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MODERN DYNAMICAL SYSTEMS AND APPLICATIONS

This volume presents an overview of the theory of dynamical systems. It contains many expository contributions by a list of leading researchers, including several Fields medalists. A broad span of topics is covered. Major areas include: hyperbolic dynamics, elliptic dynamics, mechanics, ergodic theory, group actions, rigidity, applications.

Here dynamicists will find surprising new results in their own specialty as well as surveys in others, and mathematicians from other disciplines who wish for a sample of current developments in ergodic theory and dynamical systems should look no further.

MODERN DYNAMICAL SYSTEMS AND APPLICATIONS

Dedicated to Anatole Katok on his 60th Birthday

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CONTENTS

This book gives an overview of contemporary research in dynamical systems that is clear and comprehensive enough to provide mathematicians, be they experts or not, with a useful and representative picture of current developments. The articles in this book present many facets of dynamics today. Some of them are surveys that give an account of recent progress in one subject within ergodic theory and dynamical systems. Several articles present entirely new results. These are included as being representative of the most current research and, as we hope, seminal for further research. Some articles are in between, either surveying a research area while including new results, or outlining specific new developments.

This book is written on the occasion of Anatole Katok 60th birthday, which provides a great opportunity to survey the modern theory of dynamical systems. In our opening notes we aim to convey a sense of the impact Anatole Katok has had on dynamical systems and related topics.

We, the editors, wish to thank the authors for their commitment to this project and Irina Pesin for preparing the illustrations for publication.

ANATOLE KATOK

In the spring of 1959 quite a few of the participants of the second round of the Moscow Mathematical Olympiad left before the time expired, mainly because the problems were (as usual) very hard, and the situation looked hopeless. An 8th-grader, Anatole (Tolya) Katok, left early because he had tickets to an ice show. He still ended up second; a 7th-grader, D. Kazhdan, got the first prize. This was possibly the last occasion when Katok put entertainment above mathematics.

In 1960, at the age of 16, Katok became a mathematics major at the Moscow State University. He received his master's degree in 1965 and PhD in 1968. Katok's PhD thesis advisor was Ya. Sinai, the thesis referees were A. Kolmogorov and V. Rokhlin. Mathematics blossomed in the Department of Mechanics and Mathematics (or mekh–mat) during a roughly ten year period from the end of the 50s until the end of the 60s. Brilliant and famous professors and excellent students formed a stimulating community that brought about an unmatched variety of break-through ideas and a host of future leaders in mathematics.

Seminar led by A. S. Kronrod in a classroom at Mekh-mat, Moscow State University during the 1960/61 academic year. Left/front of the aisle, from left to right: G. Margulis, L. Makar-Limanov, D. Kazhdan; on the far side of the aisle, left to right: S. Gelfand, V. Fishman, V. Pranov, A. S. Kronrod, A. Geronimus (obscured), A. Libin, M. Khanchachian, A. Katok.

By the time he completed his dissertation Katok was recognized as a prominent mathematician. He definitely deserved a position either at the Moscow State University or at the Steklov Mathematical Institute, but that was impossible due to the political atmosphere and anti-Semitism of the Soviet society at that time. He obtained, however, a position in the department of mathematical methods at the Central Economics and Mathematics Institute. It was a known "haven" for some mathematicians because it allowed them to combine their work on mathematical problems in economics, if any, with research in pure mathematics.

At this time Anosov and Katok started a joint seminar on dynamical systems at the Steklov Mathematical Institute of which Anosov was a member. (The seminar was not officially sanctioned, and several years later the administration of the institute no longer permitted the use of the building.) Katok's position did not allow him to officially have students in mathematics and therefore, he supervised several participants of the seminar informally. Two of the authors of these lines were participants of the seminar. Although they had graduated with honors from the Moscow State University and had by this time obtained new (and even published) results, their applications to the graduate school had been turned down by the communist party bureau. The guidance and support provided by the seminar and its altruistic leaders became their graduate school and were crucial for their mathematical lives.

The year 1978 marked the beginning of a new period in Katok's life – he moved to the US, bringing along his unlimited mathematical enthusiasm, optimism and energy. This opened up many new opportunities for Katok's active personality by allowing him to travel, attend and organize conferences, collaborate with other mathematicians and supervise students. He fully used these opportunities and organized more conferences, special years and other events than anybody else in the dynamics community.

Jacob Feldman, Anatole Katok and Donald Ornstein planning the 1983-4 special year at MSRI during the conference in honor of S. Kakutani at Yale University 1982.

At each of the three universities where Katok has held tenured faculty positions, the University of Maryland, Caltech and Penn State, he was a natural leader of the dynamical systems group. During his five years at Maryland Katok was instrumental in the development of their dynamical systems school and after moving to first Caltech and then Penn State he founded a strong group in dynamical systems at each of these institutions. The schools at Maryland and Penn State have become leading world centers and have successfully interacted with each other through collaborations, visits and, most importantly, the traditional joint semi-annual workshops.

Throughout the 19th century there were mathematicians who were able to maintain a knowledge of mathematics that encompassed all or most of its fields, and in the early 20th century a select few, such as Hilbert, managed to come close to this ideal. These mathematicians were important to the enterprise of mathematics, but the rapid growth of the discipline made it impossible to sustain such breadth. Today the role of universally educated mathematicians is played by those very few who have a comprehensive view of a field and an understanding of the interaction of its parts as well as its connections to other areas of mathematics. Anatole Katok belongs to this select group, representing the theory of dynamical systems. His contributions to the theory are multifaceted and have shaped large areas within dynamical systems in many ways.

While the discipline of dynamical systems is over a century old and one can trace its origin to the classical work of Poincaré, it was in the 1960s that it emerged as an independent mathematical discipline in its own right due to the seminal works of Kolmogorov, Anosov, Sinai, Smale and Moser. In this exciting time of transition, young mathematicians found this rapidly developing theory very attractive and began to work in it under the direction of the members of the founding generation. Katok stands out for his ability to be involved in essentially all areas of dynamical systems from the start. He has always been working in the theory of dynamical systems in a broad sense, and his interests range widely, both scientifically and in terms of the character of his publications and activities.

Throughout his career Katok has been extremely active in mentoring younger generations. Already during his student years he devoted much energy to high school mathematics competitions and clubs. Katok attracts many students and by all accounts is demanding but at the same time a very helpful supervisor, spending countless hours with his students. Katok has supervised more than two dozen PhD students most of whom have been very successful in their professional careers. Many have achieved prominent status in the mathematical community.

Katok's conference, colloquia and seminar presentations are always rich with ideas, wide ranging and engaging. His presence in the audience usually guarantees a lively, informed and provocative discussion.

In these few pages we can only just touch upon several areas of Katok's work. As indicated above, his work covers most major areas of dynamical systems. Already in his doctoral dissertation he developed a theory of periodic approximations of measure-preserving transformations commonly known as Katok–Stepin approximations. This theory helped solve some long-standing problems that went back to von Neumann and Kolmogorov, and it won him the prize of the Moscow Mathematical Society in 1967.

His next major achievement was the theory of monotone (or Kakutani) equivalence, which is based on a far-reaching generalization of the concept of time-change in flows. Similar lines of research were being pursued at about the same time by a group of ergodic theorists including Feldman, Ornstein, Rudolph and Weiss.

There are many constructions in the theory of dynamical systems that are due to Katok and reveal interesting and often surprising phenomena. Among these are the Anosov–Katok construction of smooth ergodic area-preserving diffeomorphisms of compact manifolds, the construction of Bernoulli diffeomorphisms with nonzero Lyapunov exponents on any surface, and the construction of a foliation for which Fubini's Theorem fails in the worst possible way, i.e., there is a set of full measure that meets every leaf in a single point. (Such foliations arise naturally in partially hyperbolic dynamical systems.)

Upon the family's emigration in 1978 Katok, like some other mathematicians who emigrated from the Soviet Union, experienced the great kindness and selfless generosity of mathematicians in Vienna, Rome, Milano and Paris, and he has never forgotten this. He first visited the University of Rome and then the IHES where, for the first time in his life, he found himself with an appointment as a mathematician at a mathematics institute, and with an office of his own. This was rewarding and immensely invigorating. Here at the IHES he obtained some of his best-known and most widely quoted results in the nonuniformly hyperbolic theory, whose development he had profoundly influenced. Among them are density of periodic points and lower bounds on their number as well as exhaustion of topological entropy by horseshoes. Those results were the topic of his lecture at the International Congress of Mathematicians in 1983 as well as the 1982 Rufus Bowen Memorial Lectures at Berkeley.

Katok is also well-known for formulating challenging conjectures and problems (for some of which he even offered prizes) that stimulated and influenced significant bodies of work in dynamical systems. The best-known of these is the Katok Entropy Conjecture, which connects important geometric and dynamical properties of geodesic flows. The story of this conjecture serves as an illustration of how Katok functions as a mathematician. He proved that the topological and Liouville entropies of a geodesic flow on a negatively curved surface agree only if the curvature is constant, i.e., the metric is locally symmetric, and he conjectured that the same holds in any dimension. He not only worked on this conjecture himself, but he put much effort into popularizing it by giving numerous talks, organizing a series of conferences that spans 2 decades, and directing the interest of mathematicians towards the conjecture. There is now a major body of work related to this conjecture that includes substantial partial results and theorems that give insight into the question. One of the high points is a result of Besson, Courtois and Gallot according to which the topological entropy of the geodesic flow is minimized only for locally symmetric metrics.

The classical theory of dynamical systems deals with actions of the groups $\mathbb{Z}$ of integers or $\mathbb{R}$ of real numbers. The study of these actions was the main subject in classical and statistical mechanics. Recently, dynamical methods have found other applications to physics and some areas in mathematics that requires dynamical systems with "more general time" – the actions of large groups. These applications include quantum field theory (that requires actions of locally compact groups), statistical physics of ferromagnetic materials (that deals with actions of multi-dimensional lattice $\mathbb{Z}^n$) and number theory where actions of some semisimple Lie groups have been used with great success.

An action of a Lie group is locally rigid if any perturbation of the action which is close to it on a compact generating set is conjugate to it up to an automorphism. Unlike in the classical case of

$\mathbb{Z}$ or $\mathbb{R}$, such structural stability for group actions is a rather widespread (although not completely understood) phenomenon.

The Katok Entropy Conjecture is one of the first rigidity statements in dynamical systems. In the last two decades Katok has been working intensely on other rigidity phenomena. In collaboration with several colleagues he made fundamental contributions to smooth rigidity and geometric rigidity, to differential and cohomological rigidity of smooth actions of higher-rank abelian groups and of lattices in Lie groups of higher rank, to measure rigidity for group actions and to nonuniformly hyperbolic actions of higher-rank abelian groups.

An important feature of Katok's mathematical work, which reflects his personality, is to always keep up with the development of the theory of dynamical systems as a whole and to express his views and understanding of it by lecturing and writing lecture notes, surveys and books. For example, his lectures in the 1971 Kiev summer school were very influential, as was his appendix to the Russian translation of Nitecki's book on Differentiable Dynamics. The former was an attempt to present a comprehensive description of the state of the hyperbolic theory after the famous starting work of Anosov, Sinai and Smale, while in the latter he gave a general version of pseudo-orbits and shadowing techniques in uniformly hyperbolic dynamics. His survey with Sinai and Stepin summarized all major results in the ergodic theory of dynamical systems obtained by that time.

During the last decade Katok has written several manuscripts and surveys where he presented his views and understanding of the modern theory of dynamical systems as a whole. Among them the eight hundred page "Introduction to the Modern Theory of Dynamical Systems" by Katok and Hasselblatt, that came out in 1995, stands out as an encyclopedic and unified account of the contemporary state of smooth dynamical systems. It became a "bible" for the field and one of the most useful sources of reference. A "baby version" of this book under the title "Dynamics: A first course — with a panorama of recent developments" appeared in 2003. Maintaining the rigor and a good portion of the scope of the previous book and illustrating the theory with numerous examples, it provides an excellent promotion of dynamical systems at the undergraduate level.

The 200 page introductory survey "Principal Structures" to Volume I of the Handbook of Dynamical Systems he edited also appeared in 2003. Aimed at mathematicians from outside of dynamical systems (but useful for and interesting to those within) it lays out a broad structural view of dynamical systems as an immensely diverse discipline with shared fundamental ideas and may serve as an example of Katok's skill in using the concrete to elucidate the general.

His 60th birthday is a suitable occasion to survey the modern theory of dynamical systems, and we dedicate this volume to our friend and mentor Anatole Katok with deep appreciation, admiration and respect, both professional and personal.