MODERN CANONICAL QUANTUM GENERAL RELATIVITY

Modern physics rests on two fundamental building blocks: general relativity and quantum theory. General relativity is a geometric interpretation of gravity, while quantum theory governs the microscopic behaviour of matter. According to Einstein's equations, geometry is curved when and where matter is localized. Therefore, in general relativity, geometry is a dynamical quantity that cannot be prescribed *a priori* but is in interaction with matter. The equations of nature are background independent in this sense; there is no space-time geometry on which matter propagates without backreaction of matter on geometry. Since matter is described by quantum theory, which in turn couples to geometry, we need a quantum theory of gravity. The absence of a viable quantum gravity theory to date is due to the fact that quantum (field) theory as currently formulated assumes that a background geometry is available, thus being inconsistent with the principles of general relativity. In order to construct quantum gravity, one must reformulate quantum theory in a background-independent way. *Modern Canonical Quantum General Relativity* is about one such candidate for a background-independent quantum gravity theory: loop quantum gravity.

This book provides a complete treatise of the canonical quantization of general relativity. The focus is on detailing the conceptual and mathematical framework, describing the physical applications, and summarizing the status of this programme in its most popular incarnation: loop quantum gravity. Mathematical concepts and their relevance to physics are provided within this book, so it is suitable for graduate students and researchers with a basic knowledge of quantum field theory and general relativity.

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Modern Canonical Quantum General Relativity

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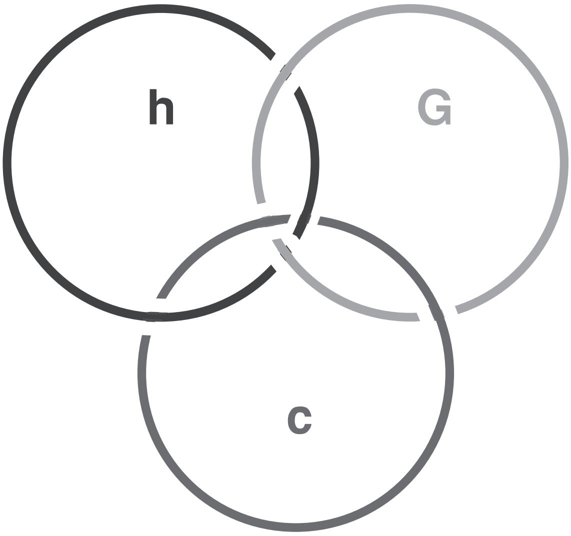

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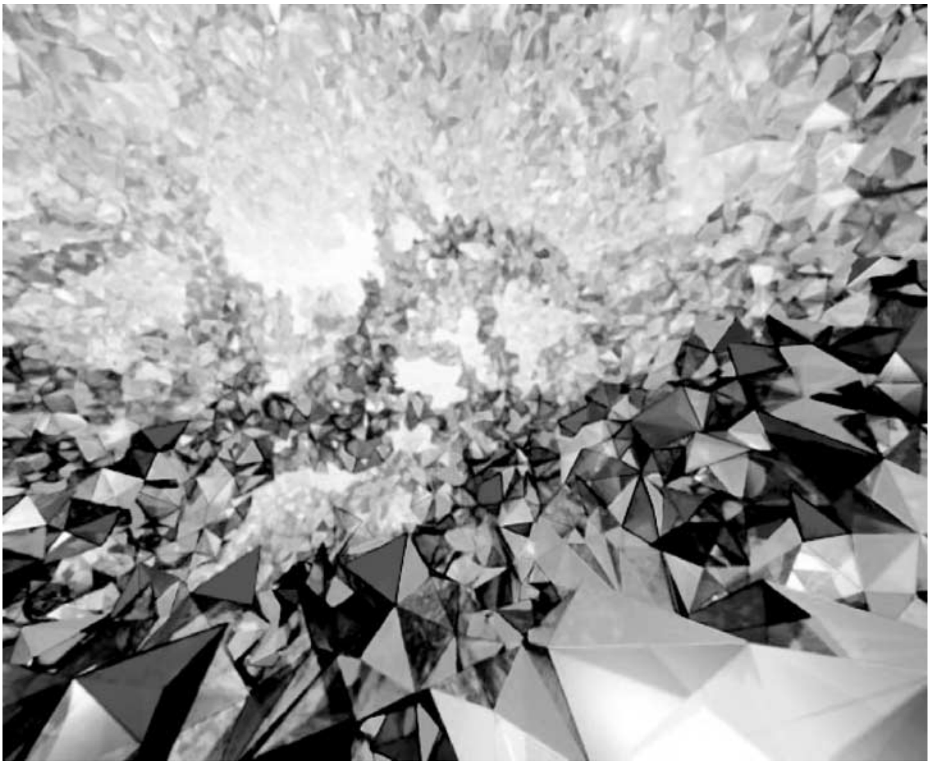

Figure 1 Copyright: Max Planck Institute for Gravitational Physics (Albert Einstein Institute), MildeMarketing Science Communication, Exozet. To see the animation, please visit the URL http://www.einstein-online.info/de/vertiefung/Spinnetzwerke/ index.html.

Quantum spin dynamics
This is a still from an animation which illustrates the dynamical evolution of quantum geometry in Loop Quantum Gravity (LQG), which is a particular incarnation of canonical Quantum General Relativity.

The faces of the tetrahedra are elementary excitations (atoms) of geometry. Each face is coloured, where red and violet respectively means that the face carries low or high area respectively. The colours or areas are quantised in units of the Planck area $\ell_P \approx 10^{-}$ cm . Thus the faces do not have area as they appear to have in the figure, rather one would have to shrink red and stretch violet faces accordingly in order to obtain the correct picture.

The faces are dual to a four-valent graph, that is, each face is punctured by an edge which connects the centres of the tetrahedra with a common face. These edges are 'charged' with half-integral spin-quantum numbers and these numbers are proportional to the quantum area of the faces. The collection of spins and edges defines a spin-network state. The spin quantum numbers are created and annihilated at each Planck time step of $\tau_P \approx 10^{-}$ s in a specific way as dictated by the quantum Einstein equations. Hence the name Quantum Spin Dynamics (QSD) in analogy to Quantum Chromodynamics (QCD).

Spin zero corresponds to no edge or face at all, hence whole tetrahedra are created and annihilated all the time. Therefore, the free space not occupied by tetrahedra does not correspond to empty (matter-free) space but rather to space without geometry, it has zero volume and therefore is a hole in the quantum spacetime. *The tetrahedra are not embedded in space, they are the space.* Matter can only exist where geometry is excited, that is, on the edges (bosons) and vertices (fermions) of the graph. Thus geometry is completely discrete and chaotic at the Planck scale, only on large scales does it appear smooth.

In this book, this fascinating physics is explained in mathematical detail.

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Foreword

Over half a century of collective study has not diminished the fascination of searching for a consistent theory of quantum gravity. I first encountered the subject in 1969 when, as a young researcher, I spent a year in Trieste working with Abdus Salam who, for a while, was very interested in the subject. In those days, the technical approaches adopted for quantum gravity depended very much on the background of the researcher: those, like myself, from a theoretical particle-physics background used perturbative quantum field theory; those whose background was in general relativity tended to use relatively elementary quantum theory, but taking full account of the background general relativity (which the other scheme did not).

The perturbative quantum field theory schemes foundered on intractable ultraviolet divergences and gave way to super-gravity – the super-symmetric extension of standard general relativity. In spite of initial optimism, this approach succumbed to the same disease and was eventually replaced by the far more ambitious superstring theories. Superstring theory is now the dominant quantum gravity programe in terms of the number of personnel involved and the number of published papers, per year, per unit researcher.

However, notwithstanding my early training as a quantum field theorist, I quickly became fascinated by the "canonical quantization", or "quantum geometry," schemes favored by those coming from general relativity. The early attempts for quantizing the metric variables were rather nave, and took on various forms according to how the intrinsic constraints of classical general relativity are handled. In the most popular approach, the constraints are imposed on the state vectors and give rise to the famous Wheeler–DeWitt equation arguably one of the most elegant equations in theoretical physics, and certainly one of the most mathematically ill-defined. Indeed, it was the very intractability of this equation that first intrigued me and prompted me to see what could be done with more sophisticated quantization methods. After much effort it became clear that the answer was "not much."

The enormous difficulty of the canonical quantum gravity scheme eventually caused it to go into something of a decline, until new life was imparted with Ashtekar's discovery of a set of variables in which the constraint equations simplify significantly. This scheme slowly morphed into "loop quantum gravity:" an approach which has, for the first time, allowed real insight into what a non-perturbative quantisation of general relativity might look like. A number of genuine results were obtained, but it became slowly apparent that the old problems with the Wheeler–DeWitt equation were still there in transmuted form, and the critical Hamiltonian constraint was still ill-defined.

It was at this point that Thomas Thiemann – the author of this book – entered the scene. I can still remember the shock I felt when I first read the papers he put onto the web dealing with the Hamiltonian constraint. Suddenly, someone with a top-rate mathematical knowledge had addressed this critical question anew, and with considerable success. Indeed, Thiemann succeeded with loop quantum gravity where I had failed with the old Wheeler–DeWitt equation, and he has gone on since that time to become one of the internationally acknowledged experts in loop quantum gravity.

Thiemann's deep knowledge of mathematics applied to quantum gravity is evident from the first page of this magnificent book. The subject is explored in considerable generality and with real mathematical depth. The author starts from first principles with a general introduction to quantum gravity, and then proceeds to give, what is by far, the most comprehensive, and mathematically precise, exposition of loop quantum gravity that is available in the literature. The reader should be warned though that, when it comes to mathematics, the author takes no hostages, and a good knowledge of functional analysis and differential geometry is assumed from the outset. Still, that is how the subject is these days, and anyone who seriously aspires to work in loop quantum gravity would be advised to gain a good knowledge of this type of mathematics. In that sense, this is a text that is written for advanced graduate students, or professionals who work in the area.

My graduate students not infrequently ask me what I think of the current status of canonical quantum gravity and, in particular, what I think the chances are of ever making proper mathematical sense of the constraints that define the theory. For some years now I have replied to the effect that, if anybody can do it, it will be Thomas Thiemann and, if he cannot do it, then probably nobody will. Anyone who reads right through this major new work will understand why I place so much trust in the author's ability to crack this central problem of quantum gravity.

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Preface

Quantum General Relativity (QGR) or Quantum Gravity for short is, by definition, a Quantum (Field) Theory of Einstein's geometrical interpretation of gravity which he himself called General Relativity (GR). It is a theory which synthesises the two fundamental building blocks of modern physics, that is, (1) the generally relativistic principle of background independence, sometimes called general covariance and (2) the uncertainty principle of quantum mechanics.

The search for a viable QGR theory is almost as old as Quantum Mechanics and GR themselves, however, despite an enormous effort of work by a vast amount of physicists over the past 70 years, we still do not have a credible QGR theory. Since the problem is so hard, QGR is sometimes called the 'holy grail of physics'. Indeed, it is to be expected that the discovery of a QGR theory revolutionises our current understanding of nature in a way as radical as both General Relativity and Quantum Mechanics did.

What we do have today are candidate theories which display some promising features that one intuitively expects from a quantum theory of gravity. They are so far candidates only because for each of them one still has to show, at the end of the construction of the theory, that it reduces to the presently known standard model of matter and classical General Relativity at low energies, which is the minimal test that any QGR theory must pass.

One of these candidates is Loop Quantum Gravity (LQG). LQG is a modern version of the canonical or Hamiltonian approach to Quantum Gravity, originally introduced by Dirac, Bergmann, Komar, Wheeler, DeWitt, Arnowitt, Deser and Misner. It is modern in the sense that the theory is formulated in terms of connections ('gauge potentials') rather than metrics. It is due to this fact that the theory was called Loop Quantum Gravity since theories of connections are naturally described in terms of Wilson loops. This also brings GR much closer to the formulation of the other three forces of nature, each of which is described in terms of connections of a particular Yang–Mills theory for which viable quantum theories exist. Consequently, the connection reformulation has resulted in rapid progress over the past 20 years.

The purpose of this book is to provide a self-contained treatise on canonical – and in particular Loop Quantum Gravity. Although the theory is still under rapid development and the present book therefore is at best a snapshot, the field has now matured enough in order to justify the publication of a new textbook. The literature on LQG now comprises more than a thousand articles scattered over a vast number of journals, reviews, proceedings and conference reports. Structures which were believed to be essential initially turned out to be negligible later on and vice versa, thus making it very hard for the beginner to get an overview of the subject. We hope that this book serves as a 'geodesic' through the literature enabling the reader to move quickly from the basics to the frontiers of current research. By definition, a geodesic cannot touch on all the subjects of the theory and we apologise herewith to our colleagues if we were unable to cover their work in this single volume manuscript. However, guides to further reading and a detailed bibliography try to compensate for this incompleteness. A complete listing of all LQG-related papers, which is periodically being updated, can be found in [1, 2].[1]

Loop Quantum Gravity is an attempt to construct a mathematically rigorous, background-independent, non-perturbative Quantum Field Theory of Lorentzian General Relativity and all known matter in four spacetime dimensions, not more and not less. In particular, no claim is made that LQG is a unified theory of everything predicting, among other things, matter content and dimensionality of the world. Hence, currently there is no restriction on the allowed matter couplings although these might still come in at a later stage when deriving the low energy limit. While the connection formulation works only in four spacetime dimensions and in that sense is a prediction, higher p-form formulations in higher dimensions are conceivable. Matter and geometry are not unified in the sense that they are components of one and the same geometrical object, however, they are unified under the four-dimensional diffeomorphism group which in perturbative approaches is broken. LQG provides a universal framework for how to combine quantum theory and General Relativity for all possible matter and in that sense is robust against the very likely discovery of further substructure of matter between the energy scales of the LHC and the Planck scale which differ by 16 orders of magnitude. This is almost the same number of orders of magnitude as between 1 mm and the length scales that the LHC can resolve, and we found a huge amount of substructure there.

The stress on mathematical rigour is here no luxurious extra baggage but a necessity: in a field where, to date, no experimental input is available, mathematical consistency is the only guiding principle to construct the theory. The strategy is to combine the presently known physical principles and to drive them to their logical frontiers without assuming any extra, unobserved structure such as extra dimensions and extra particles. This deliberately conservative approach has the advantage of either producing a viable theory or of deriving which extra structures are needed in order to produce a successful theory. Indeed, it is conceivable that at some point in the development of the theory a 'quantum leap' is necessary, similar to Heisenberg's discovery that the

[1] See also the URLs http://www.nucleares.unam.mx/corichi/lqgbib.pdf and http://www.matmor.unam.mx/corichi/lqgbib.pdf.

Bohr–Sommerfeld quantisation rules can be interpreted in terms of operators. The requirement to preserve background independence has already led to new, fascinating mathematical structures. For instance, a fundamental discreteness of spacetime at the Planck scale of 10^{-33} cm seems to be a prediction of the theory which is a first substantial evidence for a theory in which the gravitational field acts as a natural cutoff of the usual ultraviolet divergences of QFT.

Accordingly, the present text tries to be mathematically precise. We will develop in depth the conceptual and mathematical framework underlying LQG, stating exact definitions and theorems including complete proofs. Many of the calculations or arguments used during the proofs cannot be found anywhere in the literature detailed as they are displayed here. We have supplied a vast amount of mathematical background information so that the book can be read by readers with only basic prior knowledge of GR and QFT without having to consult too much additional literature. We have made an effort to stress the basic principles of canonical QGR, of which LQG is just one possible incarnation based on a specific choice of variables.

For readers who want to get acquainted first with the physical ideas and conceptual aspects of LQG before going into mathematical details, we strongly recommend the book by Carlo Rovelli [3]. The two books are complementary in the sense that they can be regarded almost as Volume I (‘Introduction and Conceptual Framework’) and Volume II (‘Mathematical Framework and Applications’) of a general presentation of QGR in general and LQG in particular. While this book also develops a tight conceptual framework, the book by Carlo Rovelli is much broader in that aspect. Recent review articles can be found in [4–14]. The status of the theory a decade ago is summarised in the books [15–17].

The present text is aimed at all readers who want to find out in detail how LQG works, conceptually and technically, enabling them to quickly develop their own research on the subject. For instance, the author taught most of the material of this book in a two-semester course to German students in physics and mathematics who were in their sixth semester of diploma studies or higher. After that they could complete diploma theses or PhD theses on the subject without much further guidance. Unfortunately, due to reasons of space, exercises and their solutions had to be abandoned from the book, see [12] for a selection. We hope to incorporate them in an extended future edition. As we have pointed out, LQG is far from being a completed theory and aspects of LQG which are at the frontier of current research and whose details are still under construction will be critically discussed. This will help readers to get an impression of what important open problems there are and hopefully encourage them to address these in their own research.

The numerous suggestions for improvements to the previous online version of this book (http://www.arxiv.org/list/gr-qc/0110034) by countless colleagues is gratefully acknowledged, in particular those by Jürgen Ehlers, Christian Fleischhack, Stefan Hofmann, Chris Isham, Jurek Lewandowski, Robert Oeckl, Hendryk Pfeiffer, Carlo Rovelli, Hanno Sahlmann and Oliver Winkler. Special

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Посвящаю своей жене Татьяне.
Ebenso gewidmet meinen Söhnen Andreas und Maximilian.

Thomas Thiemann
Berlin, Toronto 2001–2007

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Notation and conventions

Symbol	**Meaning**
$G = 6.67 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$	Newton's constant
$\kappa = 16\pi G/c^3$	gravitational coupling constant
$\ell_p = \sqrt{\hbar\kappa} \approx 10^{-33}\,\mathrm{cm}$	Planck length
$m_p = \sqrt{\hbar/\kappa}/c \approx 10^{19}\,\mathrm{GeV}/c^2$	Planck mass
Q	Yang–Mills coupling constant
$M,\ \dim(M) = D+1$	spacetime manifold
$\sigma,\ \dim(\sigma) = D$	abstract spatial manifold
Σ	spatial manifold embedded into M
G	compact gauge group
Lie(G)	Lie algebra
$N-1$	rank of gauge group
$\mu,\nu,\rho,.. = 0,1,\ldots,D$	tensorial spacetime indices
$a,b,c,.. = 1,\ldots,D$	tensorial spatial indices
$\epsilon_{a_1..a_D}$	Levi–Civita totally skew tensor pseudo density of weight -1
$g_{\mu\nu}$	spacetime metric tensor
q_{ab}	spatial (intrinsic) metric tensor of σ
K_{ab}	extrinsic curvature of σ
R	curvature tensor
$\underline{h}$	group elements for general G
$\underline{h}_{mn},\ m,n,o,.. = 1,\ldots,N$	matrix elements for general G
$I,J,K,.. = 1,2,\ldots,\dim(\mathrm{G})$	Lie algebra indices for general G
$\underline{\tau}_I$	Lie algebra generators for general G
k_{IJ}	$= -\mathrm{tr}(\underline{\tau}_I\underline{\tau}_J)/N := \delta_{IJ}$: Cartan–Killing metric for G
$[\underline{\tau}_I,\underline{\tau}_J] = 2f_{IJ}{}^K\underline{\tau}_K$	structure constants for G
$\underline{\pi}(\underline{h})$	(irreducible) representations for general G or algebra
h	group elements for SU(2)
$h_{AB},\ A,B,C,.. = 1,2$	matrix elements for SU(2)
$i,j,k,.. = 1,2,3$	Lie algebra indices for SU(2)
τ_i	Lie algebra generators for SU(2)

$k_{ij} = \delta_{ij}$	Cartan–Killing metric for SU(2)
$f_{ij}{}^{k} = \epsilon_{ijk}$	structure constants for SU(2)
$\pi_j(h)$	(irreducible) representations for SU(2) with spin j
$\underline{A}$	connection on G-bundle over σ
$\underline{A}_a^I$	pull-back of $\underline{A}$ to σ by local section
$\iota_A,\ o_A;\ A = 1, 2$	$\o_A \iota^A := \epsilon^{AB}{}_A \iota_B = 1$: spinor dyad
$\bar{\iota}_{A'},\ \bar{o}_{A'};\ A' = 1, 2$	primed (complex coinjugate) spinor dyad
g	gauge transformation or element of complexification of G
P	principal G-bundle
A	connection on SU(2)-bundle over σ
A_a^i	pull-back of A to σ by local section
$*\underline{E}$	pseudo-$(D-1)$-form in vector bundle associated to G-bundle under adjoint representation
$*\underline{E}^I_{a_1..,a_{D-1}}$	$:= k^{IJ}\epsilon_{a_1..,a_D}\underline{E}^{a_D}_J$: pull-back of $*\underline{E}$ to σ by local section
$*E$	pseudo-$(D-1)$-form in vector bundle associated to SU(2)-bundle under adjoint representation
$*E^i_{a_1..,a_{D-1}}$	$:= k^{ij}\epsilon_{a_1..,a_D}E^{a_D}_j$: pull-back of $*E$ to σ by local section
E^a_j	$:= \epsilon^{a_1..a_{D-1}}(*E)^k_{a_1..a_{D-1}}k_{jk}/((D-1)!)$: 'electric fields'
e	one-form co-vector bundle associated to the SU(2)-bundle under the defining representation (D-bein)
e_a^i	pull-back of e to σ by local section
Γ_a^i	pull-back by local section of SU(2) spin connection over σ
R, X	right-invariant vector field on G
L	Left-invariant vector field on G
$Y = iX$	momentum vector field
$\mathcal{M}$	phase space
$\mathcal{E}$	Banach manifold or space of smooth electric fields
$T_{(a_1..a_n)}$	$:= \frac{1}{n!}\sum_{\iota\in S_n} T_{a_{\iota(1)}..a_{\iota(n)}}$: symmetrisation of indices
$T_{[a_1..a_n]}$	$:= \frac{1}{n!}\sum_{\iota\in S_n} \mathrm{sgn}(\iota)\, T_{a_{\iota(1)}..a_{\iota(n)}}$: antisymmetrisation of indices
$\mathcal{A}$	space of smooth connections
$\mathcal{G}$	space of smooth gauge transformations

$\overline{\mathcal{A}}$	space of distributional connections
$\mathcal{A}^{\mathbb{C}}$	space of smooth complex connections
$\mathcal{G}^{\mathbb{C}}$	space of smooth complex gauge transformations
$\overline{\mathcal{G}}$	space of distributional gauge transformations
$\mathcal{A}/\mathcal{G}$	space of smooth connections modulo smooth gauge transformations
$\overline{\mathcal{A}}/\overline{\mathcal{G}}$	space of distributional connections modulo distributional gauge transformations
$\overline{\mathcal{A}/\mathcal{G}}$	space of distributional gauge equivalence classes of connections
$\overline{\mathcal{A}}^{\mathbb{C}}$	space of distributional complex connections
$\overline{\mathcal{A}/\mathcal{G}}^{\mathbb{C}}$	space of distributional complex gauge equivalence classes of connections
$\mathcal{C}$	set of semianalytic curves or classical configuration space
$\overline{\mathcal{C}}$	quantum configuration space
$\mathcal{P}$	set (groupoid) of semianalytic paths or set of punctures
$\mathcal{Q}$	set (group) of semianalytic closed and basepointed paths
$\mathcal{L}$	set of tame subgroupoids of $\mathcal{P}$ or general label set
$\mathcal{S}$	set of tame subgroups of $\mathcal{Q}$ (hoop group) or set of spin-network labels
l	subgroupoid
s	spin-net= spin-network label
$[s]$	(singular) knot-net= diffeomorphism equivalence class of s
Γ_0^ω	set of semianalytic, compactly supported graphs
Γ_σ^ω	set of semianalytic, countably infinite graphs
$\mathrm{Diff}(\sigma)$	group of smooth diffeomorphisms of σ
$\mathrm{Diff}^\omega_{\mathrm{sa}}(\sigma)$	group of semianalytic diffeomorphisms of σ
$\mathrm{Diff}^\omega_{\mathrm{sa},0}(\sigma)$	group of semianalytic diffeomorphisms of σ connected to the identity
$\mathrm{Diff}^\omega_0(\sigma)$	group of analytic diffeomorphisms of σ connected to the identity
$\mathrm{Diff}^\omega(\sigma)$	group of analytic diffeomorphisms of σ
φ	(semi-)analytic diffeomorphism
c	semianalytic curve
p	semianalytic path
e	entire semianalytic path (edge)
α	entire semianalytic closed path (hoop) or algebra automorphism

γ	semianalytic graph
v	vertex of a graph
$E(\gamma)$	set of edges of γ
$V(\gamma)$	set of vertexes of γ
$h_p(A) = A(p)$	holonomy of A along p
$\prec$	abstract partial order
Ω	vector state or symplectic structure or curvature two-form
F	pull-back to σ of 2Ω by a local section
ω	general state on *algebra
$\mathfrak{X},\ X,\ Y$	measure space or topological space
$L(X,Y),\ L(X)$	linear (un)bounded operators between X, Y or on X
$B(X,Y),\ B(X)$	bounded operators between X, Y or on X
$K(X)$	compact operators on X
$B_1(X)$	trace class operators on X
$B_2(X)$	Hilbert–Schmidt operators on X
$\mathcal{B}$	σ-algebra
$\mu,\ \nu,\ \rho$	measure
$\mathcal{H}$	general Hilbert space
Cyl	space of cylindrical functions
$\mathcal{D}$	dense subspace of $\mathcal{H}$ equipped with a stronger topology
$\mathcal{D}'$	topological dual of $\mathcal{D}$
$\mathcal{D}^*$	algebraic dual of $\mathcal{D}$
$\mathcal{H}^0 = L_2(\overline{\mathcal{A}}, d\mu_0)$	uniform measure L_2 space
$\mathcal{H}^{\otimes}$	infinite tensor product extension of $\mathcal{H}^0$
Cyl_l	restriction of Cyl to functions cylindrical over l
$[.], (.)$	equivalence classes
$\mathfrak{A}, \mathfrak{B}$	abstract (*-)algebra or C^*-algebra
$\Delta(\mathfrak{A})$	spectrum on Abelian C^*-algebra
χ	character (maximal ideal) of unital Banach algebra or group or characteristic function of a set
$\mathfrak{I},\ \mathfrak{J}$	ideal in abstract algebra
$\mathfrak{P}$	classical Poisson*-algebra
$\mathfrak{G}$	automorphism group (of principal fibre bundle)
$\mathfrak{D}$	Dirac or hypersurface deformation algebra
$\mathfrak{M}$	Master Constraint algebra
M	Master Constraint