AN INTRODUCTION TO UNCERTAINTY IN MEASUREMENT USING THE GUM (GUIDE TO THE EXPRESSION OF UNCERTAINTY IN MEASUREMENT)

Measurement shapes scientific theories, characterises improvements in manufacturing processes and promotes efficient commerce. Inherent in measurement is uncertainty, and students in science and engineering need to identify and quantify uncertainties in the measurements they make. This book introduces measurement and uncertainty to second- and third-year students of science and engineering. Its approach relies on the internationally recognised and recommended guidelines for calculating and expressing uncertainty (known by the acronym GUM). The statistics underpinning the methods are considered and worked examples and exercises are spread throughout the text. Detailed case studies based on typical undergraduate experiments are included to reinforce the principles described in the book. This book is also useful to professionals in industry who are expected to know the contemporary methods in this increasingly important area. Additional online resources are available to support the book at www.cambridge.org/9780521605793/.

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Cover illustration: Motorbike stunt in Sydney Harbour.
Photograph courtesy of Geoff Ambler.

To our families

Contents

And we know today – how little we know. There is never an observation made but a hundred observations are missed in the making of it; there is never a measurement but some impish truth mocks us and gets away from us in the margin of error.

H. G. Wells, *Men Like Gods* (1923)

Exact transcription from *Men Like Gods*, by H. G. Wells,
as published by The Macmillan Company, New York, 1923.

Preface

In writing this book, we address several groups of readers who require an understanding of measurement, and of uncertainty in measurement, in science and technology.

Undergraduates in science, for example, should have texts that set out the concepts and terminology of measurement in a clear and consistent manner. At present, students often encounter texts that are mutually inconsistent in several aspects. For example, some texts use the terms *error* and *uncertainty* interchangeably, whilst others assign them distinctly different meanings. Such inconsistency is liable to confuse students, who are consequently unsure about how to interpret and communicate the results of their measurements.

Until recently, a similar lack of consistency affected those whose primary occupation includes measurement, the evaluation of uncertainty in measurement, instrument and artefact calibration and the maintenance of standards of measurement – that is, professional metrologists. International trade, for example, requires mutual agreement among nations on what uncertainty is, how it is calculated and how it should be communicated; for a global economy to work efficiently, lack of such agreement cannot be tolerated. In the mid 1990s, international bodies, charged with the definition, maintenance and development of technical standards and standards of measurement in a variety of fields, published and disseminated the *Guide to the Expression of Uncertainty in Measurement* – the 'GUM'. These bodies included the Bureau International des Poids et Mesures (BIPM) or International Bureau of Weights and Measures, the International Standardisation Organisation (ISO) and the International Electrotechnical Commission (IEC). The GUM is being adopted worldwide by organisations representing a diversity of disciplines, such as calibration and testing laboratories in the physical and engineering sciences, in chemical and biochemical analytic work and related specialised areas of medical testing, in the certification of reference materials and, at the highest metrological level, in national measurement institutes.

Despite its prominence in all fields of measurement, the GUM is (in 2005) largely unknown amongst university and college academics. One of our goals in writing this book is to introduce the GUM and its essential statistical background to an undergraduate audience. We believe that adopting the methods described in the GUM at undergraduate level will confer improved clarity and consistency on the teaching and learning of errors and uncertainty, and on their expression. As use of the GUM grows in industrial and commercial laboratories, new generations of graduating students will require a working knowledge of its methods and vocabulary as well as of the statistical principles that underpin them. In this book we have attempted to anticipate and address these needs.

We include introductory material in the early chapters, the level of which is consistent with first-year university courses. However, the book as a whole is likely to be of greater benefit to second-year students who have already had some exposure to laboratory work as well as a first-year course in calculus and some basic statistics. When dealing with statistical relationships, we have not attempted the rigour normally found in mathematical statistical texts, but have preferred to introduce them in an intuitively plausible way, often by means of figures and graphs.

We have made some use of Monte Carlo simulation (MCS) in the text. One reason is that the GUM, which advocates as a standard practice the law of propagation of uncertainties involving first-order derivatives of the inputs, nonetheless recognises the need for 'other analytical or numerical methods' (when a complicated relationship exists between a measurand and its inputs). One such method is MCS, and therefore some exposure to MCS is desirable. Another important reason, in an educational context, is that MCS can make a statistical process, summarised by a theoretical equation, 'transparent' to the reader in a way that a standard theoretical approach does not. MCS bears much the same relationship to theoretical statistics as experimental physics does to theoretical physics, and can be a valuable and accessible teaching tool, since all it requires is a personal computer, random-number-generating software and some programming or spreadsheet experience.

As part of the text we have also introduced and described in some detail particular undergraduate experiments. The 'real' data from these experiments are analysed using the methods described in this book. These experiments might be suitable for adaptation to courses that deal with measurement and uncertainty. Anticipating this, we have included some suggestions on how the experiments might be developed or enhanced.

The need for an exposition of the GUM extends beyond the universities. An equally important group towards whom this book is directed consists of professionals who are not necessarily involved in making measurements or assessing uncertainties on a day-to-day basis, but who must nevertheless be familiar with contemporary international guidelines relating to measurement and uncertainty.

Specialist publications that deal with uncertainty and the GUM often assume that the reader is, or will become, a practising metrologist, and are therefore written at an advanced level. We hope that this book, with its combination of general principles and specific examples, will aid those readers who wish to know something of current guidelines, but who are less inclined to consult such publications. We have, however, included in several places some slightly more advanced material; the reason is that there occur, not uncommonly, situations where simple formulas as presented in introductory texts become invalid, and to understand why, this more advanced material is needed.

The calculation and expression of uncertainty constitute only one aspect of measurement. The subject also includes the detection, description, analysis and minimisation of errors. The minimisation of errors, moreover, can be achieved in aesthetically pleasing ways that are often pioneered at national measurement institutes. We have, therefore, included some discussion of these topics. Since in any attempt at accurate measurement there is likely to be a variety of potential sources of error, a broad familiarity with several branches of science is desirable in metrology. The best metrologists, in fact, tend to be scientific 'all-rounders'. We have given some examples of the need for this professional versatility, which contributes to the fascination and challenge of measurement in science and technology.

One of the great rewards of writing a book is the amount learned by the author or authors along the way. 'To teach is to learn' is ancient wisdom. This is certainly true for us, and we acknowledge many people who have helped us clarify our thinking. We are grateful for the assistance we have received in our attempt to bridge the gap between a textbook written by an academic for an academic audience and a specialist text written for practising metrologists by a professional in metrology.

We warmly acknowledge the contribution of our colleagues and peers who have suggested examples, problems and topics for inclusion in the text. We thank, from the National Measurement Institute of Australia (NMIA) in Sydney, Errol Atkinson, Mark Ballico, Robin Bentley, Noel Bignell, Nick Brown, Ilya Budovsky, Henry Chen, Jim Gardner, Åsa Jämting, John Peters, Steve Quigg, Brian Ricketts and Greig Small; from the University of Technology, Sydney (UTS), Nick Armstrong, Sherran Evans, Matthew Foot, Jim Franklin, Suzanne Hogg, Walter Kalceff, Geoff McCredie and Greg Skilbeck; from the Stunt Agency, Jennifer Fenton and, from Cambridge University Press, Simon Capelin and Vince Higgs. We also thank Alan Johnston for advice on cover design, and Avril Wynne for her daring flight. We also gratefully acknowledge the love, support and forbearance of our families throughout the writing of this book.

Les Kirkup and Bob Frenkel, May 2005