

Contents

<i>Preface</i>	<i>page</i> ix
PART I: LINEAR REPRESENTATIONS 1	
1 Notation and generalities	3
2 Symmetric groups I	7
2.1 Gelfand–Zetlin bases	7
2.2 Description of weights	12
2.3 Formulas of Young and Murnaghan–Nakayama	17
3 Degenerate affine Hecke algebra	24
3.1 The algebras	25
3.2 Basis Theorem	26
3.3 The center of \mathcal{H}_n	27
3.4 Parabolic subalgebras	28
3.5 Mackey Theorem	29
3.6 Some (anti) automorphisms	31
3.7 Duality	31
3.8 Intertwining elements	34
4 First results on \mathcal{H}_n-modules	35
4.1 Formal characters	36
4.2 Central characters	37
4.3 Kato’s Theorem	38
4.4 Covering modules	40
5 Crystal operators	43
5.1 Multiplicity-free socles	44
5.2 Operators \tilde{e}_a and \tilde{f}_a	47

5.3	Independence of irreducible characters	49
5.4	Labels for irreducibles	51
5.5	Alternative descriptions of ϵ_a	51
6	Character calculations	54
6.1	Some irreducible induced modules	54
6.2	Calculations for small rank	57
6.3	Higher crystal operators	60
7	Integral representations and cyclotomic Hecke algebras	64
7.1	Integral representations	65
7.2	Some Lie theoretic notation	66
7.3	Degenerate cyclotomic Hecke algebras	68
7.4	The $*$ -operation	69
7.5	Basis Theorem for cyclotomic Hecke algebras	70
7.6	Cyclotomic Mackey Theorem	73
7.7	Duality for cyclotomic algebras	74
7.8	Presentation for degenerate cyclotomic Hecke algebras	80
8	Functors e_i^λ and f_i^λ	82
8.1	New notation for blocks	83
8.2	Definitions	83
8.3	Divided powers	87
8.4	Functions φ_i^λ	90
8.5	Alternative descriptions of φ_i^λ	92
8.6	More on endomorphism algebras	99
9	Construction of $U_{\mathbb{Z}}^+$ and irreducible modules	103
9.1	Grothendieck groups	104
9.2	Hopf algebra structure	106
9.3	Contravariant form	109
9.4	Chevalley relations	112
9.5	Identification of $K(\infty)^*$, $K(\lambda)^*$, and $K(\lambda)$	115
9.6	Blocks	117
10	Identification of the crystal	120
10.1	Final properties of $B(\infty)$	120
10.2	Crystals	123
10.3	Identification of $B(\infty)$ and $B(\lambda)$	126
11	Symmetric groups II	131
11.1	Description of the crystal graph	131
11.2	Main results on S_n	136

Contents

vii

PART II: PROJECTIVE REPRESENTATIONS	149
12 Generalities on superalgebra	151
12.1 Superalgebras and supermodules	151
12.2 Schur's Lemma and Wedderburn's Theorem	157
13 Sergeev superalgebras	165
13.1 Twisted group algebras	166
13.2 Sergeev superalgebras	168
14 Affine Sergeev superalgebras	174
14.1 The superalgebras	174
14.2 Basis Theorem for \mathcal{X}_n	175
14.3 The center of \mathcal{X}_n	176
14.4 Parabolic subalgebras of \mathcal{X}_n	177
14.5 Mackey Theorem for \mathcal{X}_n	177
14.6 Some (anti) automorphisms of \mathcal{X}_n	178
14.7 Duality for \mathcal{X}_n -supermodules	179
14.8 Intertwining elements for \mathcal{X}_n	179
15 Integral representations and cyclotomic Sergeev algebras	181
15.1 Integral representations of \mathcal{X}_n	181
15.2 Some Lie theoretic notation	183
15.3 Cyclotomic Sergeev superalgebras	184
15.4 Basis Theorem for cyclotomic Sergeev superalgebras	185
15.5 Cyclotomic Mackey Theorem	187
15.6 Duality for cyclotomic superalgebras	188
16 First results on \mathcal{X}_n-modules	191
16.1 Formal characters of \mathcal{X}_n -modules	191
16.2 Central characters and blocks	193
16.3 Kato's Theorem for \mathcal{X}_n	194
16.4 Covering modules for \mathcal{X}_n	197
17 Crystal operators for \mathcal{X}_n	200
17.1 Multiplicity-free socles	200
17.2 Operators \tilde{e}_i and \tilde{f}_i	203
17.3 Independence of irreducible characters	204
17.4 Labels for irreducibles	205

18 Character calculations for \mathcal{X}_n	206
18.1 Some irreducible induced supermodules	206
18.2 Calculations for small rank	208
18.3 Higher crystal operators	216
19 Operators e_i^λ and f_i^λ	219
19.1 i -induction and i -restriction	219
19.2 Operators e_i^λ and f_i^λ	221
19.3 Divided powers	225
19.4 Alternative descriptions of ε_i	228
19.5 The $*$ -operation	229
19.6 Functions φ_i^λ	229
19.7 Alternative descriptions of φ_i^λ	230
20 Construction of $U_{\mathbb{Z}}^+$ and irreducible modules	238
20.1 Grothendieck groups revisited	238
20.2 Hopf algebra structure	239
20.3 Shapovalov form	241
20.4 Chevalley relations	244
20.5 Identification of $K(\infty)^*$, $K(\lambda)^*$, and $K(\lambda)$	246
20.6 Blocks of cyclotomic Sergeev superalgebras	247
21 Identification of the crystal	248
22 Double covers	250
22.1 Description of the crystal graph	250
22.2 Representations of Sergeev superalgebras	255
22.3 Spin representations of S_n	259
<i>References</i>	270
<i>Index</i>	275