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Introduction

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1 Background

In 1999, a major programme on turbulence was held at the Isaac Newton Institute (INI) at Cambridge, England, which was aimed at taking an overview of the current situation on turbulent flows with particular reference to the prediction of such flows in engineering systems. Though the programme spanned the range from the very fundamental to the applied, a very important feature was the involvement and support (through the UK Royal Academy of Engineering) of key players from industry. This volume, which has evolved from the INI programme, aims to address the needs of people in industry and academia who carry out calculations on turbulent systems.

It should be recognised that the prediction of turbulent flows is now of paramount importance in the development of complex engineering systems involving flow, heat and mass transfer and chemical reactions (including combustion). Whereas, in the past, the developer had to rely on experimental studies, based usually on small scale model systems, more and more emphasis is being placed nowadays on the use of computation, often through the use of commercial computational fluid dynamics (CFD) codes. Superficially, the use of such computational methods seems ideal; they allow painless extension to large scale and can often give information on fine details of the flow that are not economically accessible to experimental measurement. Furthermore, the results can be presented in an easily accessible and attractive form using the sophisticated computer graphics now generally available. Such methods have become big business!

Unfortunately, there is a major problem in the application of CFD techniques in predicting industrial turbulent flow systems, namely the inherent modelling of the turbulence itself. Though low Reynolds number turbulent flows (close to the

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transition from laminar flow) can be modelled in a reasonably fundamental way using the techniques of direct numerical simulation (DNS), such methods are of little direct relevance to industry. Firstly, the Reynolds numbers for which such calculations can be performed are very limited, and certainly well below those of prime interest to industry. Secondly, the computing resources required are enormous; whilst it is true that computing is getting ever faster and cheaper, it seems unlikely that the usual industrial range of Reynolds numbers will be accessible to DNS-type methods in the near future.

In order to achieve closure of turbulent flow predictions, therefore, it is necessary to invoke some form of *turbulence model*. There are a bewildering variety of such models available in the literature, with the two main classes being the *Reynolds averaged Navier–Stokes (RANS)* models and the *large eddy simulation (LES)* models. The first (and most numerous) class has its origin in the classical averaging of the Navier–Stokes equations due to Osborne Reynolds; here, no attempt is made to deal with the detailed structure of the turbulence. Rather, statistical quantities are obtained and models derived for their prediction based on *modelling hypotheses* that often have several (sometimes many!) adjustable constants that are optimised by comparison with experimental data. A number of distinct types of RANS model have been developed; perhaps the most widely used are based on the assumption of an effective, isotropic, turbulent viscosity whose value can be determined from the local averaged turbulence quantities. Archetypal amongst this sub-class is the k–ε model in which the turbulent viscosity is related to the turbulent kinetic energy (k) and the turbulence dissipation rate (ε). Despite the fact that the k–ε model is contradicted by a wide range of experimental data (for instance by the fact that the planes of zero shear and maximum velocity are different in channels with one rough and one smooth wall), it is still used almost universally (and perhaps often unthinkingly) in industrial CFD predictions. More advanced (though more complex) RANS models (such as the *Reynolds stress transport models*, RSTM) are available which are not limited by the assumption of small scale isotropy.

The second general class of models is the *large eddy simulation (LES)* models. Here, whilst it is recognised that the prediction of the fine details of the turbulent flow is infeasible, an attempt is made to model the temporal and spatial characteristics of the larger eddies. The smaller eddies are dealt with using *sub-grid models*. This class of models is beginning to be used in prediction of industrial systems, particularly where an understanding of the local fluctuating behaviour is important. However, the computational requirements are very large compared to the RANS models. A problem with LES models is the representation of the region near the wall.

2 Objectives

The above brief background will give an indication of the importance and difficulties of modelling turbulent systems. As a result of the intensive interactions between industrial practitioners and academic specialists which were brought about by the INI programme, the idea emerged of generating an overview of turbulent flow prediction methods and the project to produce the present volume was launched. The objectives were:

(1) To summarise current understanding of the physics of turbulent flow and implications for modelling.
(2) To review prediction methods and their applicability to various industrial prediction problems.
(3) To provide a specific set of guidelines (a 'route map') for the choice of model for a given problem.

3 Structure of the volume

The volume is structured into eight chapters (including this present one). The remaining chapters are as follows:

Chapter 2: Developments in the understanding and modelling of turbulence. This chapter essentially provides a summary of what is known about the nature of turbulent flows, particularly in the light of the work of the INI programme. The features common to all turbulent systems are discussed and those features specific to particular manifestations of turbulence are reviewed.

Chapter 3: RANS modelling of turbulent flows affected by buoyancy or stratification. Flows driven wholly or partly by buoyancy (arising from density variations caused by temperature or concentration gradients) are common in practice. Stratification arising from density gradients is also important in many systems. These effects present significant challenges in turbulence modelling and these challenges are specifically addressed in this chapter. Moreover, this chapter presents generic material on turbulence models (for instance the k–ε and second order closures for the RANS models) which can also be applied in the more general case where buoyancy is less significant.

Chapter 4: Turbulent flames. In modelling turbulent flames, it is necessary not only to model the turbulence but also to represent the interaction between the turbulent fluctuations and the chemical reaction. This provides a particular challenge since mixing by small scale turbulence has a strong effect on local chemical reaction. This chapter begins with a discussion of the two main types of combustion (non-premixed and premixed respectively). The various approaches are summarised (chemical equilibrium, flamelets, pdf methods and models, eddy breakup models etc.). Not all combustion systems fall into premixed and non-premixed types; the premixing can be partial. These cases are discussed.

Chapter 5: Boundary layers under strong distortion: an experimentalist's view. Turbulent boundary layers represent a severe challenge to prediction methods. Not only are they regions of strong distortion and change, but their interactions with shocks and vortices give rise to additional problems. This chapter reviews prediction methods, observations and measurements available for boundary layers under strong distortion and attempts to identify areas in which future research should be concentrated.

Chapter 6: Turbulence simulation. As was mentioned above, LES is being increasingly used to address industrial problems. This chapter reviews the current situation on this type of modelling, evaluating the barriers to its more widespread use. The chapter also discusses direct numerical simulation (DNS); DNS can provide useful physical insights into turbulence processes which can be used to develop closures applicable at the higher Reynolds numbers.

Chapter 7: Computational modelling of multi-phase flows. Multi-phase flows (i.e. flows involving two or more phases – gas, liquid and solid) are not only usually turbulent but also have the additional complication of having moving interfaces within the flow. In the case of flows having two or more fluid phases, these interfaces may be deformable. Nevertheless, computational modelling is playing an increasing role in the prediction of multi-phase systems. For instance, industrial systems involving dispersed flows (sewage treatment plant, agitated vessel reactors, etc.) are now widely predicted using the commercial CFD codes. This chapter reviews the background to the prediction of such dispersed flows. For flows in which interface distortion is significant, predictions are more difficult; however, there is a growing range of methods for modelling the interfacial behaviour and these are reviewed and examples of their application presented.

Chapter 8: Guidelines and criteria for the use of turbulence models in complex flows. As was stated above, the practitioner is faced with a bewildering array of turbulence models. When applied to a given system, the models can give different (and sometimes very different) results. Which is the correct model to use? Though, because of the basic limitations discussed above, there is no truly 'correct' model for most engineering turbulent systems, it is undoubtedly true that some models perform better than others for particular classes of problem. Recognising this fact, this chapter attempts to provide a reference guide to the choice of model to be used in particular circumstances.

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Developments in the understanding and modelling of turbulence

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Abstract

Recent research is making progress in framing more precisely the basic dynamical and statistical questions about turbulence and in answering them. It is helping both to define the likely limits to current methods for modelling industrial and environmental turbulent flows, and to suggest new approaches to overcome these limitations. This chapter had its basis in the new results that emerged from more than 300 presentations during the programme held in 1999 at the Isaac Newton Institute, Cambridge, UK, and on research reported elsewhere. The objective of including this material (which is a revised form of an article which appeared in the *Journal of Fluid Mechanics* – Hunt *et al.*, 2001) in the present volume is to give a background to the current state of the art. The emphasis is on the *physics* of turbulence and on how this relates to modelling. A general conclusion is that, although turbulence is not a universal state of nature, there are certain statistical measures and kinematic features of the small-scale flow field that occur in most turbulent flows, while the large-scale eddy motions have qualitative similarities within particular types of turbulence defined by the mean flow, initial or boundary conditions, and in some cases, the range of Reynolds numbers involved. The forced transition to turbulence of laminar flows caused by strong external disturbances was shown to be highly dependent on their amplitude, location, and the type of flow. Global and elliptical instabilities explain much of the three-dimensional and sudden nature of the transition phenomena. A review of experimental results shows how the structure of turbulence, especially in shear flows, continues to change as the Reynolds number of the turbulence increases well above about 10^4 in ways that current numerical simulations cannot reproduce. Studies of the dynamics of small eddy structures and

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their mutual interactions indicate that there is a set of characteristic mechanisms in which vortices develop (vortex stretching, roll-up of instability sheets, formation of vortex tubes) and another set in which they break up (through instabilities and self-destructive interactions). Numerical simulations and theoretical arguments suggest that these often occur sequentially in randomly occurring cycles. The factors that determine the overall spectrum of turbulence are reviewed. For a narrow distribution of eddy scales, the form of the spectrum can be defined by characteristic forms of individual eddies. However, if the distribution covers a wide range of scales (as in elongated eddies in the 'wall' layer of turbulent boundary layers), they collectively determine the spectra (as assumed in classical theory). Mathematical analyses of the Navier–Stokes and Euler equations applied to eddy structures lead to certain limits being defined regarding the tendencies of the vorticity field to become infinitely large locally. Approximate solutions for eigen modes and Fourier components reveal striking features of the temporal, near-wall structure such as bursting, and of the very elongated, spatial spectra of sheared inhomogeneous turbulence; but other kinds of eddy concepts are needed in less structured parts of the turbulence. Renormalized perturbation methods can now calculate consistently, and in good agreement with experiment, the evolution of second- and third-order spectra of homogeneous and isotropic turbulence. The fact that these calculations do not explicitly include high-order moments and extreme events suggests that they may play a minor role in some aspects of the basic dynamics. New methods of approximate numerical simulations of the larger scales of turbulence or 'very large eddy simulation' (VLES) based on using statistical models for the smaller scales (as is common in meteorological modelling) enable some turbulent flows with a non-local and non-equilibrium structure, such as impinging or convective flows, to be calculated more efficiently than by using large eddy simulation (LES), and more accurately than by using 'engineering' models for statistics at a single point. Generally it is shown that where the turbulence in a fluid volume is changing rapidly and is very inhomogeneous there are flows where even the most complex 'engineering' Reynolds stress transport models are only satisfactory with some special adaptation; this may entail the use of transport equations for the third moments or non-universal modelling methods designed explicitly for particular types of flow. LES methods may also need flow-specific corrections for accurate modelling of different types of very high Reynolds number turbulent flow including those near rigid surfaces.

This chapter is dedicated to the memory of George Batchelor who was the inspiration of so much research in turbulence and who died on 30th March 2000. These results were presented at the last fluid mechanics seminar in DAMTP Cambridge that he attended in November 1999.

1 Introduction

'The problem of turbulence' has been seen as one of the great challenges of mathematics, physics and engineering for more than 100 years, by Lamb, Einstein, Sommerfeld, Ishlinski and others. Much of the interest in meeting this challenge is because of its practical value; the solution of many technical, industrial and environmental problems increasingly requires improvements, both in our fundamental understanding of turbulence, and in the utilization of advances in computation to calculate, at appropriate levels of accuracy and speed, the characteristic features and statistical properties of these flows (e.g. Hunt 1995; Holmes, Lumley & Berkooz 1996).

Major centres for mathematical science and theoretical physics are holding intensive programmes on turbulence (examples being at Ascona, Monte Verita 2nd Symposium on Turbulence, Switzerland (Gyr, Kinzelbach & Tsinober 1999) and the Institute for Theoretical Physics, Santa Barbara in 2000) to complement regular summer schools and conferences, such as the European Turbulence Conference and Turbulent Shear Flow Symposia. In this chapter we draw some general conclusions about current questions and developments in research on turbulence and its practical applications, resulting from the programme at the Isaac Newton Institute at Cambridge (UK) between January and June 1999. This involved more than 400 participants, visiting for various periods, and about 300 presentations by academic and governmental researchers, and those working on problems in industrial and environmental organizations, some of which combined with the Royal Academy of Engineering to provide generous support for the programme. All three disciplines of mathematics, physics and engineering were well represented. We also refer here to other recent research developments reported in the scientific literature and at the International Congress on Industrial and Applied Mathematics held at Edinburgh in July 1999. Detailed reports on various aspects of the programme have been published by Voke, Sandham & Kleiser (1999); Launder & Sandham (2001); Vassilicos (2000b); Hunt & Vassilicos (2000).

This chapter is aimed at a broad fluid mechanical readership. It focuses, inevitably somewhat selectively and subjectively, on progress in research towards the major questions of the subject and certain practical objectives, both of which provided a framework for the programme. Although these were formulated well before the programme began, they evolved by progressive adjustment and addition during the six-month period. They essentially finally became the following.

(i) To consider broadly and in depth whether fluid turbulence in its different manifestations has some common features (in some defined statistical sense) that are universal to all kinds of fully turbulent flow, or whether any commonality only exists within certain types of turbulence (such as those driven by mean shear, or natural convection). In other words is there one 'problem of turbulence' or several?

(ii) To explore the promising directions for tackling the fundamental problems of turbulence dynamics, some of which go back to the 1930s (see Constantin 2000; Frisch 1995). Within this fell the following specific questions.

(a) Is Taylor's (1938) conjecture about turbulence correct? It is that the normalized mean rate of energy dissipation, $\hat{\varepsilon} = \varepsilon/(u_0^3/L_x)$ (where ε is the dimensional dissipation rate, u_0 is a typical r.m.s. velocity, and L_x is a typical integral length scale) of a turbulent flow field (away from a boundary) is independent of the turbulent Reynolds number $Re = u_0 L_x/v$, if the Reynolds number is sufficiently large, i.e.

$$\hat{\varepsilon} \to \text{const} \quad \text{as} \quad Re \to \infty. \tag{2.1}$$

If this is true (as is generally assumed in statistical models), what are the implications for the structure of the velocity field? If it is not, as some investigations suggest, what is the asymptotic relation between the rate of energy dissipation and the Reynolds number? These questions are central to turbulence theory and modelling: for example, Taylor's conjecture is part and parcel of such turbulence modelling, such as k–ε.

(b) Turbulence forces on mean flows are due to Reynolds stresses and arise from correlation between vorticity and velocity components. A fundamental understanding of Reynolds stresses requires, therefore, an understanding of the turbulent velocity fluctuation field. What is the nature of the 'wiggliness' and 'smoothness' of the velocity field as $Re \to \infty$, a question first raised by Richardson (1926) who wondered whether the velocity, even though its magnitude is finite, might be so 'wiggly' that it is not effectively differentiable anywhere (as with a Weierstrass function or some other fields with a non-integral Hausdorff fractal dimension). Without this wiggliness, the velocity field would not have the gradients necessary for energy dissipation to remain finite as $Re \to \infty$, Eq. (2.1). An alternative concept is that as $Re \to \infty$, turbulence is fundamentally intermittent with a finite number of distinct points where the derivatives are singular, separated by smooth regions in between. Some combinations of such distributions of near-singularities (defined as singularities in the limit as the Reynolds number tends to infinity) are necessary if Taylor's conjecture (2.1) is to be valid. Furthermore, how are such distributions consistent with the idea that velocity fields at the small scales may be self-similar over an increasing range of length scales as Re increases, a concept essential to large eddy simulations? How can deviations from self-similarity be considered in the context of multiple-scale velocity fields?

(c) Can even stronger singularities occur in which the velocity and vorticity at points in the flow tend to infinitely large values in a finite time t^*, after a finite-amplitude turbulent flow field has been initiated at $t = 0$? Although this phenomenon has never been observed, some special mathematical solutions to the Euler and the

Navier–Stokes equations suggest that it may be possible (Leray 1933; Kerr 1999; Moffatt 1999; Ohkitani & Gibbon 2000; Doering & Gibbon 2000). Are near-singularities of Navier–Stokes turbulence the remnants of finite-time singularities of the Euler equations? Does the tendency for such singular events to occur determine the 'tail' of the probability distribution of the turbulent flows and if so, how?

(d) What is the nature of the eddy transfer or 'cascade' process, in which when $Re \gg 1$ (if (2.1) is correct) the velocity fluctuations right down to the smallest scales reach a quasi-equilibrium state in the 'Lagrangian' or 'turn-over' time scale of order L_x/u_0? Also, to what extent are small-scale processes (depending on the precise definition) independent of the large-scale motions? Some physical models have suggested an infinite cascade involving vortical events at each 'eddy' scale (Tennekes & Lumley 1971; Frisch 1995), whereas others have suggested that relatively few complex events are needed (e.g. Lundgren 1982). The upscale energy transfer equally needs better understanding through study of the large-scale dynamics, which depends on how these eddy motions are correlated over large distances (Davidson 2004).

(e) To what extent do the large-scale motions of the turbulence tend to become independent of initial and boundary conditions, or, if the flow was initially laminar, of the particular process of transition to turbulence (George 1999): is this by means of internal self-organization or by chaotic interactions or both? Landau & Lifshitz (1959): 'We have seen that, whatever the initial phases β_j, over a sufficiently long interval of time the fluid passes through states arbitrarily close to any given state, defined by any possible choice of simultaneous values of the phase ϕ_j. Hence it follows that, in the consideration of turbulent flow, the actual initial conditions cease to have any effect after sufficiently long intervals of time. This shows that the theory of turbulent flows must be a statistical theory.' Batchelor's (1953) view was more conditional: '. . . we put our faith in the tendency for dynamic systems with a large number of degrees of freedom, and with coupling between these degrees of freedom, to approach a statistical state which is *independent* (partially, if not wholly) of the initial conditions. With this general property of dynamical systems in mind, rather than investigate the motion consequent upon a particular set of initial conditions, we explore the existence of solutions which are asymptotic in the sense that the further passage of time changes them in some simple way only.' This and the other fundamental questions provide a context for considering the appropriate future directions for the statistical computational models of turbulence needed for practical purposes.

(f) How are fully developed turbulent velocity fields related to their sources of energy, whether from initial conditions, continuing instabilities within a flow, or from boundary conditions such as a rigid wall?

(iii) Are certain statistical properties of fully developed inhomogeneous turbulence near plane rigid surfaces independent of the upstream or outer flow conditions and what is their form? This question refers to flows with and without a significant velocity $\bar{U}$ greater than the typical fluctuating velocity u_*; firstly, what is the mean velocity profile

$\bar{U}(x_3 u_*/v)$, whose mathematical form may be determined by the dependence on the Reynolds number of the outer flow (Barenblatt & Chorin 1998)? Secondly, under what conditions are the velocity spectra $\Phi_{11}(k_1)$ and $\Phi_{22}(k_1)$ along the streamwise direction given by

$$\Phi_{11}(k_1),\ \Phi_{22}(k_1) \propto u_*^2 k_1^{-1}, \tag{2.2}$$

when $\Lambda^{-1} < k_1 < x_3^{-1}$, for $x_3 \ll h$, where h is the thickness of the boundary layer/pipe and Λ is an outer length scale much greater than h (Marušić & Perry 1995)? Thirdly, for turbulent flows with or without a mean velocity component, how general is the self-similar form of the two-point velocity correlation of the normal components

$$R_{33}(x_3, x_3') = \overline{u_3(x_3)u_3(x_3')}/\overline{U_3^2}(x_3) = f(x_3/x_3') \quad \text{for} \quad x_3 < x_3' \tag{2.3}$$

(Hunt *et al*. 1989)?

(iv) To what extent do the asymptotic forms as $Re \to \infty$ for the statistics and characteristic eddy structures differ from those found when Re is finite? Are there distinct sub-classes of turbulence corresponding to different ranges of Re (or of Rayleigh number for natural convection (cf. Castaing *et al*. 1989))?

(v) To consider how fundamental research on turbulence might lead to improvements in turbulence-simulation methods and statistical models. The deficiencies of current models, as pointed out by industrial participants, tend to become apparent when they are applied to turbulent flows that are highly inhomogeneous and rapidly changing (over the length and time scales of the large eddies), which is to be expected since these 'non-conforming' situations do not correspond with the assumptions that underpin the models, e.g. Launder & Spalding (1972), Lumley (1978). Because industry is now familiar with the use of such models, it was requested that their rationale and limitations should be defined and explained using recent research, such as that on inhomogeneous turbulence. Since the models are often applied to 'non-conforming' flows, interest was expressed in interpreting the often puzzling results of the computations in these situations. Moreover, significant modifications are being proposed to existing modelling methods and these need to be evaluated and understood.

Questions (reviewed by Geurts 1999; see also Geurts & Leonard 1999) about the limitations of large eddy simulation methods are closely linked to those on the fundamental dynamics and statistics, since the methods involve computing the 'resolved' velocity field above a certain 'filter' scale l_{f} that is greater than that of the smallest 'Kolmogorov' eddies of the turbulence l_{k}. (Only if the Reynolds number of the turbulence is small enough, typically $Re < 10^3$, is it possible to avoid this approximation and compute the turbulence directly, e.g. Moin & Mahesh 1999.) Discussions were mainly focused on constant-density flows, though the importance of turbulence in two-phase flows (Hewitt 1999; Reeks 1999), buoyancy-dominated flows (Banerjee 1999; Launder 1999), and compressible flows (Bonnet 1999; Gatski 1999) was reviewed. There are many detailed questions about this filtering approximation; for example what happens when very small-scale, highly anisotropic and often non-Gaussian motions