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János Kollár, Karen E. Smith and Alessio Corti
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RATIONAL AND NEARLY RATIONAL VARIETIES

The most basic algebraic varieties are the projective spaces, and rational varieties are their closest relatives. In many applications where algebraic varieties appear in mathematics and the sciences, we see rational ones emerging as the most interesting examples. The authors have given an elementary treatment of rationality questions using a mix of classical and modern methods. Arising from a summer school course taught by János Kollár, this book develops the modern theory of rational and nearly rational varieties at a level that will particularly suit graduate students. There are numerous examples and exercises, all of which are accompanied by fully worked out solutions, that will make this book ideal as the basis of a graduate course. It will act as a valuable reference for researchers whilst helping graduate students to reach the point where they can begin to tackle contemporary research problems.

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University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

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www.cambridge.org

Information on this title: www.cambridge.org/9781107471740

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First published 2004

This digitally printed paperback version 2014

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-83207-6 Hardback

ISBN 978-1-107-47174-0 Paperback

Digitally Printed in Korea at Kyobo Book Centre

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