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0521837758 - The Cauchy-Schwarz Master Class: An Introduction to the Art of  
Mathematical Inequalities

J. Michael Steele

Index

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## Index

- 1-trick, 110, 144, 146, 215, 219, 227, 231  
 defined, 226  
 first used, 12  
 refinement, 205, 288
- Abel inequality, 208, 221  
 Abel, Niels Henrik, 208  
 Aczél, J., 287  
 additive bound, 4, 9, 66, 106  
 and Hölder, 137, 264  
 Åkerberg's refinement, 34  
 al-Khazin, Abu Ja'far, 286  
 Alexanderson, G., 286  
 AM-GM inequality, 20, 21  
 and Kantorovich, 247  
 Åkerberg's refinement, 34  
 algorithmic proof, 245  
 and investments, 100  
 and the exponential, 91  
 betweenness induction, 83  
 general weights, 23  
 geometric interpretation, 33  
 integral analog, 113  
 Pólya's proof, 23  
 rational weights, 22  
 smoothing proof, 244  
 stability, 35  
 via cyclic shifts, 84  
 via integral analog, 114  
 via rearrangement, 84  
 via Schur concavity, 194  
*American Mathematical Monthly*, 96,  
 192, 253  
 Andreescu, T., 251  
 anti-symmetric forms, 237  
 arithmetic mean-geometric mean  
 inequality, *see* AM-GM inequality  
*Arithmetica* of Diophantus, 40  
 arrangement of spheres, 52  
 Artin, Emil, 47  
 backtracking, 137  
 backwards induction, 236  
 baseball inequality, 82  
 Belentepe, C., vi  
 Bennett, C., 288, 289  
 Bernoulli inequality, 31  
 Bessel inequality, 71, 225  
 betweenness, exploitation of, 74  
 Birkhoff's Theorem, 207  
 birthday problem, 206  
 Bledsoe, W.W., 283  
 Bombieri, E., 284  
 box, thinking outside, 52  
 Brahmagupta, 286  
 identity, 47  
 Brunn–Minkowski inequality, 245, 287  
 Bunyakovsky, Victor Yacovlevich, 10,  
 190, 285  
 and AM-GM inequality, 115  
 Chebyshev contact, 76  
 vis-a-vis Schwarz, 11  
 Buzano, M.L., 286
- Cai, T., vi, 246  
 cancellation, origins of, 210  
 Carleman inequality, 118  
 Carleson proof, 173  
 Pólya's proof, 27  
 refined via Knopp, 128  
 Carleson inequality, 173  
 Carlson inequality, 165  
 Cauchy inequality, 1  
 and quadratic forms, 14  
 beating, 15  
 by induction, 2  
 case of equality, 37  
 in an array, 16  
 interpolation, 48  
 Pólya–Szegő converse, 83  
 three term, 13

- vector-scalar melange, 69
  - via inner product identity, 68
  - via monotone interpolation, 85
- Cauchy's induction argument
  - AM-GM inequality, 20
  - Cauchy inequality, 31
  - Jensen inequality, 101
- Cauchy, Augustin-Louis, 10
- Cauchy–Binet identity, 49
- Cauchy–Schwarz inequality, 8
  - as accidental corollary, 57
  - cross term defects, 83
  - geometric proof, 58
  - self-generalization, 16
  - via Gram–Schmidt, 71
- centered inequality, 115
- Chebyshev
  - order inequality, 76
  - tail inequality, 86
- Chebyshev, Pafnuty Lvovich, 76, 287
- Chong, K.M., 244
- Chung, F.R.K., 289
- Clevenson, M.L., 277
- closest point problem, 56
- completing the square, 57
- conjugate powers, 137
- consistency principles, 134
- convex functions, continuity of, 254
- convex minorant, 98
- convexity
  - defined, 87
  - differential criterion, 90
  - geometric characterizations, 87
  - strict, 89
  - versus J-convexity, 101
  - via transformation, 102
- Cours d'Analyse Algébrique*, 10
- Cramér–Rao inequality, 18
- Cronin–Kardon, C., vi
- crystallographic inequality, 13
- cyclic shifts, 84
- cyclic sum inequality, 104
- D'Angelo, J.P., 242
- Davis, P.J., 287
- Debeau, F., 285
- Dellacherie, C., vi, 254, 290
- determinant, 49
- Diaconis, P., vi
- Diophantus, 40, 286
- Diophantus identity
  - and Brahmagupta, 47, 237
  - and complex factorization, 237
  - and Pythagoras, 47, 237
- dissection of integrals, 106
- doubly stochastic, 196
- Dragomir, S.S., 241, 285
- Dudley, R.M., vi
- elementary symmetric function, 178
- Elliot, E.B., 290
- Enflo inequality, 225
- Engel, A., 94, 246, 251
- equality
  - in AM-GM inequality, 22
  - in Cauchy inequality, 5, 37
  - in Cauchy–Schwarz, 8
  - in Hölder, 137
  - in Jensen inequality, 89
- Erdős, P., 261
- Euclidean distance
  - in  $\mathbb{R}^d$ , 51
  - triangle inequality, 53
- exponential sums, 210
- extend and conquer, 222
- Feng, Z., 251
- Fermat, Pierre de, 40
- Fibonacci, 286
- Fisher information, 18
- Flor, P., 238
- four letter identity, 49
- Fuji, M., 287
- gamma function, 165
- Gauss, C.F., 288
- general means, 120
- generic improvement, 94
- geometric mean
  - as a limit, 120
  - as minimum, 133
  - superadditivity, 100, 133
- George, C., 231, 288
- Gram–Schmidt process, 70
- Gross–Erdmann, K.-G., 290
- Grüss inequality, 119
- Hadwiger–Finsler inequality, 102
- Hajela, D., 289
- Halmos, P., 278
- Hammer, D., 231
- Hardy's inequality, 166
  - and Hilbert, 176
  - discrete, 169
  - geometric refinement, 177
  - in  $L^p$ , 176
  - special instance, 177
- Hardy, G.H., 197, 290
- Harker–Kasper inequality, 14
- harmonic mean, 126
  - minimax characterization, 132
- harmonic series
  - divergence, 99, 255
- Heisenberg principle, 288

Cambridge University Press

0521837758 - The Cauchy-Schwarz Master Class: An Introduction to the Art of  
Mathematical Inequalities

J. Michael Steele

Index

[More information](#)

304

Hewitt, E., 243  
 Hilbert inequality, 155  
   homogeneous kernel, 164  
   integral version, 163  
   max version, 163  
   via Toeplitz method, 165  
 Hilbert's 17th problem, 46  
 Hilbert, David, 46, 55  
 HM-GM and HM-AM inequalities, 126  
 Hölder inequality, 135, 136  
   case of equality, 137  
   converse, 139  
   defect estimate, 94  
   historical form, 151  
   stability, 144, 145  
 Hölder, Otto Ludwig, 94, 135, 263, 290  
 homogeneity in  $\Sigma$ , 132  
 homogenization trick, 189  
*How to Solve It*, 30  
 humble bound, 284

inclusion radius, 148  
 inner product space  
   Cauchy-Schwarz inequality, 8  
   definition, 7  
 integral representations, 116  
 interpolation  
   in Cauchy inequality, 48  
 intuition  
   how much, 55  
   refining, 53  
 investment inequalities, 100  
 isometry, 60  
 isoperimetric property, 19  
   for the cube, 34  
 Israel, R., 253  
 iteration  
   and discovery, 149

J-convexity, 101  
 Janous, Walther, 177  
 Jensen inequality, 87, 263  
   and Schur convexity, 201  
   case of equality, 89  
   for integrals, 113  
   geometric applications, 93  
   Hölder's defect estimate, 94  
   implies Minkowski, 150  
   via Cauchy's induction, 101  
 Jensen, J.L.W.V., 101  
 Joag-Dev, K., 277

Kahane, J.-P., vi  
 Karayannakis, D., 286  
 Kedlaya, K., vi, 152, 264, 274  
 Knuth, D., 260  
 Komlós, J., 277

*Index*

Körner, T., vi  
 Kronecker's lemma, 177  
 Kubo, F., 287  
 Kufner, A., 290  
 Kuniyeda, M., 262

Lagrange identity, 39  
   continuous analogue, 48  
   simplest case, 42  
 Lagrange, Joseph Louis de, 40  
 Landau's notation, 120  
 leap-forward fall-back induction, 20, 31,  
   36, 101  
 Lee, H., vi, 247, 251  
 light cone defined, 62  
 light cone inequality, 63, 245  
 Littlewood, J.E., 197  
 looking back, 25, 26  
 Loomis-Whitney inequality, 16  
 Lorentz product, 62  
 Lovász, L., 278  
 Lozansky, E., 264  
 Lyusternik, L.A., 245

Maclaurin inequality, 285  
 Magiropoulos, M., 286  
 majorant principles, 284  
 majorization, 191  
 Maligranda, L., vi, 264, 288, 289  
 marriage lemma, 206  
 Marshall, A., 277  
 Matoušek, J., vi, 285  
 McConnell, T.R., 277  
 Meng, X., vi  
 Mengoli, P., 99, 248  
 method  
   of halves, 122  
   of parameterized parameters, 164  
 metric space, 54  
 Mignotte, M., 262  
 Milne inequality, 50  
 minimal surfaces, 10  
 Minkowski inequality, 141  
   Riesz proof, 141  
   via Jensen, 150  
 Minkowski's  
   conjecture, 44, 46  
   light cone, 62  
 Minkowski, Hermann, 44  
 Mitrinović, D.S., 277, 283  
 Möbius transformation, 242  
 moment sequences, 149  
 monotonicity  
   and integral estimates, 118  
 Montgomery, H.L., 284  
 Motzkin, T.S., 46, 286  
 Muirhead condition

Cambridge University Press

0521837758 - The Cauchy-Schwarz Master Class: An Introduction to the Art of  
Mathematical Inequalities

J. Michael Steele

Index

[More information](#)*Index*

305

- and majorization, 195
- Nakhash, A., 253
- names of inequalities, 11
- Naor, E., 259
- Needham, T., 242
- Nesbitt inequality, 84, 131, 246
- Neyman–Pearson lemma, 287
- Niculescu, C.P., 290
- Nievergelt, Y., 253
- Niven, I., 260
- nonnegative polynomials, 43
- norm
  - $p$ -norm, or  $\ell^p$ -norm, 140
  - defined, 55
- normalization method, 5, 25, 26, 66
- normed linear space, 55
- obvious
  - and not, 56
  - Hilbert story, 56
  - triangle inequality, 56
- Olkin, I., 277
- one-trick, *see* 1-trick
- Opic, B., 290
- optimality principles, 33
- order inequality, 76
- order relationship
  - systematic exploitation, 73
  - to quadratic, 76
- order-to-quadratic conversion, 78, 287
- orthogonality, definition, 58
- orthonormal, 217
- orthonormal sequence, 70
- other inequality of Chebyshev, 77
- parameterized parameters, 164
- Persson, L.E., 288, 289
- pillars, three great, 87
- Pitman, J., 190, 276
- Plummer, M.D., 278
- polarization identity, 49, 70
- Pólya's
  - dream, 23
  - questions, 30
- Pólya, George, 30, 197, 286
- Pólya–Szegő converse, 83
- positive definite, 228
- power mean continuity relations, 127
- power mean curve, 124
- power mean inequality, 123
  - simplest, 36
- Prékopa–Leindler inequality, 287
- principle of maximal effectiveness, 27
- principle of similar sides, 139
- probability model, 17
- product of linear forms, 59
- projection formula, 56
  - guessing, 58
- proportionality, 50
  - gages of, 39
- Proschan, F., 277
- Pták, V., 247
- Ptolemy inequality, 69
- Pythagorean theorem, 47, 51
- Qian, Z., vi
- quadratic form, 228
- quadratic inequality, 76
- qualitative inference principle, 3, 27
- quasilinear representation
  - geometric mean, 259
- Rademacher–Menchoff inequality, 217, 223
- ratio monotone, 189
- reflection, 60
- Reznick, B., vi
- Richberg, R., 272
- Riesz proof of Minkowski inequality, 141
- Riesz, F., 288
- Riesz, M., 288
- Rogers inequality, 152, 153
- Rogers, L.C., 135, 152, 290
- Rolle's theorem, 102, 251
- Rosset, S., 290
- Rousseau, C., 264
- rule of first finder, 12
- Schur convexity, 191
  - defined, 192
- Schur criterion, 193
- Schur inequality, 83
- Schur's lemma, 15
- Schur, Issai, 192
- Schwarz inequality, 10, 11
  - centered, 115
  - pointwise proof, 115
- Schwarz's argument, 11, 63
  - failure, 136
  - in inner product space, 15
  - in light cone, 63
- Schwarz, Hermann Amandus, 10
- Selberg inequality, 225
- self-generalization, 21
  - Hölder's inequality, 151
  - Cauchy inequality, 16
  - Cauchy–Schwarz inequality, 66
- Seymour, P.D., 289
- Shaman, P., vi
- Sharpley, R., 288, 289
- Shen, A., 231
- Shepp, L., vi
- Shparlinski, I.E., vi, 281

Cambridge University Press

0521837758 - The Cauchy-Schwarz Master Class: An Introduction to the Art of  
Mathematical Inequalities

J. Michael Steele

Index

[More information](#)

306

*Index*

Siegel's method of halves, 122

Siegel, Carl Ludwig, 122

Sigillito, V.G., 244

Simonovits, M., 277

Skillen, S., vi

slip-in trick, 117

sphere arrangement, 52

splitting trick, 106, 123, 147, 154, 227,  
263, 267

defined, 226

first used, 12

grand champion, 266

stability

in Hölder inequality, 145

of AM-GM inequality, 35

steepest ascent, 67

Ștefănescu, D., 262

stress testing, 268

Stromberg, K., 243

sum of squares, 42–44, 46

superadditivity

geometric mean, 34, 100, 133

symmetry and reflection, 62

Szegő, Gabor, 234

Szemerédi Regularity Lemma, 205

Tang, H., vi

telescoping, 29

thinking outside the box, 52

Three Chord Lemma, 104

tilde transformation, 193

Tiskin, A., 231

Toeplitz method, 165

Treibergs, A., 242

triangle inequality, 54

unbiased estimator, 17

van Dam, E.R., 230

van der Corput inequality, 214

van der Corput, J.G., 214

variance, 18, 116

Vaughn, H.E., 278

Viète identity, 133

Vince, A., 81

Vinogradov, I.M., 281

Vitale, R., vi

von Neumann, John, 51, 286

Wagoner, S.S., 238

Walker, A.W., 192

Ward, N., vi

Watkins, W., 277

Weierstrass inequality, 190

Weitzenböck inequality, 93, 102

Weyl, H., 206, 288

Wiles, Andrew, 40

Wilf, H., vi, 235

Young inequality, 136

Zuckerman, H.S., 260

Zukav, G., 286