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0521839718 - Essentials of Statistical Inference
G. A. Young and R. L. Smith
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Essentials of Statistical Inference

Essentials of Statistical Inference is a modern and accessible treatment of the procedures used to draw formal inferences from data. Aimed at advanced undergraduate and graduate students in mathematics and related disciplines, as well as those in other fields seeking a concise treatment of the key ideas of statistical inference, it presents the concepts and results underlying the Bayesian, frequentist and Fisherian approaches, with particular emphasis on the contrasts between them. Contemporary computational ideas are explained, as well as basic mathematical theory.

Written in a lucid and informal style, this concise text provides both basic material on the main approaches to inference, as well as more advanced material on modern developments in statistical theory, including: contemporary material on Bayesian computation, such as MCMC, higher-order likelihood theory, predictive inference, bootstrap methods and conditional inference. It contains numerous extended examples of the application of formal inference techniques to real data, as well as historical commentary on the development of the subject. Throughout, the text concentrates on concepts, rather than mathematical detail, while maintaining appropriate levels of formality. Each chapter ends with a set of accessible problems.

Based to a large extent on lectures given at the University of Cambridge over a number of years, the material has been polished by student feedback. Some prior knowledge of probability is assumed, while some previous knowledge of the objectives and main approaches to statistical inference would be helpful but is not essential.

G. A. YOUNG is Professor of Statistics at Imperial College London.

R. L. SMITH is Mark L. Reed Distinguished Professor of Statistics at University of North Carolina, Chapel Hill.

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G. A. Young
Department of Mathematics, Imperial College London

R. L. Smith
Department of Statistics and Operations Research, University of North Carolina, Chapel Hill

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Cover image: Analysis of z-values for gene expression data. *Solid line,* $\hat{f}(z)$; *dotted line,* theoretical null; *dashed line,* empirical null. See Figure 3.7.

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Preface

This book aims to provide a concise but comprehensive account of the essential elements of statistical inference and theory. It is designed to be used as a text for courses on statistical theory for students of mathematics or statistics at the advanced undergraduate or Masters level (UK) or the first-year graduate level (US), or as a reference for researchers in other fields seeking a concise treatment of the key concepts of and approaches to statistical inference. It is intended to give a contemporary and accessible account of procedures used to draw formal inference from data.

The book focusses on a clear presentation of the main concepts and results underlying different frameworks of inference, with particular emphasis on the contrasts among frequentist, Fisherian and Bayesian approaches. It provides a description of basic material on these main approaches to inference, as well as more advanced material on recent developments in statistical theory, including higher-order likelihood inference, bootstrap methods, conditional inference and predictive inference. It places particular emphasis on contemporary computational ideas, such as applied in bootstrap methodology and Markov chain Monte Carlo techniques of Bayesian inference. Throughout, the text concentrates on concepts, rather than mathematical detail, but every effort has been made to present the key theoretical results in as precise and rigorous a manner as possible, consistent with the overall mathematical level of the book. The book contains numerous extended examples of application of contrasting inference techniques to real data, as well as selected historical commentaries. Each chapter concludes with an accessible set of problems and exercises.

Prerequisites for the book are calculus, linear algebra and some knowledge of basic probability (including ideas such as conditional probability, transformations of densities etc., though not measure theory). Some previous familiarity with the objectives of and main approaches to statistical inference is helpful, but not essential. Key mathematical and probabilistic ideas are reviewed in the text where appropriate.

The book arose from material used in teaching of statistical inference to students, both undergraduate and graduate, at the University of Cambridge. We thank the many colleagues at Cambridge who have contributed to that material, especially David Kendall, Elizabeth Thompson, Pat Altham, James Norris and Chris Rogers, and to the many students who have, over many years, contributed hugely by their enthusiastic feedback. Particular thanks go to Richard Samworth, who provided detailed and valuable comments on the whole

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text. Errors and inconsistencies that remain, however, are our responsibility, not his. David Tranah and Diana Gillooly of Cambridge University Press deserve special praise, for their encouragement over a long period, and for exerting just the right amount of pressure, at just the right time. But it is our families who deserve the biggest 'thank you', and who have suffered most during completion of the book.

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