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Ming Liao
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LÉVY PROCESSES IN LIE GROUPS

The theory of Lévy processes in Lie groups is not merely an extension of the theory of Lévy processes in Euclidean spaces. Because of the unique structures possessed by noncommutative Lie groups, these processes exhibit certain interesting limiting properties that are not present for their counterparts in Euclidean spaces. These properties reveal a deep connection between the behavior of the stochastic processes and the underlying algebraic and geometric structures of the Lie groups themselves.

The purpose of this work is to provide an introduction to Lévy processes in general Lie groups, the limiting properties of Lévy processes in semi-simple Lie groups of noncompact type, and the dynamical behavior of such processes as stochastic flows on certain homogeneous spaces. The reader is assumed to be familiar with Lie groups and stochastic analysis, but no prior knowledge of semi-simple Lie groups is required.

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Preface

The present volume provides an introduction to Lévy processes in general Lie groups, and hopefully an accessible account on the limiting and dynamical properties of such processes in semi-simple Lie groups of non-compact type. Lévy processes in Euclidean spaces, including the famous Brownian motion, have always played a central role in probability theory. In recent times, there has been intense research activity in exploring the probabilistic connections of various algebraic and geometric structures, therefore, the study of stochastic processes in Lie groups has become increasingly important. This book is aimed at serving two purposes. First, it may provide a foundation to the theory of Lévy processes in Lie groups, as this is perhaps the first book written on the subject, and second it will present some important results in this area, revealing an interesting connection between probability and Lie groups.

Please note: when referring to a result in a referenced text, the chapter and enunciation numbering system in that text has been followed.

David Applebaum, Olav Kallenberg and Wang Longmin read portions of the manuscript and provided useful comments. Part of the book was written during the author's visit to Nankai University, Tianjin, China in the fall of 2002. I wish to take this opportunity to thank my hosts, Wu Rong and Zhou Xingwei, for their hospitality. It would be hard to imagine this work ever being completed without my wife's support and understanding.

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ad: $\text{ad}(X)Y = [X, Y]$, 40

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c_g: conjugation, 9

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$\mathfrak{g}_\alpha$: the root space of the root α, 106

g_t^e: the Lévy process g_t starting at the identity element e of a Lie group G, 7

H^+: the rate vector of a Lévy process, 186, 197

H_ρ: the element of $\mathfrak{a}$ representing the half sum of positive roots, 134

I_d: the $d \times d$ identity matrix, 27

$\text{Irr}(G, \mathbb{C})_+$: the set of equivalent classes of nontrivial irreducible representations, 82

K: fixed point set of Θ (with $\mathfrak{k}$ as Lie algebra), 104

$\mathfrak{k}$: eigenspace of θ corresponding to eigenvalue $+1$ (Lie algebra of K), 104

L_g: left translation, 9

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