

Cambridge University Press

978-0-521-83441-4 — Nonlinear Dynamics and Statistical Theories for Basic Geophysical Flows

Andrew Majda, Xiaoming Wang

Frontmatter

[More Information](#)

Non-linear dynamics and statistical theories
for basic geophysical flows

Non-linear dynamics and statistical theories for basic geophysical flows

ANDREW J. MAJDA

New York University

XIAOMING WANG

Florida State University



Cambridge University Press
978-0-521-83441-4 — Nonlinear Dynamics and Statistical Theories for Basic Geophysical Flows
Andrew Majda, Xiaoming Wang
Frontmatter
[More Information](#)

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India
103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9780521834414

© Cambridge University Press 2006

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2006

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-83441-4 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Contents

<i>Preface</i>	<i>page xi</i>
1 Barotropic geophysical flows and two-dimensional fluid flows: elementary introduction	1
1.1 Introduction	1
1.2 Some special exact solutions	8
1.3 Conserved quantities	33
1.4 Barotropic geophysical flows in a channel domain – an important physical model	44
1.5 Variational derivatives and an optimization principle for elementary geophysical solutions	50
1.6 More equations for geophysical flows	52
References	58
2 The response to large-scale forcing	59
2.1 Introduction	59
2.2 Non-linear stability with Kolmogorov forcing	62
2.3 Stability of flows with generalized Kolmogorov forcing	76
References	79
3 The selective decay principle for basic geophysical flows	80
3.1 Introduction	80
3.2 Selective decay states and their invariance	82
3.3 Mathematical formulation of the selective decay principle	84
3.4 Energy–enstrophy decay	86
3.5 Bounds on the Dirichlet quotient, $\Lambda(t)$	88
3.6 Rigorous theory for selective decay	90
3.7 Numerical experiments demonstrating facets of selective decay	95
References	102

vi	<i>Contents</i>	
A.1	Stronger controls on $\Lambda(t)$	103
A.2	The proof of the mathematical form of the selective decay principle in the presence of the beta-plane effect	107
4	Non-linear stability of steady geophysical flows	115
4.1	Introduction	115
4.2	Stability of simple steady states	116
4.3	Stability for more general steady states	124
4.4	Non-linear stability of zonal flows on the beta-plane	129
4.5	Variational characterization of the steady states	133
	References	137
5	Topographic mean flow interaction, non-linear instability, and chaotic dynamics	138
5.1	Introduction	138
5.2	Systems with layered topography	141
5.3	Integrable behavior	145
5.4	A limit regime with chaotic solutions	154
5.5	Numerical experiments	167
	References	178
	Appendix 1	180
	Appendix 2	181
6	Introduction to information theory and empirical statistical theory	183
6.1	Introduction	183
6.2	Information theory and Shannon's entropy	184
6.3	Most probable states with prior distribution	190
6.4	Entropy for continuous measures on the line	194
6.5	Maximum entropy principle for continuous fields	201
6.6	An application of the maximum entropy principle to geophysical flows with topography	204
6.7	Application of the maximum entropy principle to geophysical flows with topography and mean flow	211
	References	218
7	Equilibrium statistical mechanics for systems of ordinary differential equations	219
7.1	Introduction	219
7.2	Introduction to statistical mechanics for ODEs	221
7.3	Statistical mechanics for the truncated Burgers–Hopf equations	229
7.4	The Lorenz 96 model	239
	References	255

8	Statistical mechanics for the truncated quasi-geostrophic equations	256
8.1	Introduction	256
8.2	The finite-dimensional truncated quasi-geostrophic equations	258
8.3	The statistical predictions for the truncated systems	262
8.4	Numerical evidence supporting the statistical prediction	264
8.5	The pseudo-energy and equilibrium statistical mechanics for fluctuations about the mean	267
8.6	The continuum limit	270
8.7	The role of statistically relevant and irrelevant conserved quantities	285
	References	285
	Appendix 1	286
9	Empirical statistical theories for most probable states	289
9.1	Introduction	289
9.2	Empirical statistical theories with a few constraints	291
9.3	The mean field statistical theory for point vortices	299
9.4	Empirical statistical theories with infinitely many constraints	309
9.5	Non-linear stability for the most probable mean fields	313
	References	316
10	Assessing the potential applicability of equilibrium statistical theories for geophysical flows: an overview	317
10.1	Introduction	317
10.2	Basic issues regarding equilibrium statistical theories for geophysical flows	318
10.3	The central role of equilibrium statistical theories with a judicious prior distribution and a few external constraints	320
10.4	The role of forcing and dissipation	322
10.5	Is there a complete statistical mechanics theory for ESTMC and ESTP?	324
	References	326
11	Predictions and comparison of equilibrium statistical theories	328
11.1	Introduction	328
11.2	Predictions of the statistical theory with a judicious prior and a few external constraints for beta-plane channel flow	330
11.3	Statistical sharpness of statistical theories with few constraints	346
11.4	The limit of many-constraint theory (ESTMC) with small amplitude potential vorticity	355
	References	360

12	Equilibrium statistical theories and dynamical modeling of flows with forcing and dissipation	361
12.1	Introduction	361
12.2	Meta-stability of equilibrium statistical structures with dissipation and small-scale forcing	362
12.3	Crude closure for two-dimensional flows	385
12.4	Remarks on the mathematical justifications of crude closure	405
	References	410
13	Predicting the jets and spots on Jupiter by equilibrium statistical mechanics	411
13.1	Introduction	411
13.2	The quasi-geostrophic model for interpreting observations and predictions for the weather layer of Jupiter	417
13.3	The ESTP with physically motivated prior distribution	419
13.4	Equilibrium statistical predictions for the jets and spots on Jupiter	423
	References	426
14	The statistical relevance of additional conserved quantities for truncated geophysical flows	427
14.1	Introduction	427
14.2	A numerical laboratory for the role of higher-order invariants	430
14.3	Comparison with equilibrium statistical predictions with a judicious prior	438
14.4	Statistically relevant conserved quantities for the truncated Burgers–Hopf equation	440
	References	442
A.1	Spectral truncations of quasi-geostrophic flow with additional conserved quantities	442
15	A mathematical framework for quantifying predictability utilizing relative entropy	452
15.1	Ensemble prediction and relative entropy as a measure of predictability	452
15.2	Quantifying predictability for a Gaussian prior distribution	459
15.3	Non-Gaussian ensemble predictions in the Lorenz 96 model	466
15.4	Information content beyond the climatology in ensemble predictions for the truncated Burgers–Hopf model	472

Contents

ix

15.5	Further developments in ensemble predictions and information theory	478
	References	480
16	Barotropic quasi-geostrophic equations on the sphere	482
16.1	Introduction	482
16.2	Exact solutions, conserved quantities, and non-linear stability	490
16.3	The response to large-scale forcing	510
16.4	Selective decay on the sphere	516
16.5	Energy enstrophy statistical theory on the unit sphere	524
16.6	Statistical theories with a few constraints and statistical theories with many constraints on the unit sphere	536
	References	542
	Appendix 1	542
	Appendix 2	546
	<i>Index</i>	550

Preface

This book is an introduction to the fascinating and important interplay between non-linear dynamics and statistical theories for geophysical flows. The book is designed for a multi-disciplinary audience ranging from beginning graduate students to senior researchers in applied mathematics as well as theoretically inclined graduate students and researchers in atmosphere/ocean science. The approach in this book emphasizes the serendipity between physical phenomena and modern applied mathematics, including rigorous mathematical analysis, qualitative models, and numerical simulations. The book includes more conventional topics for non-linear dynamics applied to geophysical flows, such as long time selective decay, the effect of large-scale forcing, non-linear stability and fluid flow on the sphere, as well as emerging contemporary research topics involving applications of chaotic dynamics, equilibrium statistical mechanics, and information theory. The various competing approaches for equilibrium statistical theories for geophysical flows are compared and contrasted systematically from the viewpoint of modern applied mathematics, including an application for predicting the Great Red Spot of Jupiter in a fashion consistent with the observational record. Novel applications of information theory are utilized to simplify, unify, and compare the equilibrium statistical theories and also to quantify aspects of predictability in non-linear dynamical systems with many degrees of freedom. No previous background in geophysical flows, probability theory, information theory, or equilibrium statistical mechanics is needed to read the text. These topics and related background concepts are all introduced and developed through elementary examples and discussion throughout the text as they arise. The book is also of wider interest to applied mathematicians and other scientists to illustrate how ideas from statistical physics can be applied in novel ways to inhomogeneous large-scale complex non-linear systems.

The material in the book is based on lectures of the first author given at the Courant Institute in 1995, 1997, 2001, and 2004. The first author thanks Professor

Pedro Embid as well as his former Ph.D. students Professor Pete Kramer and Seuyung Shim for their help with early versions of Chapters 1, 2, 3, 4, and 6 of the present book. Joint research work with Professors Richard Kleeman and Bruce Turkington as well as Majdas former Courant post docs, Professors Marcus Grote, Ilya Timofeyev, Rafail Abramov, and Mark DeBattista have been incorporated into the book; their explicit and implicit contributions are acknowledged warmly. The authors acknowledge generous support of the National Science Foundation and the Office of Naval Research during the development of this book, including partial salary support for Xiaoming Wangs visit to Courant in the spring semester of 2001.