## Introduction

## Gabrielle Demange and Myrna Wooders

Diverse activities are conducted within and by organized groups of individuals, including political, economic, and social activities. These activities have increasingly become subjects of intense game-theoretic research. A sample of the subjects treated are trade networks, coauthorship networks, buyer-seller networks with differentiated products, and networks of information sharing. Other subjects are social norms are punctuality, the adoption of new technologies, clubs and the provision of club goods and public goods, and collusive alliances, among corporations, international alliances, and trading agreements. This volume, which has three main parts, is an introduction to game-theoretic treatments of organized groups, with networks, clubs, and coalitions, including some illustrations and applications.

Part One is an introduction to recent studies on network formation with bilateral relationships as its principal focus. Although this promising approach is not yet fully developed and is still primarily theoretical, the literature is already vast. There is a wide literature in sociology and economics that makes the importance of social networks clear. One topic of this literature, for example, is the crucial role of personal contacts in obtaining information about job opportunities. Networks of relationships also underlie the trade and exchange of goods in noncentralized markets, and the provision of mutual insurance in developing countries. The aim of much recent research on networks in economics and game theory is to give strategic foundations to network formation.

Part Two discusses societies that may be partitioned into multiple groups. The situations treated range from ones with social activities, public goods, and competition under increasing returns to scale to formation of nations and secession. The literature on these topics, inspired by seminal works of Charles Tiebout, James Buchanan, and Frank Westhoff is, by itself, already large. This part focuses on collective activities, such as those carried out by firms, clubs, and jurisdictions.

Part Three studies strategic approaches to group formation and cooperation in political and economic contexts. The areas include the formation of parties, design of constitutions, alliances among firms, growth and environmental economics, and informal arrangements for risk sharing and trade. This part stresses the role of protocols (rules governing the coalition formation process), the extent of spillovers

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between coalitions, and the possibilities of enforcing rules governing coalition formation.

At this point, let us make our vocabulary more precise, and clarify how we distinguish between groups, networks, clubs, and coalitions. The term *group* is the most neutral – it does not suggest that the group members engage in any collective or cooperative activity. Coalitions are groups of players whose members decide to cooperate, possibly by establishing binding agreements. The set of coalitions and the payoffs they can achieve constitute the primitives of a cooperative game, as first described by von Neumann and Morgenstern in a transferable utility setting. Networks are described by bilateral links between decision makers (nodes or players). Although links are decided bilaterally, a group of connected decision makers may act collectively as a coalition. The precise structure of the links connecting a coalition may influence the payoffs to its members. A club allows a group of players to engage in some collective activity, such as the provision of a public good subject to congestion. The club members may also engage in other activities (e.g., trade of private goods) with other individuals and other groups or clubs.

Although the parts differ widely in many aspects, a premise of all chapters is that whether groups are small or large, whether they are many or few, and whether they form or threaten to form coalitions, should depend on the context and should be determined endogenously. In line with the distinction just described, Part One focuses on predicting which networks may form, whereas Part Two and Part Three focus on the expected coalition structures (partitions of the players into coalitions) and their properties.

Turning to the predictions of group formation, these presumably depend on the modeling approach. One of the goals of the analysis is to gain a better understanding of this dependence and to identify when and how the "rules of the game" matter. Two approaches underpin much of the research presented. The first approach is based on an explicit game in strategic form, most often a two-stage game. Loosely speaking, the first stage determines the structure of collaboration based on individuals' strategy choices. In the second stage, given the collaboration structure, the distribution of benefits among individuals is determined. The second approach is more closely aligned with core theory, cooperative games and price- (or tax-) taking equilibrium. Core theory takes equilibrium as a situation in which no group of individuals could do better for themselves, using their own resources. From a club theoretic perspective, no group of participants can do better by forming a new club, given the choices of other individuals. These first and second approaches are, however, not entirely distinct (e.g., the core or equal cost sharing may be used in the second stage of a strategic game). Let us illustrate the differences with two examples.

In environmental problems, sovereign countries must decide whether to improve environmental quality – namely to provide an environmental global public good – on a voluntary basis. In practice, countries negotiate on an international agreement that defines emission targets for each signatory and often the way to achieve these targets as well. Applying the first approach, the negotiation protocol is modeled as a twostage game. In the first stage nations decide on whether to join an agreement. Then, in the second stage, nations within the agreement abide by its rules but otherwise all behaviour is noncooperative.

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In the second approach, for instance, individuals with diverse preferences may decide to share some public good and allocate the costs. Or countries may form clubs to invest jointly in a growth-enhancing technology. In equilibrium, no coalition of individuals would prefer to form a new group and allocate costs differently among themselves.

Clearly, the chapters differ in the situations and issues addressed and in the analytical tools used, but all advance our understanding of the impact of groups on social outcomes.

#### Part One: Network Formation, Communication, and Learning

The four chapters in this part acquaint the reader with recent developments in gametheoretic models of network formation. They survey the results that have been obtained so far, ranging from learning to stability and efficiency of networks; provide an overview of several directions in the literature; and present a diverse and informative collection of examples.

"A Survey of Network Formation Models: Stability and Efficiency" by Matthew Jackson introduces the reader to the diversity of situations described, in the recent literature, by networks. Indeed, networks can be many things. The main focus of the chapter is to relate network formation to allocation rules. For example, if any group of connected players must equally share the surplus generated by the group, then individuals have incentives to link to more productive players. The question is whether there are allocation rules that lead to equilibrium networks with desirable properties. The focus is on the compatibility of efficiency with individual incentives to form and sever links.

"Models of Network Formation in Cooperative Games" by Anne van den Nouweland, surveys the first works that address coalition formation through formation of bilateral links. In these works, payoffs to coalitions are derived from a cooperative game and are invariant to the precise structure of links between the coalition members. The chapter starts with the work of Aumann-Myerson, who model network formation through an extensive game in which once a link is established it cannot be broken. The chapter then discusses a one-shot static game, in which any individual can break a link with another individual and any two players, if they agree, can form a link. The pattern of equilibrium networks is studied as a function of the properties of the underlying game (e.g., convexity or whether a coalition can do at least as well for its membership as any partition of the coalition into smaller coalitions) and the solution concept employed.

In "Farsighted Stability in Network Formation," by Frank Page and Samir Kamat the main two issues are the rules governing network formation and what networks are likely to emerge and persist if individuals are farsighted. Farsighted individuals anticipate possible reactions to their link formation activities. The chapter treats directed networks in which a link may go from one node to another but not in the reverse direction. Moreover, links are allowed to vary in intensity. To illustrate some applications, citations are not generally bilateral and, while A may list B among his friends, B does not necessarily list A among his; moreover, the intensities of their affections may differ. Also loops are allowed, and links may be used in multiple

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ways. A rumor for example may go from A to B to C and then back to A. The framework presented accommodates all these situations. Remarkably, farsightedly stable networks exist; some examples demonstrate their properties.

In "Learning in Networks," Sanjeev Goyal discusses the relationships between learning and the structure of interaction among decision makers. In a wide range of situations, individuals make decisions without being fully informed about their consequences, and take advantage of their previous experiences and their observations of the experiences of others. The precise ways in which individuals interact can influence the generation and dissemination of information. The chapter addresses this issue, modeling the interaction structure as a network in which individuals learn from their neighbors. Two sorts of situations are considered. In the first, some network linking decision makers is given. The effects of the form of networks on learning are studied. In the second, the dissemination of information feeds back on the network structure, thus making link formation endogenous. A central application is the adoption of new technologies.

#### Part Two: On Equilibrium Formation of Groups: A Theoretical Assesment

Our focus turns to coalitions and clubs. The typical motivation for coalition formation and clubs is increasing returns to cooperation in some aspects and decreasing returns in others. Such situations abound both in politics and in economics. Classic examples are shared facilities such as swimming pools. The benefits of cost sharing imply increasing returns to larger club membership, but congestion, diversity in individual tastes, or both – some swimmers may like company whereas others may prefer doing laps in solitude – may hamper full exploitation of increasing returns to cost sharing and lead to the splitting of the society into smaller groups. Whether such splitting is optimal depends on the strength of the increasing returns, the mechanism by which the gains to cooperation are divided, and the compromises required by heterogeneous individuals in belonging to one group, club, or coalition. If congestion effects are sufficiently strong, then multiple groups may be required for the achievement of socially optimal outcomes. The chapters in this section treat firm formation, the financing of public projects, local public goods, and clubs.

There are two major premises: free mobility and free entry. Under free mobility, agents are free to join any group, provided they abide by the rules governing group formation. This is a natural economic assumption. For example, it is common to be able to "buy into" clubs by paying a membership fee or into communities by paying the price of a house. A theoretical motivation for free mobility comes from large coalitions in large economies, where it is natural to assume that individuals have nearly negligible influence on other members of a coalition. Free entry describes a situation in which coalitions can freely form. When there are trade-offs between the size of coalitions and diversity, free entry or some price system taking into account all possibilities is required to ensure that a sufficient variety of coalitional activities is on offer or can be provided. All chapters in this section concern both free entry and free mobility but from differing perspectives. A major issue is the distribution

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of costs and benefits from group formation, the tax or admission price systems, or profits and their effects on optimality and stability.

In "Group Formation: The Interaction of Increasing Returns and Preferences Diversity," Gabrielle Demange focuses on situations in which the diversity in individuals, either in their tastes or in their income, is the driving force behind the splitting of the society. An important application is the analysis of oligopoly in a differentiated goods economy. Despite increasing returns to scale, several firms may be active, each one attracting a different set of customers through a different policy. Free entry introduces the threat that a successful entrant will trigger a reallocation of the customers among the firms. In view of the theory of contestable markets, a main issue is whether free entry results in a stable and efficient oligopoly, or instead is "destructive," calling for regulation. After examining this issue, the chapter discusses the crucial features that shape group formation in more general models. Finally group formation is addressed in situations in which individuals may have a negative impact on a group they join. Free mobility places a major constraint on group formation. For example, jurisdictions face such constraints on their redistributive policies, which generate negative externalities on some individuals, typically the wealthy one.

In "Games and Economies with Near Exhaustion of Gains to Scale," Alexander Kovalenkov and Myrna Wooders concentrate on the marketlike properties of situations with many participants, in which almost all gains to collective activities are exhausted, including situations in which the whole group is optimal. The gametheoretic model encompasses clubs, local public goods, and other deviations from the standard exchange economy model. The authors argue that, if there are some small frictions preventing free and completely costless formation of coalitions, then an economy has stable and near optimal group structures – "almost free" entry equilibrium exists and equilibrium outcomes are optimal or nearly optimal. Free mobility of individuals between groups is expressed as price-taking equilibrium in which the commodities are the players themselves. With sufficiently many players, free mobility and free entry lead to the same set of outcomes. Numerous examples are presented to demonstrate the broad applicability of the results. There may be combinations of positive or negative externalities, and there may be many or few large or small groups.

In "Coalitions and Clubs: Tiebout Equilibrium in Large Economies," John Conley and Stefani Smith survey research studying how the worth of players to groups is determined. The crucial determinants of a player's worth are those characteristics of the player that directly influence others (his or her "crowding type"). If these characteristics are observable, such as gender, personality, occupation, and so on, then, remarkably, competitive admission prices to clubs that depend only on the crowding types of players yield optimal outcomes. The price-taking equilibrium concept allows free mobility of individuals between groups and also free entry of new clubs given prices. Two different models are treated: one in which the crowding types of agents are exogenous (such as gender or height) and another in which agents choose their crowding types, perhaps at some cost, such as the cost of an education. Although they may hold in special cases, if all individuals face the same educational

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cost function for instance, equilibria exhibiting population stratification in general are not possible.

In "Secession-Proof Cost Allocations and Stable Group Structures in Models of Horizontal Differentiation," Michel Le Breton and Shlomo Weber focus on the impact of cost-sharing schemes on public decisions and their stability. There may be institutional constraints, constraints on information, or ideals of fairness and equity that impede the use of a competitive cost sharing. Despite these considerations, in practice, a government has to select an explicit scheme to cover the costs of a project. This chapter seeks to deepen our understanding of the importance of the cost allocation scheme in determining whether "secession" can be avoided. The analysis is carried out in a horizontal differentiation model in which individuals have different rankings over elements of policy space (in opposition to vertical differentiation models in which individuals differ only in their incomes).

### Part Three: Groups, Clubs, and Alliances in Political and Economic Environments

This part analyzes group formation and cooperation in various contexts mainly from a strategic perspective. The first two chapters in this part bear on political issues and coalition formation. The first chapter focuses on strategic issues and the second concerns assessment of coalitional power and the relationship to institutions.

In "Political Parties and Coalition Formation," Amrita Dhillon surveys a recent literature that analyzes coalition formation in politics. Coalitions are crucial at different stages in politics: at the preelectoral stage, a coalition of individuals or of candidates forms a party that campaigns for election whereas at the postelectoral stage, a coalition of candidates or of parties may try to form a government. Instead of treating a political party as a unitary actor, recent works take into account the diversity of preferences in political party formation. Accordingly, in that respect, the models here share many features of the chapters in the previous part: forming a party entails costs, in particular the costs of compromise. On the other hand, the expected benefits depend on the stage being modeled and on individual motivations, which are most often classified as office seeking, vote seeking, and policy seeking. Broadly speaking, forming a party would increase the possibilities of satisfying these motivations. Traditional questions about the size and number of parties are revisited: Are parties minimal winning coalitions? Do more parties form under majority than plurality rule (Duverger's hypothesis)? An important insight gained from the analysis is that some traditional results based on a unidimensional set of alternatives do not extend to more general setups.

In "Power in the Design of Constitutional Rules" Mika Widgrén addresses the crucial question of measuring decision power in real-life institutions. A debate on the European Union has recently been sparked between scholars who study cooperative approaches and power index models, on the one hand, and scholars who study noncooperative approaches and spatial voting models on the other. The verdict of the latter group is that power indices exclude important variables (such as institutions and strategies) and are influenced by computational formulas and

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hidden assumptions. The justification of the former group is that power indices deal with institutional design without the need for explicit decision procedures and specific preferences. However, institutional design often has to consider procedures and interinstitutional aspects. Widgren's chapter examines models from both sides of the debate. He argues that the two sides can be seen as extremes of a unified approach. Like spatial voting games, this approach is based on a posterior analysis but, when necessary information on players' preferences is not available, it can be used for a priori analysis, like the power indices of cooperative games. Using EU decision making as an example, the chapter demonstrates what elements of procedural voting the two so far very distinct approaches can accommodate. It also discusses how the unified approach is able to contribute to the analysis both theoretically and empirically.

The following two chapters treat environmental agreements and industrial organization. Both entail spillovers or, in other words, externalities across coalitions, which are excluded in most analyses of cooperative game theory. In the presence of spillovers, the payoff to a coalition depends on the coalitions formed by other players. This has several consequences. First, the basic representation of coalitional gains is a game in partition function form, meaning that the payoffs to a coalition depend on the entire coalition structure and not only on the membership of the coalition. Second, whenever a coalition forms, its members must anticipate how other players organize themselves into coalitions. This is a very important distinctive feature of the formation of groups in Chapters 11 and 12.

In "Group and Network Formation in Industrial Organization: A survey," Francis Bloch describes recent analyses that seek to understand the forms of cooperation that arise among firms or among traders. Three domains are investigated: (i) the formation of collusive agreements, (ii) the formation of cost-reducing alliances, and (iii) the formation of trade networks. The study of collusion focuses on endogenous cartel formation, both in oligopolistic markets and in auctions. Typically a free rider problem arises in a quantity-setting market: each firm prefers other firms to collude to restrict output but is itself better off outside of the cartel. In the analysis of strategic alliances, this feature is no longer true. Assuming that firms cooperate, in reducing production costs for instance, but compete on the market, the alliances (or networks of cooperative links) that emerge are characterized. Finally the formation of trade markets in which strategic traders can choose their trading partners is analyzed, both in strategic market games and in buyer-seller networks.

In "Institution Design for Managing Global Commons: Lessons from Coalition Theory," Carlo Carraro discusses the impact of institutional design on the emergence of cooperative international agreements to manage global commons. The chapter studies how different accession rules, minimum participation rules, and negotiation rules affect a country's decision to sign a treaty to protect a global common (i.e. to join a coalition that provides a global public good). The chapter also analyzes what the outcome of the negotiations would be if treaty design (for example, the minimum participation rule or the negotiation agenda) were endogenized and strategically chosen by the negotiating countries. In this latter case, players not only choose whether to join a coalition, but, in a preceding stage of the game also chose the rules of the coalition game.

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In "Inequality and Growth Clubs," Fernando Jaramillo, Hubert Kempf, and Fabien Moizeau apply coalition theory to macroeconomics. Indeed various institutions shape and organize the functioning of an economy at the macroeconomic level. That the growth process across countries does not converge is now a wellestablished fact but contradicts the standard neoclassical model. The convergence debate and the attempts based on various types of segmentation (through education, for instance) to reconcile theory with empirics is first summarized. Although the link between the dynamics of inequality and growth rates has been emphasized, the difference in aggregate human capital across countries is left unexplained. The second part of the chapter builds up a model, based on coalition theory, to fill this gap. Countries voluntarily form clubs so as to share some resources that are necessary for growth. The formation of endogenous growth clubs therefore depends on the initial distribution of resources across countries. The model provides a microfoundation for the divergence in growth rates (the so-called twin peaks or multipeaks phenomena).

The final two chapters focus on the enforceability problem of cooperative agreements. The notion that binding cooperative agreements can be enforced has underpinned the cooperative approach to coalition formation. For instance, in Chapter 12, once a treaty has been signed, it is assumed that the signatories abide by the rules of the treaty. The following two chapters treat situations in which cooperative agreements are informal and must be self-enforcing.

In "Informal Insurance, Enforcement Constraints, and Group Formation," Garance Genicot and Debraj Ray discuss informal insurance arrangements in developing countries. Although there is considerable evidence of mutual insurance in village communities, the same studies reveal a large departure from perfect insurance. Apart from asymmetry of information and moral hazard, the lack of enforceability could impede widespread risk sharing. The starting point of the chapter is the following observation: if a large group – a village community for instance – could foresee the benefits of mutual risk sharing, smaller groups could do the same and deviate from insurance arrangements created by larger groups. The chapter formulates a recursive definition of group stability and applies it to the insurance problem. In contrast to the predictions of the standard insurance model, stable groups are always bounded in size, and an increase in environmental riskiness can lower the overall provision of insurance.

In "Spontaneous Market Emergence and Social Networks," Marcel Fafchamps investigates how market exchange is partly shaped by social networks and equilibrium dynamics. Markets at early stages of development are characterized by trade based on mutual trust and on the sharing of information among acquaintances. Also in credit, labor, and business markets, courts are seldom used to enforce commercial contracts. Other forms of punishment, such as stigmatization of defectors from an informal arrangement, may be employed. This chapter explains how different sorts of patterns of exchange can take place in a dynamic setting when no formal market institutions exist that exclude or punish cheaters. The role of social networks, which are predetermined by nonmarket forces (such as ethnic, religious, and family affiliation) in providing valuable information on partners, is emphasized.

## PART ONE

# Network Formation, Communication, and Learning

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# A Survey of Network Formation Models: Stability and Efficiency

Matthew O. Jackson

## 1.1. Introduction

The set of economic situations in which network structures play an important role is wide and varied. For instance, personal contacts play critical roles in obtaining information about job opportunities.<sup>1</sup> Such networks of relationships also underlie the trade and exchange of goods in noncentralized markets,<sup>2</sup> the provision of mutual insurance in developing countries,<sup>3</sup> research and development and collusive alliances among corporations,<sup>4</sup> and international alliances and trading agreements<sup>5</sup> to mention just a few examples.

Given both the prevalence of situations in which networks of relationships play a role and their importance in determining the outcome of the interaction, it is essential to have theories about how such network structures matter and how they form. To get a feeling for what kinds of issues arise and why we might be interested, consider a brief example. We know from extensive research in both the sociology and labor economics literature that social connections are the leading source of information about jobs and that ultimately many (and in some professions most) jobs are obtained through personal contacts.<sup>6</sup> The reason we might care

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<sup>&</sup>lt;sup>1</sup> See, for example, Rees (1966), Granovetter (1973, 1974), Boorman (1975), Montgomery (1991), Topa (2001), Arrow and Borzekowski (2000), Calvo-Armengol (2000), Calvo-Armengol and Jackson (2001, 2004), Cahuc and Fontaine (2002), and Ioannides and Datcher Loury (2002).

<sup>&</sup>lt;sup>2</sup> See, for example, Tesfatsion (1997, 1998); Corominas-Bosch (1999); Weisbuch, Kirman, and Herreiner (2000); Charness, Corominas-Bosch, and Frechette (2001); Kranton and Minehart (2001); and Wang and Watts (2002).

<sup>&</sup>lt;sup>3</sup> See Fafchamps and Lund (2000) and De Weerdt (2002).

<sup>&</sup>lt;sup>4</sup> See Bloch (2001), Belleflamme and Bloch (2002), Goyal and Moraga (2001), Goyal and Joshi (2000), and Billard and Bravard (2002).

<sup>&</sup>lt;sup>5</sup> See Goyal and Joshi (1999), Casella and Rauch (2001), and Furusawa and Konishi (2002).

<sup>&</sup>lt;sup>6</sup> The introduction in Montgomery (1991) provides a nice and quick overview of some of the studies on this. Some of the seminal references are Granovetter (1973, 1974), who found that over 50 percent of surveyed residents of a Massachusetts town had obtained their jobs through social contacts, and Rees (1966), who found, in a similar study, that over 60 percent had done so.