AN INTRODUCTION TO STATISTICAL SIGNAL PROCESSING

This book describes the essential tools and techniques of statistical signal processing. At every stage theoretical ideas are linked to specific applications in communications and signal processing using a range of carefully chosen examples. The book begins with a development of basic probability, random objects, expectation, and second-order moment theory followed by a wide variety of examples of the most popular random process models and their basic uses and properties. Specific applications to the analysis of random signals and systems for communicating, estimating, detecting, modulating, and other processing of signals are interspersed throughout the book. Hundreds of homework problems are included and the book is ideal for graduate students of electrical engineering and applied mathematics. It is also a useful reference for researchers in signal processing and communications.

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Preface

The origins of this book lie in our earlier book *Random Processes: A Mathematical Approach for Engineers* (Prentice Hall, 1986). This book began as a second edition to the earlier book and the basic goal remains unchanged – to introduce the fundamental ideas and mechanics of random processes to engineers in a way that accurately reflects the underlying mathematics, but does not require an extensive mathematical background and does not belabor detailed general proofs when simple cases suffice to get the basic ideas across. In the years since the original book was published, however, it has evolved into something bearing little resemblance to its ancestor. Numerous improvements in the presentation of the material have been suggested by colleagues, students, teaching assistants, and reviewers, and by our own teaching experience. The emphasis of the book shifted increasingly towards examples and a viewpoint that better reflected the title of the courses we taught using the book for many years at Stanford University and at the University of Maryland: *An Introduction to Statistical Signal Processing.* Much of the basic content of this course and of the fundamentals of random processes can be viewed as the analysis of statistical signal processing systems: typically one is given a probabilistic description for one random object, which can be considered as an *input signal.* An operation is applied to the input signal (*signal processing*) to produce a new random object, the *output signal.* Fundamental issues include the nature of the basic probabilistic description, and the derivation of the probabilistic description of the output signal given that of the input signal and the particular operation performed. A perusal of the literature in statistical signal processing, communications, control, image and video processing, speech and audio processing, medical signal processing, geophysical signal processing, and classical statistical areas of time series analysis, classification and regression, and pattern recognition shows a wide variety of probabilistic models for input processes and for operations on those processes, where the operations might be deterministic or random, natural or artificial, linear or nonlinear, digital or analog, or beneficial or harmful. An introductory course focuses on the fundamentals underlying the analysis of such systems: the theories of probability, random processes, systems, and signal processing.

When the original book went out of print, the time seemed ripe to convert the manuscript from the prehistoric troff format to the widely used LaTeX format and to undertake a serious revision of the book in the process. As the revision became more extensive, the title changed to match the course name and content. We reprint the original preface to provide some of the original motivation for the book, and then close this preface with a description of the goals sought during the many subsequent revisions.

Preface to Random Processes: An Introduction for Engineers

> Nothing in nature is random ... A thing appears random only through the incompleteness of our knowledge.
>
> *Spinoza, Ethics I*

> I do not believe that God rolls dice.
>
> *attributed to Einstein*

Laplace argued to the effect that given complete knowledge of the physics of an experiment, the outcome must always be predictable. This metaphysical argument must be tempered with several facts. The relevant parameters may not be measurable with sufficient precision due to mechanical or theoretical limits. For example, the uncertainty principle prevents the simultaneous accurate knowledge of both position and momentum. The deterministic functions may be too complex to compute in finite time. The computer itself may make errors due to power failures, lightning, or the general perfidy of inanimate objects. The experiment could take place in a remote location with the parameters unknown to the observer; for example, in a communication link, the transmitted message is unknown a priori, for if it were not, there would be no need for communication. The results of the experiment could be reported by an unreliable witness – either incompetent or dishonest. For these and other reasons, it is useful to have a theory for the analysis and synthesis of processes that behave in a random or unpredictable manner. The goal is to construct mathematical models that lead to reasonably accurate prediction of the long-term average behavior of random processes. The theory should produce good estimates of the average behavior of real processes and thereby correct theoretical derivations with measurable results.

In this book we attempt a development of the basic theory and applications of random processes that uses the language and viewpoint of rigorous mathematical treatments of the subject but which requires only a typical bachelor's degree level of electrical engineering education including elementary discrete and continuous time linear systems theory, elementary probability, and transform theory and applications. Detailed proofs are presented only when within the scope of this background. These simple proofs, however, often provide the groundwork for "handwaving" justifications of more general and complicated results that are semi-rigorous in that they can be made rigorous by the appropriate delta-epsilontics of real analysis or measure theory. A primary goal of this approach is thus to use intuitive arguments that accurately reflect the underlying mathematics and which will hold up under scrutiny if the student continues to more advanced courses. Another goal is to enable the student who might not continue to more advanced courses to be able to read and generally follow the modern literature on applications of random processes to information and communication theory, estimation and detection, control, signal processing, and stochastic systems theory.

Revisions

Through the years the original book has continually expanded to roughly double its original size to include more topics, examples, and problems. The material has been significantly reorganized in its grouping and presentation. Prerequisites and preliminaries have been moved to the appendices. Major additional material has been added on jointly Gaussian vectors, minimum mean squared error estimation, linear and affine least squared error estimation, detection and classification, filtering, and, most recently, mean square calculus and its applications to the analysis of continuous time processes. The index has been steadily expanded to ease navigation through the book. Numerous errors reported by reader email have been fixed and suggestions for clarifications and improvements incorporated.

This book is a work in progress. Revised versions will be made available through the World Wide Web page http://ee.stanford.edu/~gray/sp.html. The material is copyrighted by Cambridge University Press, but is freely available as a pdf file to any individuals who wish to use it provided only that the contents of the entire text remain intact and together. Comments, corrections, and suggestions should be sent to rmgray@stanford.edu. Every effort will be made to fix typos and take suggestions into account on at least an annual basis.

Acknowledgements

We repeat our acknowledgements of the original book: to Stanford University and the University of Maryland for the environments in which the book was written, to the John Simon Guggenheim Memorial Foundation for its support of the first author during the writing in 1981–2 of the original book, to the Stanford University Information Systems Laboratory Industrial Affiliates Program which supported the computer facilities used to compose this book, and to the generations of students who suffered through the ever changing versions and provided a stream of comments and corrections. Thanks are also due to Richard Blahut and anonymous referees for their careful reading and commenting on the original book. Thanks are due to the many readers who have provided corrections and helpful suggestions through the Internet since the revisions began being posted. Particular thanks are due to Yariv Ephraim for his continuing thorough and helpful editorial commentary. Thanks also to Sridhar Ramanujam, Raymond E. Rogers, Isabel Milho, Zohreh Azimifar, Dan Sebald, Muzaffer Kal, Greg Coxson, Mihir Pise, Mike Weber, Munkyo Seo, James Jacob Yu, and several anonymous reviewers for Cambridge University Press. Thanks also to Philip Meyler, Lindsay Nightingale, and Joseph Bottrill of Cambridge University Press for their help in the production of the final version of the book. Lastly, the first author would like to acknowledge his debt to his professors who taught him probability theory and random processes, especially Al Drake and Wilbur B. Davenport Jr. at MIT and Tom Pitcher at USC.

Glossary

$\{\ \}$	a collection of points satisfying some property, e.g. $\{r : r \leq a\}$ is the collection of all real numbers less than or equal to a value a
$[\ ]$	an interval of real points including the end points, e.g. for $a \leq b$ $[a, b] = \{r : a \leq r \leq b\}$. Called a *closed interval*
$(\ )$	an interval of real points excluding the end points, e.g. for $a \leq b$ $(a, b) = \{r : a < r < b\}$. Called an *open interval*. Note this is empty if $a = b$
$(\ ]$, $[\ )$	denote intervals of real points including one endpoint and excluding the other, e.g. for $a \leq b$ $(a, b] = \{r : a < r \leq b\}$, $[a, b) = \{r : a \leq r < b\}$
$\emptyset$	the empty set, the set that contains no points.
$\forall$	for all
Ω	the sample space or universal set, the set that contains all of the points
$\#(F)$	the number of elements in a set F
$\stackrel{\Delta}{=}$	equal by definition
exp	the exponential function, $\exp(x) \stackrel{\Delta}{=} e^x$, used for clarity when x is complicated
$\mathcal{F}$	sigma-field or event space
$\mathcal{B}(\Omega)$	Borel field of Ω, that is, the sigma-field of subsets of the real line generated by the intervals or the Cartesian product of a collection of such sigma-fields
iff	if and only if
l.i.m.	limit in the mean
$o(u)$	function of u that goes to zero as $u \to 0$ faster than u

P	probability measure
P_X	distribution of a random variable or vector X
p_X	probability mass function (pmf) of a random variable X
f_X	probability density function (pdf) of a random variable X
F_X	cumulative distribution function (cdf) of a random variable X
$E(X)$	expectation of a random variable X
$M_X(ju)$	characteristic function of a random variable X
$\oplus$	addition modulo 2
$1_F(x)$	indicator function of a set F: $1_F(x) = 1$ if $x \in F$ and 0 otherwise
Φ	Φ-function (Eq. (2.78))
Q	complementary Phi function (Eq. (2.79))
$\mathcal{Z}_k$	$\triangleq \{0, 1, 2, \ldots, k-1\}$
$\mathcal{Z}_+$	$\triangleq \{0, 1, 2, \ldots\}$, the collection of nonnegative integers
$\mathcal{Z}$	$\triangleq \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, the collection of all integers