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978-0-521-83324-0 - From Classical to Quantum Mechanics: An Introduction to the Formalism, Foundations and Applications
Giampiero Esposito, Giuseppe Marmo and George Sudarshan
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