

## Contents

<i>Preface</i>	<i>page xi</i>
1 Classical Banach Spaces	1
The Sequence Spaces $\ell_p$ and $c_0$	1
Finite-Dimensional Spaces	2
The $L_p$ Spaces	3
The $C(K)$ Spaces	4
Hilbert Space	6
“Neoclassical” Spaces	7
The Big Questions	7
Notes and Remarks	9
Exercises	9
2 Preliminaries	11
Continuous Linear Operators	11
Finite-Dimensional Spaces	12
Continuous Linear Functionals	13
Adjoints	15
Projections	16
Quotients	17
A Curious Application	20
Notes and Remarks	20
Exercises	20
3 Bases in Banach Spaces	24
Schauder’s Basis for $C[0, 1]$	28
The Haar System	30
Notes and Remarks	32
Exercises	33
4 Bases in Banach Spaces II	34
A Wealth of Basic Sequences	34
Disjointly Supported Sequences in $L_p$ and $\ell_p$	35

viii	<i>Contents</i>	
	Equivalent Bases	38
	Notes and Remarks	41
	Exercises	42
5	Bases in Banach Spaces III	44
	Block Basic Sequences	44
	Subspaces of $\ell_p$ and $c_0$	47
	Complemented Subspaces of $\ell_p$ and $c_0$	49
	Notes and Remarks	51
	Exercises	54
6	Special Properties of $c_0$ , $\ell_1$ , and $\ell_\infty$	55
	True Stories About $\ell_1$	55
	The Secret Life of $\ell_\infty$	60
	Confessions of $c_0$	63
	Notes and Remarks	65
	Exercises	65
7	Bases and Duality	67
	Notes and Remarks	71
	Exercises	72
8	$L_p$ Spaces	73
	Basic Inequalities	73
	Convex Functions and Jensen's Inequality	74
	A Test for Disjointness	77
	Conditional Expectation	78
	Notes and Remarks	82
	Exercises	83
9	$L_p$ Spaces II	85
	The Rademacher Functions	85
	Khinchine's Inequality	87
	The Kadec–Pełczyński Theorem	91
	Notes and Remarks	97
	Exercises	98
10	$L_p$ Spaces III	99
	Unconditional Convergence	99
	Orlicz's Theorem	101
	Notes and Remarks	106
	Exercises	106
11	Convexity	107
	Strict Convexity	108
	Nearest Points	112
	Smoothness	113

	<i>Contents</i>	ix
	Uniform Convexity	114
	Clarkson's Inequalities	117
	An Elementary Proof That $L_p^* = L_q$	119
	Notes and Remarks	122
	Exercises	122
12	$C(K)$ Spaces	124
	The Cantor Set	124
	Completely Regular Spaces	125
	Notes and Remarks	134
	Exercises	134
13	Weak Compactness in $L_1$	136
	Notes and Remarks	141
	Exercises	141
14	The Dunford–Pettis Property	142
	Notes and Remarks	146
	Exercises	147
15	$C(K)$ Spaces II	148
	The Stone–Čech Compactification	148
	Return to $C(K)$	153
	Notes and Remarks	155
	Exercises	155
16	$C(K)$ Spaces III	156
	The Stone–Čech Compactification of a Discrete Space	156
	A Few Facts About $\beta N$	157
	“Topological” Measure Theory	158
	The Dual of $\ell_\infty$	161
	The Riesz Representation Theorem for $C(\beta D)$	162
	Notes and Remarks	165
	Exercises	165
	Appendix: Topology Review	166
	Separation	166
	Locally Compact Hausdorff Spaces	167
	Weak Topologies	169
	Product Spaces	170
	Nets	171
	Notes and Remarks	172
	Exercises	172
	<i>References</i>	173
	<i>Index</i>	181