Cambridge University Press 978-0-521-84258-7 - The Calculus of Retirement Income: Financial Models for Pension Annuities and Life Insurance Moshe A. Milevsky Table of Contents <u>More information</u>

Contents

List of Figures and Tables			page x
		I MODELS OF ACTUARIAL FINANCE	
1	Intro	duction and Motivation	3
	1.1	The Drunk Gambler Problem	3
	1.2	The Demographic Picture	5
	1.3	The Ideal Audience	9
	1.4	Learning Objectives	10
	1.5	Acknowledgments	12
	1.6	Appendix: Drunk Gambler Solution	14
2	Modeling the Human Life Cycle		17
	2.1	The Next Sixty Years of Your Life	17
	2.2	Future Value of Savings	18
	2.3	Present Value of Consumption	20
	2.4	Exchange Rate between Savings and Consumption	22
	2.5	A Neutral Replacement Rate	26
	2.6	Discounted Value of a Life-Cycle Plan	27
	2.7	Real vs. Nominal Planning with Inflation	28
	2.8	Changing Investment Rates over Time	30
	2.9	Further Reading	32
	2.10	Problems	33
3	Models of Human Mortality		34
	3.1	Mortality Tables and Rates	34
	3.2	Conditional Probability of Survival	35
	3.3	Remaining Lifetime Random Variable	37
	3.4	Instantaneous Force of Mortality	38
	3.5	The ODE Relationship	39
	3.6	Moments in Your Life	41

v

vi

Cambridge University Press
978-0-521-84258-7 - The Calculus of Retirement Income: Financial Models for Pension
Annuities and Life Insurance
Moshe A. Milevsky
Table of Contents
More information

Contents

	3.7	Median vs. Expected Remaining Lifetime	44
	3.8	Exponential Law of Mortality	45
		Gompertz-Makeham Law of Mortality	46
		Fitting Discrete Tables to Continuous Laws	49
		General Hazard Rates	51
		Modeling Joint Lifetimes	53
		Period vs. Cohort Tables	55
		Further Reading	59
		Notation	60
		Problems	60
		Technical Note: Incomplete Gamma Function in Excel	61
	3.18	Appendix: Normal Distribution and Calculus Refresher	62
4	Valu	ation Models of Deterministic Interest	64
	4.1	Continuously Compounded Interest Rates?	64
	4.2	Discount Factors	66
	4.3	How Accurate Is the Rule of 72?	67
	4.4	Zero Bonds and Coupon Bonds	68
	4.5	Arbitrage: Linking Value and Market Price	70
	4.6	Term Structure of Interest Rates	72
	4.7	Bonds: Nonflat Term Structure	73
		Bonds: Nonconstant Coupons	74
		Taylor's Approximation	75
		Explicit Values for Duration and Convexity	76
		Numerical Examples of Duration and Convexity	78
		Another Look at Duration and Convexity	80
		Further Reading	81
		Notation	82
	4.15	Problems	82
5	Mod	els of Risky Financial Investments	83
	5.1	Recent Stock Market History	83
	5.2	Arithmetic Average Return versus Geometric Average	
		Return	86
	5.3	A Long-Term Model for Risk	88
	5.4	Introducing Brownian Motion	91
	5.5	Index Averages and Index Medians	97
	5.6	The Probability of Regret	98
	5.7	Focusing on the Rate of Change	100
	5.8	How to Simulate a Diffusion Process	101
	5.9	Asset Allocation and Portfolio Construction	102
		Space–Time Diversification	104
		Further Reading	107
		Notation	108
	5.13	Problems	108

Cambridge University Press
978-0-521-84258-7 - The Calculus of Retirement Income: Financial Models for Pension
Annuities and Life Insurance
Moshe A. Milevsky
Table of Contents
More information

	Contents	vii
6	Models of Pension Life Annuities	110
	6.1 Motivation and Agenda	110
	6.2 Market Prices of Pension Annuities	110
	6.3 Valuation of Pension Annuities: General	110
	6.4 Valuation of Pension Annuities: Exponential	115
	6.5 The Wrong Way to Value Pension Annuities	115
	6.6 Valuation of Pension Annuities: Gompertz–Makeham	116
	6.7 How Is the Annuity's Income Taxed?	119
	6.8 Deferred Annuities: Variation on a Theme	121
	6.9 Period Certain versus Term Certain	123
	6.10 Valuation of Joint and Survivor Pension Annuities	125
	6.11 Duration of a Pension Annuity	128
	6.12 Variable vs. Fixed Pension Annuities	130
	6.13 Further Reading	134
	6.14 Notation	136
	6.15 Problems	136
7	Models of Life Insurance	
	7.1 A Free (Last) Supper?	138
	7.2 Market Prices of Life Insurance	138
	7.3 The Impact of Health Status	139
	7.4 How Much Life Insurance Do You Need?	140
	7.5 Other Kinds of Life Insurance	142
	7.6 Value of Life Insurance: Net Single Premium	143
	7.7 Valuing Life Insurance Using Pension Annuities	145
	7.8 Arbitrage Relationship	147
	7.9 Tax Arbitrage Relationship	148
	7.10 Value of Life Insurance: Exponential Mortality	149
	7.11 Value of Life Insurance: GoMa Mortality	149
	7.12 Life Insurance Paid by Installments	150
	7.13 NSP: Delayed and Term Insurance	150
	7.14 Variations on Life Insurance	151
	7.15 What If You Stop Paying Premiums?	154
	7.16 Duration of Life Insurance	157
	7.17 Following a Group of Policies	159
	7.18 The Next Generation: Universal Life Insurance	160
	7.19 Further Reading	162
	7.20 Notation	162
	7.21 Problems	162
8	Models of DB vs. DC Pensions	164
	8.1 A Choice of Pension Plans	164
	8.2 The Core of Defined Contribution Pensions8.3 The Core of Defined Benefit Pensions	165 169
	o.5 The Core of Defined Defield Pensions	109

Cambridge University Press
978-0-521-84258-7 - The Calculus of Retirement Income: Financial Models for Pension
Annuities and Life Insurance
Moshe A. Milevsky
Table of Contents
More information

viii	Contents	
	 8.4 What Is the Value of a DB Pension Promise? 8.5 Pension Funding and Accounting 8.6 Further Reading 8.7 Notation 8.8 Problems 	172 176 180 181 182
	II WEALTH MANAGEMENT: APPLICATIONS AND IMPLICATIONS	
9	 Sustainable Spending at Retirement 9.1 Living in Retirement 9.2 Stochastic Present Value 9.3 Analytic Formula: Sustainable Retirement Income 9.4 The Main Result: Exponential Reciprocal Gamma 9.5 Case Study and Numerical Examples 9.6 Increased Sustainable Spending <i>without</i> More Risk? 9.7 Conclusion 9.8 Further Reading 9.9 Problems 9.10 Appendix: Derivation of the Formula 	 185 185 187 190 192 193 202 206 208 208 209
10	 Longevity Insurance Revisited 10.1 To Annuitize or Not To Annuitize? 10.2 Five 95-Year-Olds Playing Bridge 10.3 The Algebra of Fixed and Variable Tontines 10.4 Asset Allocation with Tontines 10.5 A First Look at Self-Annuitization 10.6 The Implied Longevity Yield 10.7 Advanced-Life Delayed Annuities 10.8 Who Incurs Mortality Risk and Investment Rate Risk? 10.9 Further Reading 10.10 Notation 10.11 Problems 	 215 215 216 218 220 225 226 234 241 244 245 245
11	Options within Variable Annuities 11.1 To Live and Die in VA 11.2 The Value of Paying by Installments 11.3 A Simple Guaranteed Minimum Accumulation Benefit 11.4 The Guaranteed Minimum Death Benefit 11.5 Special Case: Exponential Mortality 11.6 The Guaranteed Minimum Withdrawal Benefit 11.7 Further Reading 11.8 Notation	249 249 252 257 258 259 262 268 268 269

Cambridge University Press	
978-0-521-84258-7 - The Calculus of Retirement Income: Financial Models for Pension	n
Annuities and Life Insurance	
Moshe A. Milevsky	
Table of Contents	
More information	

Contents	ix
12 The Utility of Annuitization	270
12.1 What Is the Protection Worth?	270
12.2 Models of Utility, Value, and Price	271
12.3 The Utility Function and Insurance	272
12.4 Utility of Consumption and Lifetime Uncertainty	274
12.5 Utility and Annuity Asset Allocation	278
12.6 The Optimal Timing of Annuitization	281
12.7 The Real Option to Defer Annuitization	282
12.8 Advanced RODA Model	287
12.9 Subjective vs. Objective Mortality	289
12.10 Variable vs. Fixed Payout Annuities	290
12.11 Further Reading	291
12.12 Notation	292
13 Final Words	293
14 Appendix	295
Bibliography	
Index	