>10</td>
</tr>
<tr>
<td>Product of prevarieties</td>
<td>521</td>
</tr>
<tr>
<td>Product theorem</td>
<td>226</td>
</tr>
<tr>
<td>Product variety</td>
<td>522, 524</td>
</tr>
<tr>
<td>of affine varieties</td>
<td>519</td>
</tr>
<tr>
<td>Projection formula</td>
<td>555, 556</td>
</tr>
<tr>
<td>Projective distance</td>
<td>69</td>
</tr>
<tr>
<td>Projective linear subspace</td>
<td>560</td>
</tr>
<tr>
<td>Projective space</td>
<td>531</td>
</tr>
<tr>
<td>Projective space over function field</td>
<td>499</td>
</tr>
<tr>
<td>Projective variety</td>
<td>531</td>
</tr>
<tr>
<td>is complete</td>
<td>536</td>
</tr>
<tr>
<td>Proper intersection product</td>
<td></td>
</tr>
<tr>
<td>with a divisor</td>
<td>556</td>
</tr>
<tr>
<td>Proper morphism</td>
<td>535–536</td>
</tr>
<tr>
<td>Proximity function</td>
<td>447</td>
</tr>
<tr>
<td>in diophantine approximation</td>
<td>481</td>
</tr>
<tr>
<td>in diophantine geometry</td>
<td>483</td>
</tr>
<tr>
<td>of a holomorphic curve</td>
<td>466, 476</td>
</tr>
<tr>
<td>over number field</td>
<td>499</td>
</tr>
<tr>
<td>Pull-back</td>
<td></td>
</tr>
<tr>
<td>of Cartier divisor</td>
<td>551</td>
</tr>
</tbody>
</table>
of coherent sheaf, 568
of divisor class, 556
of $k$-forms, 543
of vector bundle, 529
Push-forward
of cycle, 553
of principal divisor, 554
on Chow groups, 554
Quadratic form, 258
Quadratic function, 258
associated bilinear form, 258
associated linear form, 259
associated quadratic form, 259
Quasi-homogeneous function, 285
Quasicompact, xv, 517
Quotient sheaf, 564
Quotient vector bundle, 529
Radical ideal, 515
Radical of an ideal, 515
Radical of an integer, 402
Ramification defect
of a meromorphic function, 452
Ramification divisor, 473, 599–601
Ramification index, 4
Rank of an abelian group, 297
Rational equivalence
of cycles, 553
of Weil divisors, 547
Rational function, 524
Rational map, 574–577
image of, 574
on curve, 577
Rational point, 522
of an affine variety, 516
Real embedding, 7
Reciprocity law, 309
Reduced divisor, 600
Reduced $K$-algebra, 515
Reduction, 3
Refinement of open covering, 565
Regular curve, 581
Regular function, 515, 521
Regular in codimension 1, 546
Regular local ring, 538
Regular point, 538
Regular variety, 538
is normal, 546
Residue degree, 4
Residue field, 3
of a point, 522
Restriction map for presheaves, 520
Restriction of a vector bundle, 529
Resultant, 588–589
Riemann form, 239
Riemann surface, 585
Riemann theta function, 262
Riemann’s existence theorem, 412, 585
Grothendieck’s generalization, 415
Riemann–Roch theorem
for abelian varieties, 262
for curves, 582
for surfaces, 583
Rigidity line bundle, 302
pull-back, 302
tensor product, 302
Roth’s lemma, 159
generalized, 207
Roth’s theorem, 152–156, 164, 179, 181, 406
with moving targets, 170
Runge’s theorem, 317, 326
$S$-integer, 17, 348
$S$-integral point, 183
$S$-unit, 17, 348
Schanuel’s theorem, 393
Scheme, 522, 555, 567, 573, 598–599
affine, 516
Schmidt–Struppeck–Vaaler theorem, 69
Schwarz’s lemma, 366
Second main theorem, 451
Ahlfors’s proof, 460
Cartan’s generalization, 471
over function field, 505
Cartan–Vojta version, 473, 484
equidimensional case, 475
for projective curve, 474, 476
truncated version, 476
Griffiths’s conjecture, 473
in diophantine approximation, 482
section of vector bundle, 527
Seesaw principle, 247
Segre embedding, 531, 535
Semistable reduction of elliptic curve, 427
Separable degree of morphism, 579
Separable morphism, 581
Serre duality, 571
Sheaf, 520, 528, 563–574
of $k$-forms, 542
on singular varieties, 543
of modules, 566
Index 651

of relative differentials, 543
Short exact sequence of sheaves, 564
Shuffle of type $(d, e)$, 31
Siegel’s lemma, 72
Bombieri–Vaaler version, 74
relative version, 79
Siegel’s theorem, 184, 484
Singular point, 538
Size of approximation class, 164
Skyscraper sheaf, 569
Small point, 296
Small solution, 138
Smooth point, 539
and regularity, 539
Smooth variety, 539–543
Smyth’s theorem, 29, 116
Special divisor, 270
Special set of a variety, 486
Spherical proximity function, 458
Standard affine open subsets, 531
Standard metric, 58
Standard étale morphism, 581
for schemes, 598
Stein factorization, 578
Stereographic projection, 457
Stewart–Tijdeman theorem, 419
Stothers–Mason theorem, 418, 503
Strictly convergent power series, 364
Strong approximation theorem, 11
Subbundle of vector bundle, 526
Subfamily of $\text{Pic}^0(X)$, 248
Subsheaf, 563
Subspace theorem, 177
absolute, 228
affine, 178
basic inequality, 199
Evertse’s lemma, 217
general, 180
primitive solutions, 199
Successive minimum, 76, 611
Superelliptic equation, 145
Support
of a cycle, 551
of Cartier divisor, 550
of coherent sheaf, 571
of Weil divisor, 544
Szpiro conjecture, 431
Tangent bundle, 542
of affine space, 540
Tangent space, 536–540
Tate uniformization
of complex elliptic curve, 312
Tate’s elliptic curve, 314
Tate’s limit argument, 286
Tautological line bundle, 532
Taylor coefficients, 366
Taylor series, 366
Tensor product of vector bundles, 528
Theorem of the cube, 260
complex analytically, 262
for varieties, 261
Theorem of the square, 253
complex analytically, 257
Theta divisor, 272
is ample, 280
Theta function, 261
normalized, 261
trivial, 261
Thue equation, 150, 189
Thue’s theorem, 150
Thue–Mahler equation, 140
Topological covering, 411
finite, 412
Torsion coset, 83, 399
Torsion sheaf, 599
Torus
coset, 82
linear, 82
Totally real algebraic number, 107
Trace, 586
Transition function, 529
Transition matrix, 526, 527
Translation, 235
Transversal intersection, 557
Triangle group, 437
Triangle inequality, 1
ultrametric, 2
Trivial metric, 58
Trivial vector bundle, 526
of rank 1, 528
Trivialization of vector bundle, 526
Truncated counting function
higher, 508
of a holomorphic curve, 475
of a meromorphic function, 455
Unit equation, 125–140
general, 186
Unit of a number field, 17
Universal covering, 412
Unramified field extension, 591–597
<table>
<thead>
<tr>
<th>Term</th>
<th>Page(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>outside $S$, 348</td>
<td></td>
</tr>
<tr>
<td>over $M$, 346</td>
<td></td>
</tr>
<tr>
<td>Unramified morphism, 580</td>
<td></td>
</tr>
<tr>
<td>for schemes, 598–599</td>
<td></td>
</tr>
<tr>
<td>local behaviour, 581, 598</td>
<td></td>
</tr>
<tr>
<td>$\nu$-topology, 55</td>
<td></td>
</tr>
<tr>
<td>Vaaler’s cube-slicing theorem, 618</td>
<td></td>
</tr>
<tr>
<td>Valiron’s result on deficiencies, 455</td>
<td></td>
</tr>
<tr>
<td>Valuation, 346, 593</td>
<td></td>
</tr>
<tr>
<td>discrete, 544</td>
<td></td>
</tr>
<tr>
<td>Valuation ring, 3</td>
<td></td>
</tr>
<tr>
<td>discrete, 544</td>
<td></td>
</tr>
<tr>
<td>Value group, 4, 593</td>
<td></td>
</tr>
<tr>
<td>Variety, 522</td>
<td></td>
</tr>
<tr>
<td>associated complex analytic variety, 584</td>
<td></td>
</tr>
<tr>
<td>associated scheme, 522</td>
<td></td>
</tr>
<tr>
<td>basis of topology, 522</td>
<td></td>
</tr>
<tr>
<td>codimension of a closed subset, 523</td>
<td></td>
</tr>
<tr>
<td>dimension, 523</td>
<td></td>
</tr>
<tr>
<td>equidimensional, 523</td>
<td></td>
</tr>
<tr>
<td>general type, 486</td>
<td></td>
</tr>
<tr>
<td>geometrically irreducible, 522, 524</td>
<td></td>
</tr>
<tr>
<td>geometrically reduced, 522, 524</td>
<td></td>
</tr>
<tr>
<td>of pure dimension, 523</td>
<td></td>
</tr>
<tr>
<td>rationally connected, 237</td>
<td></td>
</tr>
<tr>
<td>Vector bundle, 525–530</td>
<td></td>
</tr>
<tr>
<td>generated by global sections, 530</td>
<td></td>
</tr>
<tr>
<td>on projective variety, 533</td>
<td></td>
</tr>
<tr>
<td>rank, 526</td>
<td></td>
</tr>
<tr>
<td>sheaf of sections, 528, 566</td>
<td></td>
</tr>
<tr>
<td>Vector field, 540, 542</td>
<td></td>
</tr>
<tr>
<td>Very ample line bundle, 533</td>
<td></td>
</tr>
<tr>
<td>on curve, 583</td>
<td></td>
</tr>
<tr>
<td>Vinogradov symbols, xv</td>
<td></td>
</tr>
<tr>
<td>Vojta divisor, 356–359</td>
<td></td>
</tr>
<tr>
<td>with small height, 378</td>
<td></td>
</tr>
<tr>
<td>Vojta’s conjecture, 483</td>
<td></td>
</tr>
<tr>
<td>over function field, 503</td>
<td></td>
</tr>
<tr>
<td>with ramification, 488</td>
<td></td>
</tr>
<tr>
<td>for curves, 494</td>
<td></td>
</tr>
<tr>
<td>Vojta’s height inequality, 483</td>
<td></td>
</tr>
<tr>
<td>Vojta’s theorem, 387</td>
<td></td>
</tr>
<tr>
<td>Waring’s problem, 153, 393</td>
<td></td>
</tr>
<tr>
<td>Weak Mordell–Weil theorem, 341, 349</td>
<td></td>
</tr>
<tr>
<td>for elliptic curves, 329–335</td>
<td></td>
</tr>
<tr>
<td>Weierstrass equation, 242</td>
<td></td>
</tr>
<tr>
<td>global minimal form, 429</td>
<td></td>
</tr>
<tr>
<td>minimal local form, 426</td>
<td></td>
</tr>
<tr>
<td>Weierstrass $\wp$-function, 246, 453</td>
<td></td>
</tr>
<tr>
<td>Weil divisor, 544</td>
<td></td>
</tr>
<tr>
<td>component, 544</td>
<td></td>
</tr>
<tr>
<td>effective, 544</td>
<td></td>
</tr>
<tr>
<td>of invertible meromorphic section, 554</td>
<td></td>
</tr>
<tr>
<td>of rational function, 545, 547, 553</td>
<td></td>
</tr>
<tr>
<td>of section, 547</td>
<td></td>
</tr>
<tr>
<td>Weil height, 42–45</td>
<td></td>
</tr>
<tr>
<td>Weil’s theorem of decomposition, 63</td>
<td></td>
</tr>
<tr>
<td>Weyl sum, 393</td>
<td></td>
</tr>
<tr>
<td>Wieferich pair, 404</td>
<td></td>
</tr>
<tr>
<td>Wronskian, 450, 471</td>
<td></td>
</tr>
<tr>
<td>Wronskian criterion, 160</td>
<td></td>
</tr>
<tr>
<td>Ye’s error term, 460</td>
<td></td>
</tr>
<tr>
<td>Zariski topology, 515</td>
<td></td>
</tr>
<tr>
<td>on projective space, 530</td>
<td></td>
</tr>
<tr>
<td>Zariski’s tangent space, 538</td>
<td></td>
</tr>
<tr>
<td>Zero set, 515, 530</td>
<td></td>
</tr>
<tr>
<td>Zero-divisor, 544</td>
<td></td>
</tr>
<tr>
<td>Zhang’s theorem, 94</td>
<td></td>
</tr>
</tbody>
</table>