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978-0-521-84201-3 - Elasticity with Mathematica®: An Introduction to Continuum Mechanics and Linear Elasticity
Andrei Constantinescu and Alexander Korsunsky
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Elasticity with MATHEMATICA®

This book gives an introduction to the key ideas and principles in the theory of elasticity with the help of symbolic computation. Differential and integral operators on vector and tensor fields of displacements, strains, and stresses are considered on a consistent and rigorous basis with respect to curvilinear orthogonal coordinate systems. As a consequence, vector and tensor objects can be manipulated readily, and fundamental concepts can be illustrated and problems solved with ease. The method is illustrated using a variety of plane and three-dimensional elastic problems. General theorems, fundamental solutions, displacements, and stress potentials are presented and discussed. The Rayleigh-Ritz method for obtaining approximate solutions is introduced for elastostatic and spectral analysis problems. The book contains more than 60 exercises and solutions in the form of MATHEMATICA notebooks that accompany every chapter. Once the reader learns and masters the techniques, they can be applied to a large range of practical and fundamental problems.

Andrei Constantinescu is currently Directeur de Recherche at CNRS: the French National Center for Scientific Research in the Laboratoire de Mecanique des Solides, and Associated Professor at École Polytechnique, Palaiseau, near Paris. He teaches courses on continuum mechanics, elasticity, fatigue, and inverse problems at engineering schools from the ParisTech Consortium. His research is in applied mechanics and covers areas ranging from inverse problems and the identification of defects and constitutive laws to fatigue and lifetime prediction of structures. The results have applied through collaboration and consulting for companies such as the car manufacturer Peugeot-Citroen, energy providers Électricité de France and Gaz de France, and the aeroengine manufacturer MTU.

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AN INTRODUCTION TO CONTINUUM MECHANICS AND LINEAR ELASTICITY

Andrei Constantinescu

CNRS and Ecole Polytechnique

Alexander Korsunsky

Trinity College, University of Oxford

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