POLITICAL GAME THEORY

*Political Game Theory* is a self-contained introduction to game theory and its applications to political science. The book presents choice theory, social choice theory, static and dynamic games of complete information, static and dynamic games of incomplete information, repeated games, bargaining theory, mechanism design, and a mathematical appendix covering logic, real analysis, calculus, and probability theory. The methods employed have many applications in various subdisciplines including comparative politics, international relations, and American politics. *Political Game Theory* is tailored to students without extensive backgrounds in mathematics and traditional economics; however, many special sections present technical material appropriate for more advanced students. A large number of exercises are also provided for practice with the skills and techniques discussed.

Nolan McCarty is Associate Dean and Professor of Politics and Public Affairs at the Woodrow Wilson School at Princeton University. His recent publications include *Polarized America: The Dance of Ideology and Unequal Riches* (2006, with Keith Poole and Howard Rosenthal) and *The Realignment of National Politics and the Income Distribution* (1997, with Keith Poole and Howard Rosenthal), as well as many articles in periodicals such as the *American Political Science Review* and the *American Journal of Political Science*.

Adam Meirowitz is Associate Professor of Politics and Jonathan Dicken- son Bicentennial Preceptor at Princeton University. He has published in periodicals such as the *American Political Science Review*, the *American Journal of Political Science, Games and Economic Behavior*, and *Social Choice and Welfare*.
ANALYTICAL METHODS FOR SOCIAL RESEARCH

*Analytical Methods for Social Research* presents texts on empirical and formal methods for the social sciences. Volumes in the series address both the theoretical underpinnings of analytical techniques and their application in social research. Some series volumes are broad in scope, cutting across a number of disciplines. Others focus mainly on methodological applications within specific fields such as political science, sociology, demography, and public health. The series serves a mix of students and researchers in the social sciences and statistics.

**Series Editors:**
R. Michael Alvarez, *California Institute of Technology*
Nathaniel L. Beck, *New York University*
Lawrence L. Wu, *New York University*

**Other Titles in the Series:**
*Event History Modeling: A Guide for Social Scientists*, by Janet M. Box-Steffensmeier and Bradford S. Jones
*Ecological Inference: New Methodological Strategies*, edited by Gary King, Ori Rosen, and Martin A. Tanner
*Spatial Models of Parliamentary Voting*, by Keith T. Poole
*Essential Mathematics for Political and Social Research*, by Jeff Gill
*Data Analysis Using Regression and Multilevel/Hierarchical Models*, by Andrew Gelman and Jennifer Hill
Political Game Theory

AN INTRODUCTION

NOLAN McCARTY
Princeton University

ADAM MEIROWITZ
Princeton University
To Moms and Dads, Liz, Janis, Lachlan, and Delaney
Contents

Acknowledgments page xiii

1 Introduction ............................. 1
   1. Organization of the Book 3

2 The Theory of Choice .......................... 6
   1. Finite Sets of Actions and Outcomes 7
   2. Continuous Choice Spaces* 11
   3. Utility Theory 18
   4. Utility Representations on Continuous Choice Spaces* 20
   5. Spatial Preferences 21
   6. Exercises 25

3 Choice Under Uncertainty ...................... 27
   1. The Finite Case 27
   2. Risk Preferences 38
   3. Learning 46
   4. Critiques of Expected Utility Theory 51
   5. Time Preferences 57
   6. Exercises 62

4 Social Choice Theory .......................... 66
   1. The Open Search 66
   2. Preference Aggregation Rules 68
   3. Collective Choice 76
   4. Manipulation of Choice Functions 82
   5. Exercises 85

5 Games in the Normal Form ...................... 87
   1. The Normal Form 89
## Contents

2. Solutions to Normal Form Games .... 93
3. Application: The Hotelling Model of Political Competition .... 101
4. Existence of Nash Equilibria .... 107
5. Dominance and Mixed Strategies .... 113
6. Calculating Nash Equilibria .... 115
7. Application: Interest Group Contributions .... 117
8. Application: International Externalities .... 119
9. Computing Equilibria with Constrained Optimization .... 121
10. Proving the Existence of Nash Equilibria** .... 123
11. Comparative Statics .... 126
12. Refining Nash Equilibria .... 138
14. Exercises .... 145

6 Bayesian Games in the Normal Form ............. 150
1. Formal Definitions .... 152
2. Application: Trade Restrictions .... 154
3. Application: Jury Voting .... 156
4. Application: Jury Voting with a Continuum of Signals .... 159
5. Application: Public Goods and Incomplete Information .... 161
6. Application: Uncertainty About Candidate Preferences .... 164
7. Application: Campaigns, Contests, and Auctions .... 166
8. Existence of Bayesian Nash Equilibria .... 168
9. Exercises .... 169

7 Extensive Form Games ....................... 171
1. Backward Induction .... 175
2. Dynamic Games of Complete but Imperfect Information .... 177
3. The Single-Deviation Principle .... 184
4. A Digression on Subgame Perfection and Perfect Equilibria .... 185
5. Application: Agenda Control .... 186
6. Application: A Model of Power Transitions .... 192
7. Application: A Model of Transitions to Democracy .... 193
8. Application: A Model of Coalition Formation .... 197
9. Exercises .... 201

8 Dynamic Games of Incomplete Information ........ 204
1. Perfect Bayesian Equilibria .... 208
2. Signaling Games .... 214
3. Application: Entry Deterrence in Elections .... 219
4. Application: Information and Legislative Organization .... 227
5. Application: Informational Lobbying .... 232
6. Refinements of Perfect Bayesian Equilibrium* .... 236
7. Exercises .... 248
Contents

9 Repeated Games ........................................... 251
1. The Repeated Prisoner's Dilemma 252
2. The Grim Trigger Equilibrium 253
3. Tit-for-Tat Strategies 256
4. Intermediate Punishment Strategies 258
5. The Folk Theorem 260
6. Application: Interethnic Cooperation 263
7. Application: Trade Wars 269
8. Exercises 273

10 Bargaining Theory ........................................... 275
1. The Nash Bargaining Solution 275
2. Noncooperative Bargaining 281
4. The Baron-Ferejohn Model Under Open Rule 291
5. Bargaining with Incomplete Information 294
6. Application: Veto Bargaining 296
7. Application: Crisis Bargaining 307
8. Exercises 318

11 Mechanism Design and Agency Theory ................. 320
1. An Example 321
2. The Mechanism Design Problem 323
3. Application: Polling 326
4. Auction Theory 328
5. Application: Electoral Contests and All-Pay Auctions* 334
6. Incentive Compatibility and Individual Rationality 339
7. Constrained Mechanism Design 342
8. Mechanism Design and Signaling Games 361
9. Exercises 366

12 Mathematical Appendix .................................... 369
1. Mathematical Statements and Proofs 370
2. Sets and Functions 372
3. The Real Number System 376
4. Points and Sets 378
5. Continuity of Functions 380
6. Correspondences** 383
7. Calculus 384
8. Probability Theory 404

Bibliography 417
Index 423
Acknowledgments

The origin of this book is the utter inability of either of its authors to write legibly on a blackboard (or any other surface, for that matter). To save our students from what would have been the most severe form of pedagogical torture, we were forced to commit our lecture notes to an electronic format. Use of this medium also compensated for our inability to spell without the aid of a spell checker. Ultimately we decided that all of the late nights spent typesetting game theory notes should not go in vain. So we undertook to turn them into this book, which, of course, led to more late nights spent typing. We hope these weren’t wasted either.

We are most grateful to our students at Columbia and Princeton, on whom we inflicted early versions of our notes and manuscript. Puzzled looks and panicked office hours helped us learn how to convey game theory to students of politics. We also benefited from early conversations with Chris Achen, Scott Ashworth, Larry Bartels, Cathy Hafer, Keith Krehbiel, David Lewis, Kris Ramsay, and Thomas Romer on what a book on political game theory ought to look like. Along the way Stuart Jordan and Natasha Zharinova have provided valuable assistance and feedback. We especially thank John Londregan and Mark Fey for noting mistakes in earlier drafts. Finally, our greatest debts are to those who taught us political game theory:

---

1 Our misspelling styles are quite distinctive, however. For a given word, McCarty uses completely random spellings whereas Meirowitz consistently misspells the word in exactly the same way.
Acknowledgments

David Austen-Smith, Jeffrey Banks, David Baron, Bruce Bueno de Mesquito, Thomas Romer, and Howard Rosenthal.

Nolan McCarty
Adam Meirowitz

Princeton, NJ