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0521836875 - An Introduction to Noncommutative Noetherian Rings, Second Edition

K. R. Goodearl and R. B. Warfield

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