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S. Nemat-Nasser

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PLASTICITY

A Treatise on Finite Deformation of Heterogeneous Inelastic Materials

Providing a basic foundation for advanced graduate study and research in the mechanics of solids, this treatise contains a systematic development of the fundamentals of finite inelastic deformations of heterogeneous materials. The book combines the mathematical rigor of solid mechanics with the physics-based micro-structural understanding of the materials science, to present a coherent picture of finite inelastic deformation of single and polycrystalline metals, over broad ranges of strain rates and temperatures. It also includes a similarly rigorous and experimentally based development of the quasi-static deformation of cohesionless granular materials that support the applied loads through contact friction. Every effort has been made to provide a thorough treatment of the subject, rendering the book accessible to students in solid mechanics and in the mechanics of materials. This is the only book that integrates rigorous mathematical description of finite deformations seamlessly with mechanisms based on micromechanics in order to produce useful results with relevance to practical problems.

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Inelastic Materials

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PREFACE

This book is a treatise on the mathematical and experimental foundations of the finite deformation of crystalline metals and granular materials. It combines the mathematical rigor of solid mechanics with the physics-based microstructural understanding of the materials science, to present a coherent picture of finite inelastic deformation of single- and poly-crystalline metals, over broad ranges of strain rates and temperatures. It also includes a similarly rigorous and experimentally-based development of the quasi-static deformation of cohesionless granular materials that support the applied loads through contact friction.

The book has grown out of my lecture notes for graduate-level courses on the fundamentals of continuum plasticity and the mechanics of large deformations. These courses were initiated at the University of California, San Diego (UCSD), in the late 60's, and were then continued at Northwestern University in the 70's and early 80's. During this period at Northwestern University, it became clear to me that true advances in the basic understanding of the mechanics of materials, and particularly the inelastic deformation of metals and geomaterials, can be achieved only by moving beyond the traditional phenomenological approach to plasticity models that are based on the classical concepts of rate-independent yielding and empirical data fitting, and to exploring the basic micromechanisms of the phenomena through direct observations, coordinated with indirect systematic experimentation and micromechanical modeling. This then necessitates integrating the traditional mathematical and computational modeling of the solid mechanics community, with extensive macroscopic and microstructural experimental characterization of real materials, in the spirit of the materials science community, demanding novel approaches to both teaching and research in this area, and requiring, at a minimum, new experimental techniques and facilities to investigate the underpinning micromechanisms of deformation.

Hence, in the mid 80's, I spearheaded the creation of a unified mechanics and materials program at UCSD, and sought to develop there a state-of-the-art materials characterization laboratory that now includes many novel testing facilities and techniques. At the same time, an integrated materials science program was launched at UCSD, that built on the contributions of researchers from the physical, natural, and basic engineering and oceanographic sciences, who had interest in materials science and technology. Serving as the founding director of this program, I taught several courses that combined mechanics and materials, and directed a Ph.D. research program that included many outstanding graduate students eager to learn the fundamentals of both solid mechanics and materials, and who were willing to undertake both experimental and theoretical research in the mechanics of materials. It is in this integrated environment that the present version of this book has now emerged.

In addition to my own work, the book liberally draws (with commensurate citation) from my collaborative research with former graduate students, post-doctoral associates, visiting scientists, and other coworkers. Many of these students started their graduate education knowing little about mechanics and materials, but ended their study making significant new experimental and theoretical/computational contributions that have helped to improve my knowledge of the subject. They all have studied at least some sections of one of the several early versions of this book, and have made contributions that have served to improve the work. I am most grateful to them.

There are several former students and coworkers whose work and contributions have been pivotal to a number of topics covered in this book. The idea of writing a book on the micromechanics of large deformations started with my collaboration with Dr. Monte Mehrabadi in the late 70's at Northwestern University, where a draft of a chapter on kinematics was started but was not fully completed. This work was later published,¹ and its results together with further extensions, are included in Section 1.5 of Chapter 1 and in various sections of Chapter 2. In the late 70's, I was most fortunate to work with Professor Jes Christoffersen and Dr. Mehrabadi to seek to relate the continuum concepts of stress and strain to the micromechanics of contact forces and sliding and rolling of the grains within a granular mass. This effort was indeed successful, leading to significant results² that are summarized in Section 7.2.

Regarding the content of Chapter 7, it was Professor M. Oda who worked with Mr. John Schmidt (the technician working in my laboratory) and me to produce the biaxial testing system shown in Figure 7.4.2, page 547, and together with Dr. Konishi and Mr. Zong-Lian Qiu, to perform and analyze many of the tests on biaxial deformation of photoelastic granules. These and related contributions are liberally used and referenced in Chapter 7, including some theoretical work with both Mehrabadi and Oda.³

Another significant contributor to my work, was my former student and post-doctoral fellow, Dr. B. Balendran, who worked on both computational algorithms and micromechanics of granular materials, and helped with many aspects of the related materials in Chapters 5 and 7, as well as some of the basic mathematical results presented in Chapters 1 and 2.⁴

¹ Mehrabadi, M. M. and Nemat-Nasser, S. (1987), Some basic kinematical relations for finite deformations of continua, *Mech. Mat.*, Vol. 6, 127-138.

² Christoffersen, J., Mehrabadi, M. M., and Nemat-Nasser, S. (1981), A micromechanical description of granular material behavior, *J. Appl. Mech.*, Vol. 48, 339-344. [This work was completed in the fall of 1979, and was subsequently distributed as a report dated January, 1980. I presented the results at the 1980 Congress of Theoretical and Applied Mechanics, in Toronto, Canada.]

³ Mehrabadi, M. M., Nemat-Nasser, S., and Oda, M. (1982), On Statistical description of stress and fabric in granular materials, *Int. J. Num. Anal. Meth. Geomech.*, Vol. 6, 95-108.

Oda, M., Nemat-Nasser, S., and Mehrabadi, M. M. (1982), A statistical study of fabric in a random assembly of spherical granules, *Int. J. Num. Anal. Methods Geomech.*, Vol. 6, 77-94.

Oda, M., Konishi, J., and Nemat-Nasser, S. (1982), Experimental micromechanical evaluation of strength of granular materials: effects of particle rolling, *Mech. Mat.*, Vol. 1, No. 4, 269-283.

⁴ Balendran, B. and Nemat-Nasser, S. (1993), Double sliding model of cyclic deformation of granular materials, including dilatancy effects, *J. Mech. Phys. Solids*, Vol. 41, No. 3, 573-612.

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Others with intellectual contributions that have affected the contents of this book, include my former graduate students Professors T. Iwakuma, M. Obata, G. Subhash, M. Zikry, M. Rashid, and Y-F. Li, and Drs. A. Thakur, N. Okada, T. Okinaka, and R. Kapoor. I am also grateful to Professor Benjamin Lorent for proofreading Chapters 4 and 7, and to Mr. L. Ni and Dr. J. Zhang who have read and commented on the contents of Chapter 5. Mr. Ni's collaborative work with me on constitutive algorithms, is included in this chapter.⁵ I am especially thankful to Dr. M. Scheidler of Army Research Laboratory, who read and commented on the contents of Chapters 1, 2, 3, and 4,⁶ which resulted in much improvement.

The manuscript was proofread in its various versions, several times, by my wife Éva, resulting in important consistency and grammatical improvements. I, however, take full responsibility for any errors that remain. In addition, my daughter Shiba and my assistant Lauri Jacobs helped with various aspects of the word processing, and Dr. Masoud Beizaie assisted in preparing some of the figures and helped with several other tasks. I am most grateful for all these contributions. Except for Chapter 9, I have formatted the book using *ditroff*. Most of the figures and the graphs are constructed by *pic* and *grap*. In this connection, I wish to thank Frank Dwyer who helped to move old ditroff editing and formatting tools from an old Sun system to a new one, which allowed me to continue to use many macros that were constructed particularly to tailor the figures and equations to my taste. Chapter 9 was formatted using *grof*, and I wish to thank my graduate student Mr. Sai Sarva who helped to solve a number of formatting problems, thereby rendering this chapter consistent with the rest of the book.

Sia Nemat-Nasser, La Jolla, California

June, 2004

Balendran, B. and Nemat-Nasser, S. (1995), Integration of inelastic constitutive equations for constant velocity gradient with large rotation, *Appl. Math. Comput.*, Vol. 67, No. 1-3, 161-195.

Balendran, B. and Nemat-Nasser, S. (1996), Derivative of a function of a nonsymmetric second-order tensor, *Q. Appl. Math.*, Vol. LIV, No. 3, 583-600.

⁵ Nemat-Nasser, S. and Ni, L. (1994), Effective constitutive algorithms in elastoplasticity and elastoviscoplasticity, *European J. Appl. Math.*, Vol. 5, Part 3, 313-336.

⁶ Dr. Scheidler's comments on the material contained in the original version of Section 4.9, led to a complete rewriting of this section which now includes new results on elastic anisotropy in continuum metal plasticity, and clarification of the notion of plastic spin.