Elementary Probability

2nd Edition

Now available in a fully revised and updated new edition, this well-established textbook provides a straightforward introduction to the theory of probability. The presentation is entertaining without any sacrifice of rigour; important notions are covered with the clarity that the subject demands.

Topics covered include conditional probability, independence, discrete and continuous random variables, basic combinatorics, generating functions and limit theorems, and an introduction to Markov chains. This edition includes an elementary approach to martingales and the theory of Brownian motion, which supply the cornerstones for many topics in modern financial mathematics such as option and derivative pricing. The text is accessible to undergraduate students, and provides numerous worked examples and exercises to help build the important skills necessary for problem solving.

'[T]he author succeeds in combining the utmost succinctness with clarity and genuine readability. . . . This textbook can be recommended unreservedly.'

Internationale Mathematische Nachrichten

'[T]his book is a superb resource of theory and application, which should be on every lecturer's shelves, and those of many students. You may never need to buy another book on probability.'

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'Excellent! A vast number of well-chosen worked examples and exercises guide the reader through the basic theory of probability at the elementary level . . . an excellent text which I am sure will give a lot of pleasure to students and teachers alike.'

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'[W]ould make a fine addition to an undergraduate library. A student with a solid background in calculus, linear algebra, and set theory will find many useful tools of elementary probability here.'

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'The book would make an excellent text for the properly prepared class, a solid instructor's reference for both probability applications and problems, as well as a fine work for purposes of self-study.'

J. Philip Smith, *School Science and Mathematics*

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by
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Preface to the Second Edition

> The calculus of probabilities, in an appropriate form, should interest equally the mathematician, the experimentalist, and the statesman. . . . It is under its influence that lotteries and other disgraceful traps cunningly laid for greed and ignorance have finally disappeared.
>
> *Francois Arago, Eulogy on Laplace, 1827*

> Lastly, one of the principal uses to which this Doctrine of Chances may be applied, is the discovering of some truths, which cannot fail of pleasing the mind, by their generality and simplicity; the admirable connexion of its consequences will increase the pleasure of the discovery; and the seeming paradoxes wherewith it abounds, will afford very great matter of surprize and entertainment to the inquisitive.
>
> *Abraham de Moivre, The Doctrine of Chances, 1756*

This book provides an introduction to elementary probability and some of its simple applications. In particular, a principal purpose of the book is to help the student to solve problems. Probability is now being taught to an ever wider audience, not all of whom can be assumed to have a high level of problem-solving skills and mathematical background. It is also characteristic of probability that, even at an elementary level, few problems are entirely routine. Successful problem solving requires flexibility and imagination on the part of the student. Commonly, these skills are developed by observation of examples and practice at exercises, both of which this text aims to supply.

With these targets in mind, in each chapter of the book, the theoretical exposition is accompanied by a large number of examples and is followed by worked examples incorporating a cluster of exercises. The examples and exercises have been chosen to illustrate the subject, to help the student solve the kind of problems typical of examinations, and for their entertainment value. (Besides its practical importance, probability is without doubt one of the most entertaining branches of mathematics.) Each chapter concludes with problems: solutions to many of these appear in an appendix, together with the solutions to most of the exercises.

The ordering and numbering of material in this second edition has for the most part been preserved from the first. However, numerous alterations and additions have been included to make the basic material more accessible and the book more useful for self-study. In particular, there is an entirely new introductory chapter that discusses our informal and intuitive ideas about probability, and explains how (and why) these should be incorporated into the theoretical framework of the rest of the book. Also, all later chapters now include a section entitled, "Review and checklist," to aid the reader in navigation around the subject, especially new ideas and notation.

Furthermore, a new section of the book provides a first introduction to the elementary properties of martingales, which have come to occupy a central position in modern probability. Another new section provides an elementary introduction to Brownian motion, diffusion, and the Wiener process, which has underpinned much classical financial mathematics, such as the Black–Scholes formula for pricing options. Optional stopping and its applications are introduced in the context of these important stochastic models, together with several associated new worked examples and exercises.

The basic structure of the book remains unchanged; there are three main parts, each comprising three chapters.

The first part introduces the basic ideas of probability, conditional probability, and independence. It is assumed that the reader has some knowledge of elementary set theory. (We adopt the now conventional formal definition of probability. This is not because of high principles, but merely because the alternative intuitive approach seems to lead more students into errors.) The second part introduces discrete random variables, probability mass functions, and expectation. It is assumed that the reader can do simple things with functions and series. The third part considers continuous random variables, and for this a knowledge of the simpler techniques of calculus is desirable.

In addition, there are chapters on combinatorial methods in probability, the use of probability (and other) generating functions, and the basic theory of Markov processes in discrete and continuous time. These sections can be omitted at a first reading, if so desired.

In general, the material is presented in a conventional order, which roughly corresponds to increasing levels of knowledge and dexterity on the part of the reader. Those who start with a sufficient level of basic skills have more freedom to choose the order in which they read the book. For example, you may want to read Chapters 4 and 7 together (and then Chapters 5 and 8 together), regarding discrete and continuous random variables as two varieties of the same species (which they are). Also, much of Chapter 9 could be read immediately after Chapter 5, if you prefer.

In particular, the book is structured so that the first two parts are suitable to accompany the probability component of a typical course in discrete mathematics; a knowledge of calculus is not assumed until the final part of the book. This layout entails some repetition of similar ideas in different contexts, and this should help to reinforce the reader's knowledge of the less elementary concepts and techniques.

The ends of examples, proofs, and definitions are indicated by the symbols ●, ■, and ▲, respectively.

Finally, you should note that the book contains a random number of errors. I entreat readers to inform me of all those they find.

D.S.
Oxford, January 2003