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Elements of the Representation Theory of Associative Algebras

Volume 2 Tubes and Concealed Algebras of Euclidean type

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India

103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9780521836104

First published 2007

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-83610-4 Hardback
ISBN 978-0-521-54420-7 Paperback

To our Wives

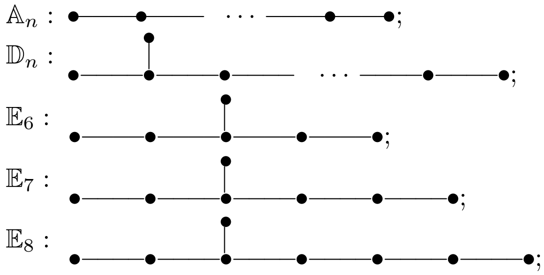
Sabina and Mirosława

Contents

Introduction

The first volume serves as a general introduction to some of the techniques most commonly used in representation theory. The quiver technique, the Auslander–Reiten theory and the tilting theory were presented with some application to finite dimensional algebras over a fixed algebraically closed field.

In particular, a complete classification of those hereditary algebras that are representation-finite (that is, admit only finitely many isomorphism classes of indecomposable modules) is given. The result, known as Gabriel's theorem, asserts that a basic connected hereditary algebra A is representation-finite if and only if the quiver Q_A of A is a Dynkin quiver, that is, the underlying non-oriented graph $\overline{Q}_A$ of Q_A is one of the Dynkin diagrams



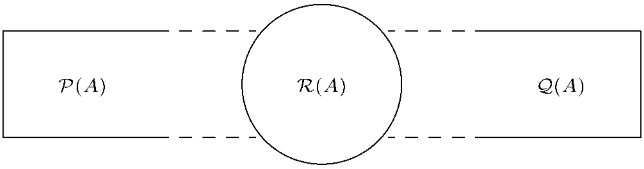
(n vertices, $n \geq 1$);

(n vertices, $n \geq 4$);

We also study in Volume 1 the class of hereditary algebras that are representation-infinite. It is shown in Chapter VIII that if B is a representation-infinite hereditary algebra, or B is a tilted algebra of the form

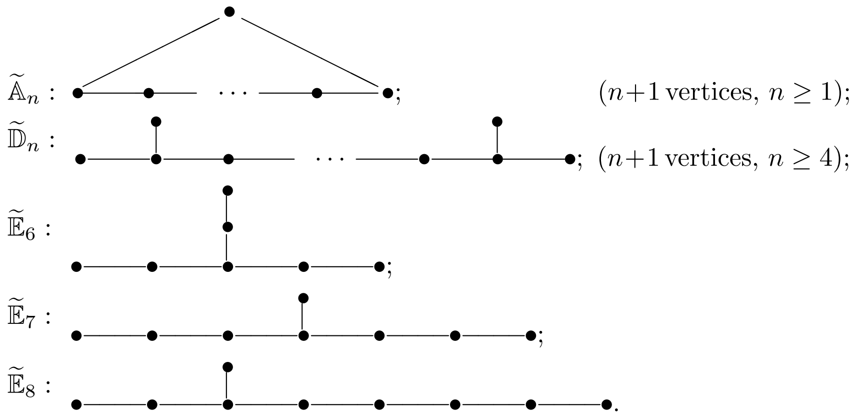
$$B = \operatorname{End} T_{KQ},$$

where KQ is a representation-infinite hereditary algebra and T_{KQ} is a postprojective tilting KQ-module, then B is representation-infinite and the Auslander–Reiten quiver $\Gamma(\operatorname{mod} B)$ of B has the shape



where $\mathrm{mod}\,B$ is the category of finite dimensional right B-modules, $\mathcal{P}(B)$ is the unique postprojective component of $\Gamma(\mathrm{mod}\,B)$ containing all the indecomposable projective B-modules, $\mathcal{Q}(B)$ is the unique preinjective component of $\Gamma(\mathrm{mod}\,B)$ containing all the indecomposable injective B-modules, and $\mathcal{R}(B)$ is the (non-empty) regular part consisting of the remaining components of $\Gamma(\mathrm{mod}\,B)$.

A prominent rôle in the representation theory is played by the class of hereditary algebras that are representation-infinite and minimal with respect to this property. They are just the hereditary algebras of Euclidean type, that is, the path algebras KQ, where Q is a connected acyclic quiver whose underlying non-oriented graph $\overline{Q}$ is one of the following Euclidean diagrams



It is shown in Chapter VII that the underlying graph $\overline{Q}$ of a finite connected quiver $Q = (Q_0, Q_1)$ is a Dynkin diagram, or a Euclidean diagram, if and only if the associated quadratic form $q_Q : \mathbb{Z}^{|Q_0|} \longrightarrow \mathbb{Z}$ is positive definite, or positive semidefinite and not positive definite, respectively.

The main aim of Volumes 2 and 3 is to study the representation-infinite tilted algebras $B = \mathrm{End}\,T_{KQ}$ of a Euclidean type Q and, in particular, to give a fairly complete description of their indecomposable modules, their module categories $\mathrm{mod}\,B$, and the Auslander–Reiten quivers $\Gamma(\mathrm{mod}\,B)$.

For this purpose, we introduce in Chapter X a special type of components in the Auslander–Reiten quivers of algebras, namely stable tubes, and study their behaviour in module categories. In particular, we present a handy criterion on the existence of a standard self-hereditary stable tube, due to Ringel [215], and a characterisation of generalised standard stable tubes, due to Skowroński [246], [247], [254].

In Chapters XI and XII, we present a detailed description and properties of the regular part $\mathcal{R}(B)$ of the Auslander–Reiten quiver $\Gamma(\mathrm{mod}\, B)$ of any concealed algebra B of Euclidean type, that is, a tilted algebra

$$B = \mathrm{End}\, T_{KQ}$$

of a Euclidean type Q defined by a postprojective tilting KQ-module T_{KQ}.

In particular, it is shown that:

- the regular part $\mathcal{R}(B)$ of the Auslander–Reiten quiver $\Gamma(\mathrm{mod} B)$ is a disjoint union of the $\mathbb{P}_1(K)$-family
$$\boldsymbol{\mathcal{T}}^B = \{\mathcal{T}_\lambda^B\}_{\lambda \in \mathbb{P}_1(K)}$$
of pairwise orthogonal standard stable tubes $\mathcal{T}_\lambda^B$, where $\mathbb{P}_1(K)$ is the projective line over K,
- the family $\boldsymbol{\mathcal{T}}^B$ separates the postprojective component $\mathcal{P}(B)$ from the preinjective component $\mathcal{Q}(B)$,
- the module category $\mathrm{mod} B$ is controlled by the Euler quadratic form $q_B : K_0(B) \longrightarrow \mathbb{Z}$ of the algebra B.

A crucial rôle in the investigation is played by the canonical algebras of Euclidean type, introduced by Ringel [215]. As an application of the developed theory, we present in Chapter XIII a complete list of indecomposable regular KQ-modules over any path algebra KQ of a canonically oriented Euclidean quiver Q, and we show how a simple tilting process allows us to construct the indecomposable regular modules over any path algebra KQ of a Euclidean type Q.

In Chapter XIV, we give the Happel–Vossieck [112] characterisation of the minimal representation-infinite algebras B having a postprojective component in the Auslander–Reiten quiver $\Gamma(\mathrm{mod}\, B)$. As a consequence, we get a finite representation type criterion for algebras. We also present a complete classification, by means of quivers with relations, of all concealed algebras of Euclidean type, due independently by Bongartz [29] and Happel–Vossieck [112].

In Volume 3, we introduce some concepts and tools that allow us to give there a complete description of arbitrary representation-infinite tilted algebras B of Euclidean type and the module category $\mathrm{mod}\, B$, due to Ringel [215]. We also investigate the wild hereditary algebras $A = KQ$, where Q is an acyclic quiver such that the underlying graph is neither a Dynkin nor a Euclidean diagram. We describe the shape of the components of the regular part $\mathcal{R}(A)$ of $\Gamma(\mathrm{mod}\, A)$ and we establish a wild behaviour of the category $\mathrm{mod}\, A$, for any such an algebra A. Finally, we introduce in Volume 3 the concepts of tame representation type and of wild representation type for algebras, and we discuss the tame and the wild nature of module categories mod B. Also, we present (without proofs) selected results of the representation theory of finite dimensional algebras that are related to the material discussed in the book.

It was not possible to be encyclopedic in this work. Therefore many important topics from the theory have been left out. Among the most notable omissions are covering techniques, the use of derived categories and partially ordered sets. Some other aspects of the theory presented here are discussed in the books [10], [15], [16], [91], [121], [235], and especially [215].

We assume that the reader is familiar with Volume 1, but otherwise the exposition is reasonably self-contained, making it suitable either for courses and seminars or for self-study. The text includes many illustrative examples and a large number of exercises at the end of each of the Chapters X-XIV.

The book is addressed to graduate students, advanced undergraduates, and mathematicians and scientists working in representation theory, ring and module theory, commutative algebra, abelian group theory, and combinatorics. It should also, we hope, be of interest to mathematicians working in other fields.

Throughout this book we use freely the terminology and notation introduced in Volume 1. We denote by K a fixed algebraically closed field. The symbols $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$ mean the sets of natural numbers, integers, rational, real, and complex numbers. The cardinality of a set X is denoted by $|X|$. Given a finite dimensional K-algebra A, the A-module means a finite dimensional right A-module. We denote by $\operatorname{Mod} A$ the category of all right A-modules, by $\operatorname{mod} A$ the category of finite dimensional right A-modules, and by $\Gamma(\operatorname{mod} A)$ the Auslander–Reiten translation quiver of A. The ordinary quiver of an algebra A is denoted by Q_A. Given a matrix $C = [c_{ij}]$, we denote by C^t the transpose of C.

A finite quiver $Q = (Q_0, Q_1)$ is called a **Euclidean quiver** if the underlying graph $\overline{Q}$ of Q is any of the Euclidean diagrams $\widetilde{\mathbb{A}}_m$, with $m \geq 1$, $\widetilde{\mathbb{D}}_m$, with $m \geq 4$, $\widetilde{\mathbb{E}}_6$, $\widetilde{\mathbb{E}}_7$, and $\widetilde{\mathbb{E}}_8$. Analogously, Q is called a **Dynkin quiver** if the underlying graph $\overline{Q}$ of Q is any of the Dynkin diagrams $\mathbb{A}_m$, with $m \geq 1$, $\mathbb{D}_m$, with $m \geq 4$, $\mathbb{E}_6$, $\mathbb{E}_7$, and $\mathbb{E}_8$.

We take pleasure in thanking all our colleagues and students who helped us with their comments and suggestions. We wish particularly to express our appreciation to Ibrahim Assem, Sheila Brenner, Otto Kerner, and Kunio Yamagata for their helpful discussions and suggestions. Particular thanks are due to Dr. Jerzy Białkowski and Dr. Rafał Bocian for their help in preparing a print-ready copy of the manuscript.