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Contents

Preface

These are notes for a graduate topics course offered on several occasions to a rather diverse group of doctoral students at Bowling Green State University. An earlier version of these notes was available through my Web pages for some time and, judging from the e-mail I've received, has found its way into the hands of more than a few readers around the world. Offering them in their current form seemed like the natural thing to do.

Although my primary purpose for the course was to train one or two students to begin doing research in Banach space theory, I felt obliged to present the material as a series of compartmentalized topics, at least some of which might appeal to the nonspecialist. I managed to cover enough topics to suit my purposes and, in the end, assembled a reasonable survey of at least the rudimentary tricks of the trade.

As a prerequisite, the students all had a two-semester course in real analysis that included abstract measure theory along with an introduction to functional analysis. While abstract measure theory is only truly needed in the final chapter, elementary facts from functional analysis, such as the Hahn–Banach theorem, the Open Mapping theorem, and so on, are needed throughout. Chapter 2, "Preliminaries," offers a brief summary of several key ideas from functional analysis, but it is far from self-contained. This chapter also features a large set of exercises I used as the basis for additional review, when necessary. A modest background in topology is also helpful but, because many of my students needed review here, I included a brief appendix containing most of the essential facts.

I make no claims of originality here. In fact, the presentation borrows heavily from several well-known sources. I tried my best to document these sources fully in the references and in the brief Notes and Remarks sections at the end of each chapter. You will also see that I've included a few exercises to accompany each chapter. These only scratch the surface, of course. Energetic readers may want to seek out greater challenges through the readings suggested in the Notes and Remarks.

My goal was a quick survey of what I perceive to be the major topics in classical Banach space theory: Basis theory, L_p spaces, $C(K)$ spaces, and a brief introduction to the geometry of Banach spaces. But the emphasis here is on *classical*; most of this material is more than thirty years old and, indeed, a great deal of it is more than fifty years old. Readers interested in contemporary research topics in Banach space theory are sure to be disappointed with this modest introduction and are encouraged to look elsewhere.

Finally, I should point out that the course has proven to be of interest to more students than I had originally imagined. Basis theory, for example, has enjoyed a resurgence in certain modern arenas, and such chestnuts as the so-called gliding hump argument frequently resurface in a variety of contemporary research venues. From this point of view, the course has much to offer students interested in operator theory, frames and wavelets, and even in certain corners of algebra such as lattice theory. More important, at least from my point of view, is that the early history of Banach space theory is loaded with elegant, insightful arguments and clever techniques that are not only worthy of study in their own right but are also deserving of greater publicity. It is in this spirit that I offer these notes.

Neal Carothers
Bowling Green, Ohio
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